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A COMPARISON OF METHODS USED FOR OBTAINING
ELECTRON CONTENT FROM SATELLITE OBSERVATIONS

by

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ABSTRACT

Measurements of the effects of the ionosphere on the polarization and the Doppler shift of radio transmissions from earth satellites permit the electron content of the ionosphere to be calculated. Thirty-five passages of Transit 2A have been analyzed in a variety of ways in order to permit the accuracy of the several methods to be estimated. The most accurate method is a hybrid analysis using both Faraday and Doppler data simultaneously. Methods based on the rate of polarization rotation, on the number of rotations between two times and on best-fitting polynomials to either the Faraday or the Doppler data are compared and their errors estimated.

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I. INTRODUCTION

The radio transmissions of orbiting earth satellites have been found very useful in the determination of ionospheric electron content. Most authors have based their calculations on the observed polarization rotation (the Faraday effect) or on the small change in Doppler shift imposed by the ionosphere. Refraction measurements are also possible in theory Al'pert (1958), Weekes (1958), Titheridge (1961) , but they have not found widespread application as yet.

A variety of methods have been used in the analysis of satellite recordings and different assumptions are necessary in each case. In this paper, 35 passages of Transit 2A will be analyzed, each in a number of different ways. Then, the results of each method are compared for internal consistency or with other methods in order to estimate their individual accuracies. A similar procedure has been used by Burgess (1963) who has compared several different analysis methods and has shown the results of his comparisons for one passage of Transit 4A. In Section II each of the analysis methods to be used in this paper are described. They consist of (1) two methods based on the observed rate of polarization rotation, (2) a "rotation angle" method, obtained by measuring the change in the angle of polarization rotation between two times (3) several methods using only differential Doppler data or only Faraday rotation data, which will obtain polynomial expressions for the electron content as a function of time with minimum mean-square error, (4) an independent method based on Doppler data alone, and (5) a hybrid Faraday-Doppler technique using both types of observational data. Finally, these techniques are applied to the 35 passages of Transit 2A in Section III and the results are considered.

The satellite records were obtained between July 23 and October 13, 1960 at Stanford University. The received 54-Mc signal was compared with a sub-harmonic of the received 324-Mc signal (using phase-locked receivers) and the small effect of the ionosphere was determined. Polarization rotation was measured by the amplitude fluctuation of the 54-Mc signal. These two types of observations and the satellite ephemeris provided by the Applied Physics Laboratory of the Johns Hopkins University are the basic input data necessary to calculate electron content.

II. ANALYSIS PROCEDURES

A. ROTATION RATE METHODS

The total angle of polarization rotation is usually expressed as

$$\Omega = (K/f^2) H \cos \theta \sec \chi \cdot I \quad (1)$$

where

Ω = angle of polarization rotation, radians

f = wave frequency, cps

H = magnetic field intensity, amp-turns/m

θ = angle between wave normal and magnetic field

χ = angle between the ray and the vertical

$I = \int_0^{h_s} N dh$, the electron content, that is, the electron density integrated along the ray path up to the satellite height

K = a constant, equal to 2.97×10^{-2} in mks units

The equation assumes quasi-longitudinal propagation, high frequencies and a single ray path for both modes but these approximations are quite satisfactory at the mentioned frequencies. It is convenient for us to abbreviate this equation as,

$$\Omega = \eta M I \quad (2)$$

where $\eta = K/f^2$ and $M = H \cos \theta \sec \chi$. The factor M may be computed at any point in space about the observer as has been done by Yeh and Gonzales (1960). In eq. 2 the value of M should be determined near the height of the centroid of the electron density profile. Therefore in all of the work to follow, M was evaluated at a level approximately 50 km above the height of maximum density which was determined from a true height analysis of an ionogram recorded near the time of the satellite passage.

Since the speed of a satellite is much higher than the drift velocities to be expected in the ionosphere, one may consider that N (and consequently I) is a function of position only, e. g.,

$N = N(\text{height, latitude, longitude})$

$I = I(\text{latitude, longitude})$

However, due to the motion of the satellite, the coordinates of the points along the ray path are functions of time. Thus it is possible to consider I an explicit function of time. In this paper, whenever reference is made to the time variation of I or M , it is to be understood in this context.

In order to obtain the rotation rate, Eq. 2 is differentiated with respect to time, giving

$$\dot{\Omega} = \eta [M \dot{I} + I \dot{M}] \quad (3)$$

in which dots imply a time derivative. The analysis of Bowhill (1958) shows that for a flat earth and a uniform magnetic dip, the term M is a constant. Furthermore, when horizontal gradients in the ionosphere are neglected ($\dot{I} = 0$), the rotation rate becomes proportional to the electron content,

$$I = \dot{\Omega} / \eta \dot{M} \quad (4)''$$

Many authors Garriott (1960), Hame and Stuart (1960), Yeh and Swenson (1961) have used this expression or its near equivalent because of the ease with which the satellite observations are related to electron content. In the calculations to be shown in Section III, M was determined from a table of M values supplied by Mr. L. J. Blumle, at the Goddard Space Flight Center and $\dot{\Omega}$ was determined by a five point differentiation formula using the times of the Faraday nulls. Both terms are evaluated at the "proximal point" of the passage, which will be defined more precisely in Section II C.

If the electron content is constrained to vary linearly with time Eq. 3 may be differentiated to give

$$\ddot{\Omega} = [2\dot{M} \dot{I} + I \ddot{M}] \quad (5)$$

Then Eqs. 3 and 5 may be solved simultaneously (") for values of I and \dot{I} at the proximal point. This method has the considerable advantage of eliminating the restriction to a horizontally stratified ionosphere but the disadvantage of requiring the second derivative of Ω to be evaluated.

B. ROTATION ANGLE METHOD

Perhaps the most widely used method has been based upon the change in the rotation angle between two times. When horizontal stratification of the ionosphere is once again assumed, the change in the rotation angle between times t_1 and t_2 is

$$\Delta \Omega = \Omega_1 - \Omega_2 = \eta (M_1 - M_2) I \quad (6)''$$

"Footnote: The equations identified by a quotation mark are those for which calculations of electron content have been made and the calculated values are discussed in Section III.

from which I may be calculated. The change $\Delta\Omega$ is π times the number of Faraday fades between t_1 and t_2 . Numerous authors have used equations closely equivalent to (6), although some improvement is obtained when propagation approaches the transverse direction [Garriott (1960), Blackband (1960)], when allowance is made for refraction and the higher order terms in the expressions for the index of refraction [Yeh (1960)], and when path splitting is minimized by computer ray tracing [Lawrence, et al (1963)]. The principal disadvantage to this method is the neglect of horizontal gradients.

C. BEST FITTING POLYNOMIALS

If the time variation of electron content is represented by a power series, $I(t) = a_0 + a_1t + a_2t^2 + \dots$, the coefficients a_0, a_1, a_2, \dots may be evaluated such that the polynomial provides the least mean-square error when compared with the observed data. This method is similar to that of de Mendonça (1962) and has been developed more fully by Burgess (1963). To obtain the coefficient several equations must be solved simultaneously and matrix notation provides a convenient way to display the results and to formalize the method for machine computation. The method is applied to the Faraday rotation data as follows. Eq. 2 may be written as

$$\Omega = \eta M (a_0 + a_1t + a_2t^2 + a_3t^3 + \dots) \quad (7)$$

in which it should be remembered that M is also a function of time. The value of Ω at the proximal point which serves as the time reference is

$$\Omega_0 = \eta M_0 a_0 \quad (8)$$

Subtracting (8) from (7) leads to

$$(\Omega - \Omega_0)/\eta = (M - M_0)a_0 + (Mt)a_1 + (Mt^2)a_2 + (Mt^3)a_3 + \dots \quad (9)$$

The left hand side of (9) may be determined from observation and the right hand side predicts a value dependent upon the choices of a_0, a_1, \dots . To select these coefficients, the mean-square error is first found by subtracting the RHS from the LHS, and summing up the errors at each of the n times at which Eq. 9 has been evaluated:

$$\epsilon = \frac{1}{n} \sum_1^n \left\{ [(\Omega - \Omega_0)/\eta] - (M - M_0)a_0 - (Mt)a_1 - \dots \right\}^2 \quad (10)$$

For least error, the coefficients are determined by simultaneous solutions of the set of equations obtained from

$$\frac{\partial \epsilon}{\partial a_0} = \frac{\partial \epsilon}{\partial a_1} = \frac{\partial \epsilon}{\partial a_2} = \text{etc.} = 0 \quad (11)$$

This set of equations may be represented in matrix form by

$$[\underline{F}] [\underline{a}] = [\underline{g}] \quad (12)$$

with

$$\underline{F} = \begin{bmatrix} \sum (M - M_0)^2 & \sum M(M - M_0)t & \sum M(M - M_0)t^2 & \dots \\ \sum M(M - M_0)t & \sum M^2 t^2 & \sum M^2 t^3 & \dots \\ \sum M(M - M_0)t^2 & \sum M^2 t^3 & \sum M^2 t^4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (1)$$

$$[\underline{g}] = \begin{bmatrix} \sum (\Omega - \Omega_0)(M - M_0)/\eta \\ \sum (\Omega - \Omega_0)(Mt)/\eta \\ \sum (\Omega - \Omega_0)(Mt^2)/\eta \\ \vdots \end{bmatrix}$$

The unknown coefficients are obtained by multiplying both members of Eq. 12 by the inverse of matrix $[\underline{F}]$, giving

$$[\underline{a}] = [\underline{F}]^{-1} [\underline{g}] \quad (14)''$$

In the calculations described in Section III, slightly different matrices $[\underline{F}]^{-1}$ and $[\underline{g}]$ were used in order to reduce the tendency of $[\underline{F}]^{-1}$ to approach singularity. The altered matrices were obtained by multiplying each term of Eq. 9 by $\sqrt{t/M}$ then proceeding

through Eqs. 10 - 14. Values of $[a]$ were obtained for three power series, e.g., third, second and first degree polynomials with all higher powers set equal to zero.

A similar procedure may be used for the evaluation of the Doppler observations. It may be shown readily that the reduction in phase path length owing to propagation through the ionosphere is related to the electron content by

$$\Delta P = (40.3/f^2) \sec \alpha \cdot I \quad (\text{meters}) \quad (15)$$

Again, α should be evaluated near the centroid of the electron distribution and we have used a height about 50 km greater than the height of maximum density in all cases. Dividing through by the free space wavelength λ and expressing I as a polynomial, one obtains

$$(\Delta P / \lambda) = (40.3/cf) \sec \alpha (b_0 + b_1 t + b_2 t^2 + \dots) \quad (16)$$

We may now define precisely the proximal point (which determines our time reference) as that point along the satellite path at which the phase path defect ΔP given by Eq. 15 is a minimum. This point is usually quite obvious on the record showing the beat (differential Doppler) between the 54 Mc signal and the (1/6) sub-harmonic of the 324 Mc signal. At the proximal point the beat frequency will go to zero and reverse the direction of the relative phase shift of the two signals. This point is usually quite near the point of minimum geometrical range but either a vertical component in the satellite velocity or horizontal ionospheric gradients may shift the two points apart by 30 seconds or even more in time. At the proximal point,

$$(\Delta P / \lambda) = (\Delta P_0 / \lambda) = (40.3/cf) \sec \alpha_0 b_0 \quad (17)$$

Proceeding as before, (17) is subtracted from (16), giving

$$\left(\frac{\Delta P - \Delta P_0}{\lambda} \right) = \left(\frac{40.3}{cf} \right) \left[(\sec \alpha - \sec \alpha_0) b_0 + \sec \alpha b_1 t + \dots \right] \quad (18)$$

The LHS of (18) may be determined from the satellite observations. Each cycle of phase change measured from the proximal point contributes one wave length to Eq. 18 (a small correction must be included to account for the phase path change at 324 Mc).

The values of b_0, b_1, \dots are to be selected such that the RHS of (18) agrees as closely as possible with the observations. It is convenient to multiply both sides of (18) by $\cos x$ and then to make the substitutions $\beta = (cf)/40.3$ and $\varphi \equiv (\cos x_0 - \cos x)/\cos x_0$. Then the mean-square error is

$$\epsilon = \frac{1}{n} \sum_{i=1}^n \left[\left(\frac{\Delta P - \Delta P_0}{\lambda} \right) \beta \cdot \cos x - [b_0 + b_1 t + b_2 t^2 + \dots] \right]^2 \quad (19)$$

A set of linear equations is obtained by forming the partial derivatives of (19), summarized as

$$[\underline{D}] [\underline{b}] = [\underline{h}] \quad (20)$$

in which

$$[\underline{D}] = \begin{bmatrix} \sum \varphi^2 & \sum \varphi t & \sum \varphi t^2 & \dots \\ \sum \varphi t & \sum t^2 & \sum t^3 & \dots \\ \sum \varphi t^2 & \sum t^3 & \sum t^4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$[\underline{h}] = \begin{bmatrix} \sum \left(\frac{\Delta P - \Delta P_0}{\lambda} \right) (\cos x) \\ \sum \left(\frac{\Delta P - \Delta P_0}{\lambda} \right) (\cos x)t \\ \sum \left(\frac{\Delta P - \Delta P_0}{\lambda} \right) (\cos x)t^2 \\ \vdots \end{bmatrix}$$

As with the Faraday data, solutions for

$$[\underline{b}] = [\underline{D}]^{-1} [\underline{h}] \quad (21)''$$

were obtained for third, second and first degree polynomials.

D. Doppler Method

This method uses only the differential Doppler data as is also true of the method just described above. It is approximately the same as that used by de Mendonça (1962). In this method the electron content is assumed to vary linearly with time ($b_2 = b_3 = \dots = 0$) and the value of $(dI/dt) = b_1$ is obtained by calculating the change in electron content between two times. After I has been established the best fitting value of I_0 is determined by minimizing the mean-square error. The necessary equations are obtained below.

Two times (t_j and t_k) are established at which the zenith angles (α) of the satellite are the same. (In the calculations shown in Section III we have arbitrarily selected these times approximately ± 50 seconds on either side of the time of minimum α). Eq. 16 is then written for each of these times and their difference taken:

$$\left(\frac{\Delta P_j - \Delta P_k}{\lambda} \right) = \frac{\sec \alpha}{\beta} b_1 (t_j - t_k) \quad (22)$$

The LHS is the number of cycles of phase shift between t_j and t_k (corrected for the ionospheric effect at 324 Mc). From this measurement b_1 may be calculated. It should be expected that the value of b_1 will fluctuate somewhat dependent upon the time interval chosen, owing to the irregularities in the ionosphere and to the higher order terms in $I(t)$ which were neglected.

To evaluate b_0 we find the mean-square error to be

$$\epsilon = \frac{1}{n} \sum_1^n \left[\left(\frac{\Delta P - \Delta P_0}{\lambda} \right) \beta \cos \alpha_0 - \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right) b_0 - \left(\frac{\cos \alpha_0}{\cos \alpha} \right) b_1 t \right]^2 \quad (23)$$

Forming $(\partial \epsilon / \partial b_0) = 0$, we solve for b_0 as

$$b_0 = \frac{\sum \left(\frac{\Delta P - \Delta P_0}{\lambda} \right) \beta \cos \alpha_0 \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right) - \sum \left(\frac{\cos \alpha_0}{\cos \alpha} \right) \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right) b_1 t}{\sum \left(1 - \frac{\cos \alpha_0}{\cos \alpha} \right)^2} \quad (24)''$$

E. Hybrid Faraday - Doppler Method

This method makes use of both types of data simultaneously and has the great advantage that it is not necessary to assume horizontal stratification of the ionosphere. The method was first described by Burgess (1962) and improved upon by Golton (1962). A somewhat more general approach was used by de Mendonça and Garriott (1962) in analyzing the same data which will be discussed in this paper. Reference may be made to these papers for a detailed description of the methods. We need only, point out here that there are no assumptions necessary in the application of this method which are likely to introduce an error greater than a few per cent in the final value of electron content.

III. Calculations of Electron Content

The most accurate of the analyses described above is expected to be the one described last, the hybrid method. Therefore it will be examined first for internal consistency by performing the calculations based upon different sections of the data in a single satellite passage. These "sub-sets" were usually spaced about ± 1 , ± 2 and ± 3 minutes on either side of the proximal point; in a number of other cases the data were located asymmetrically about the proximal point, from each sub-set, the value of electron content at the proximal point was calculated. From the group of answers so obtained pertaining to a single passage, a value near the average of the group was arbitrarily assigned as the "correct" value of electron content at the proximal point. Then the percent deviation of each value was calculated from the assumed "correct" value of electron content. This procedure was repeated for each passage considered. The histogram showing the number of values in each one-percent error interval is shown in Fig. 1.

It is clear that the method shows very good internal consistency in that the calculated value of electron content at the proximal point is largely independent of the data segment used in the calculation. Over 80% of the values lie within $\pm 4\%$ of the "correct" value. When values obtained from asymmetric data sets are considered separately, the results are nearly the same as shown in Fig. 1, although occasionally a larger deviation is encountered. Excluding these sets, over 85% of the values fall within $\pm 4\%$ of the "correct" value.

The same assumed "correct" values will now be used to estimate the accuracy of the other methods. It should be noted that in all of the comparisons to follow, the electron content has been evaluated at the same time in each satellite passage, e. g., the proximal point. The error in each of the two rotation rate methods (Sec. IIA) has been calculated and Fig. 2 is a histogram showing the number of the values in each five-percent error interval for the case in which horizontal gradients are neglected (Eq. 4). The figure shows that there is a systematic error involved in the use of Eq. 4 such that the average of the values is about 20% too high. The ionosphere above Stanford usually exhibits a decreasing electron density towards the north and a gradient in this direction should result in an overestimation

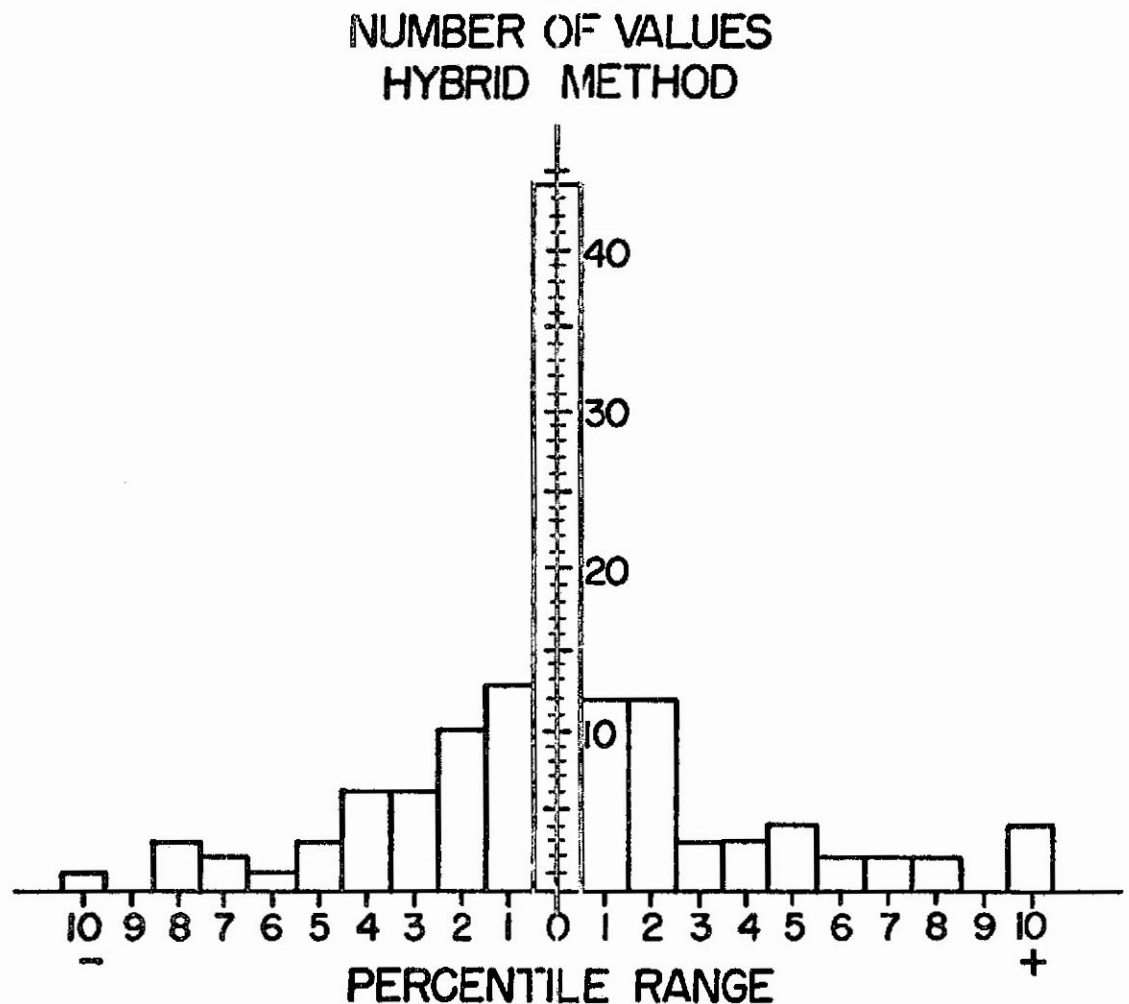


Figure 1

The percentage error for each calculation made with the hybrid Faraday - Doppler equations. The "Correct" value of electron content was assumed to be near the mean of all measurements made on an individual passage and the percentage error was calculated from this value. The number of values in each 1% interval are shown in the histogram. The Figure illustrates the internal consistency of the method.

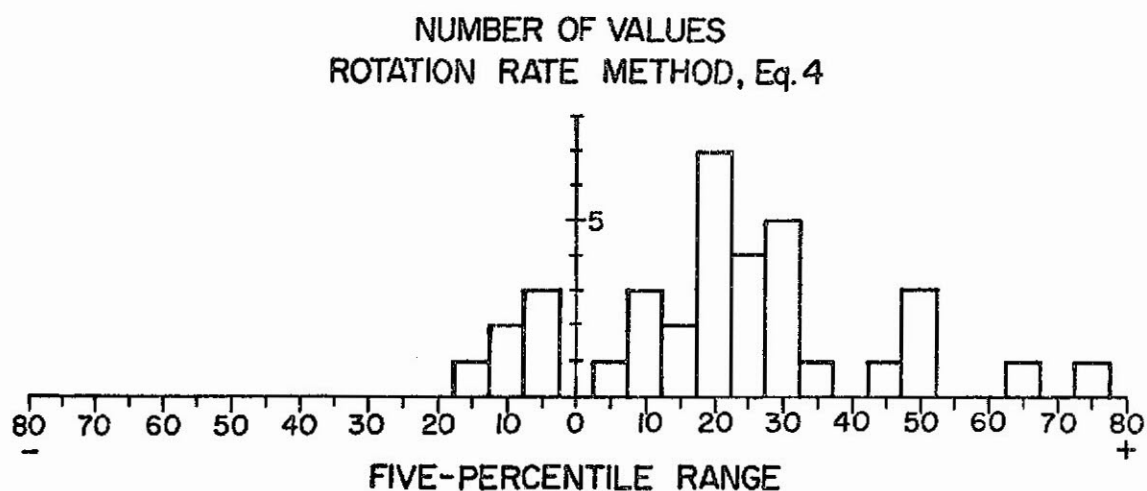


Figure 2 The number of values in each five-percent error interval for the rotation rate method of Eq. 4. One value was obtained for each of the 35 passages. The error was measured from the assumed "correct" value previously obtained.

of electron content when the gradient is ignored, as the calculations have confirmed. The distribution is much broader as well, only 65% of the values lying within $\pm 15\%$ of the mean.

It might be expected that calculations made by solving Eqs. 3 and 5 simultaneously would be appreciably better than those shown in Fig. 2 since a gradient in the electron content would be allowed. In fact, they are so much worse that they are not even presented in a figure. The difficulty seems to lie in the determination of $\dot{\Omega}$. Five point differentiation formulas were used to obtain $\dot{\Omega}$ and then again to find Ω , and it is likely that some improvement could be achieved in a more refined analysis. Some error is certainly involved in scaling the time of the Faraday nulls and these errors are amplified by the double differentiation. Perhaps even more important are the large scale irregularities in the ionosphere [Little and Lawrence (1960)]. These irregularities slightly shift the regular period of the Faraday fading and thereby contribute to the very erratic estimates of $\dot{\Omega}$. These results suggest that very little success may be expected from calculations requiring an accurate estimate of $\dot{\Omega}$.

The results of the rotation angle calculation (Sec. IIB, Eq. 6) when compared with the "correct" values of electron content obtained by the hybrid method are shown in Fig. 3. A larger total number of values are shown here and in most of the following figures than are shown in Fig. 2 because each satellite passage contributes three or four values, one for each data subset used in the calculations. Again, the values appear to be too high by about 25% and have appreciable scatter. Only 50% of the results are within $\pm 10\%$ of the mean and only 65% are within $\pm 20\%$ of the mean value. The overestimation is consistent with a decreasing electron content toward the north as in the rotation rate analysis. Some improvement should be expected in both the positive bias and in the scatter of the results when the additional precautions that some authors have used are incorporated (Sec. IIB). However, the method clearly suffers from the necessity of neglecting horizontal gradients.

Next, the results of the polynomial methods will be described. Fig. 4 shows the number of values in each five-percent error interval based on the Faraday fading data alone when the electron content is limited to a linear time variation, e. g., $I = a_0 + a_1 t$. It is observed that nearly all of the systematic error has been removed and the scatter of the results is comparable with that shown in Figs. 2 and 3. About 60% of the data are within $\pm 15\%$ of the mean. This method therefore appears superior to the other much more widely adopted Faraday analysis techniques. However, when the polynomial expression for I is allowed to have third or even second degree terms, the errors become very large and the results are quite useless. Although this may to some extent be due to scaling inaccuracies and relatively few data points (only 10 to 30 Faraday nulls in most cases), it is believed that ionospheric irregularities also play a major factor in the wide scatter of the results.

Sec. IIC also described the method of best fitting a polynomial to the differential Doppler data. For these calculations the number of beats

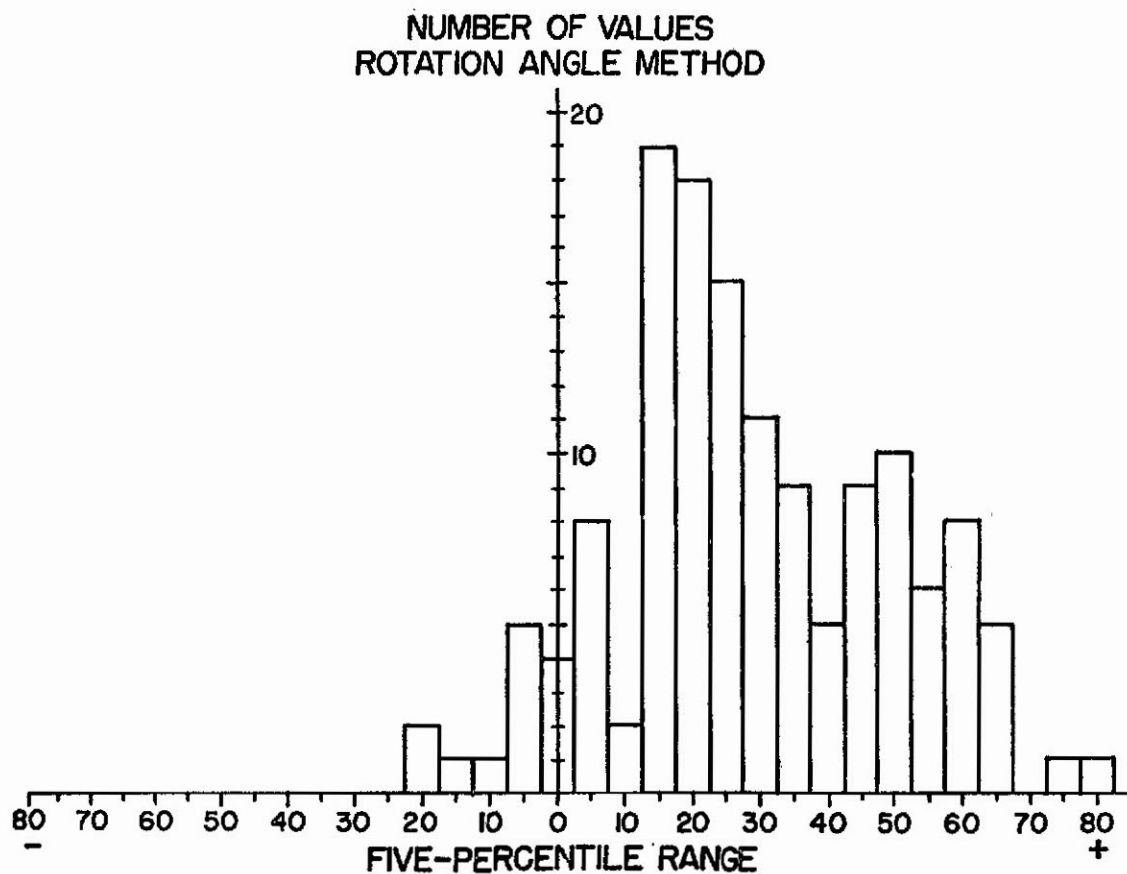


Figure 3

The number of values in each five-percent error interval for the rotation angle method of Eq. 6. A greater total number of values are obtained than were shown in Fig. 2 because each passage yields several estimates of electron content.

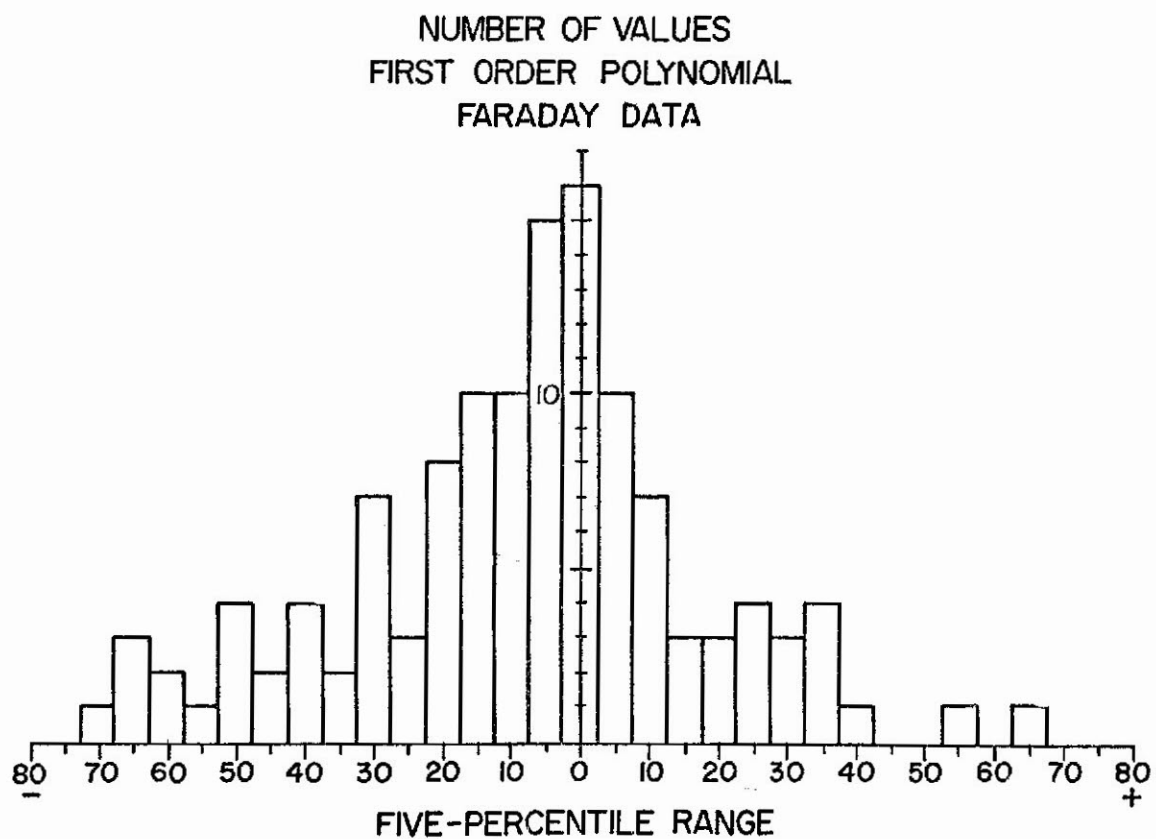


Figure 4 The number of values in each five-percent error interval based upon a best-fitting linear polynomial obtained with the Faraday data alone (Eq. 14).

or wavelengths of phase change were tabulated approximately every 5 or 10 seconds. Thus, there were more data points involved in the summations representing the elements of the matrices in Eq. 21 than there were in the Faraday method, Eq. 14. Figs. 5 a, b, and c show the number of values in each five-percent error interval for polynomials of third, second and first degree, respectively. There is a small positive systematic error in all of the figures, but only 5% or 10%. The scatter of the data consistently improves as the degree of the polynomial is reduced. In Fig. 5c almost 80% of the data falls within $\pm 15\%$ of the mean. Just as with the Faraday data, too many degrees of freedom in $I(t)$ degrades the accuracy with which the electron content is determined. When the value obtained from data sets located asymmetrically about the proximal point are plotted by themselves, the results are almost identical to Fig. 5c.

An additional Doppler method was described in Sec. IID and an expression for electron content was obtained in Eq. 24. The results of this method are shown in Fig. 6. Just as in the preceding method, there is a positive bias of 5% or 10% and about 70% of the data fall within $\pm 10\%$ of the mean. These latter two methods should correlate very closely since they both permit a linear time variation of electron content only. When their values are compared for each passage, they differ by an average of only 1% and they have an rms deviation of only 3%.

IV. Discussion

It has not been possible to directly verify the accuracy of the hybrid method because there are no more accurate methods available with which to compare it. However, its internal consistency has been demonstrated and the lack of unrealistic approximations encourages considerable confidence.

The most commonly employed methods of reducing Faraday data (rotation rate and rotation angle) have been known to suffer from the neglect of horizontal gradients. The results shown above imply a systematic error of about + 20% for these methods when used at latitudes near Stanford (43°N geomagnetic). It is likely that some improvement is possible by manually estimating the rotation rate from a careful plot of the fading record, rather than using a computer, by selecting the times for rotation angle calculations in order to minimize the error resulting from horizontal gradients and by using ray tracing methods when evaluating the required geometrical quantities. Nevertheless, the best-fitting linear polynomial is found to give reasonably good values without the above complexities, although a computer is still required if very large amounts of data are to be handled.

Both Doppler methods are found to be reasonably good and nearly free from systematic error as long as the electron content is restricted to a linear time variation.

These results differ with those found by Burgess (1963) principally in that we have found the best-fitting polynomial methods to be relatively

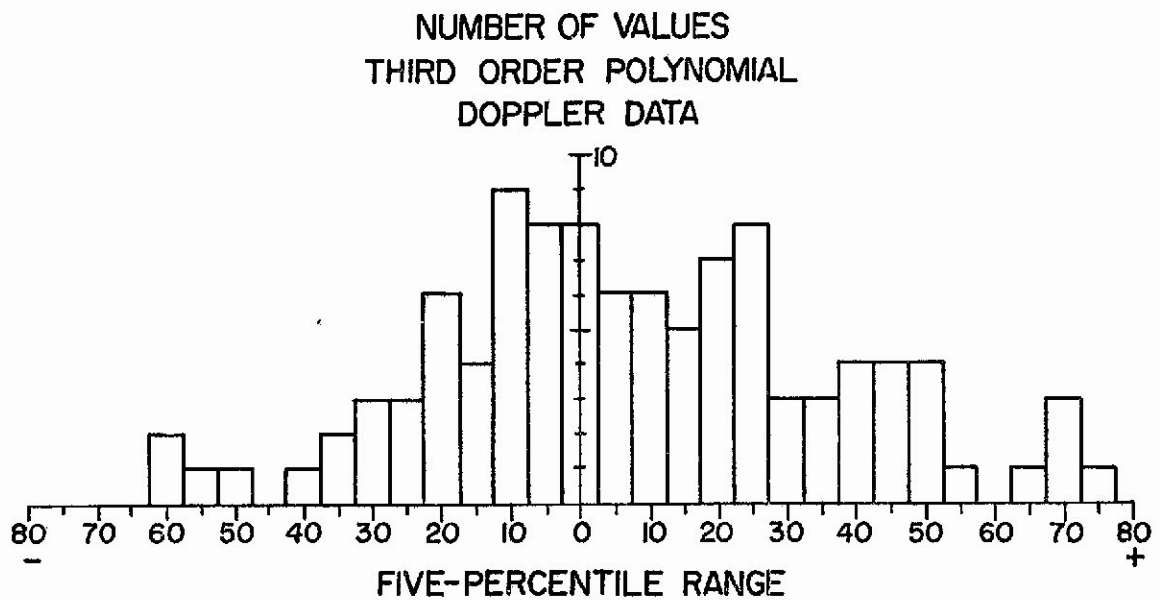


Figure 5a A histogram similar to that in the previous Figures, obtained from a polynomial of the form $I = b_0 + b_1 t + b_2 t^2 + b_3 t^3$, using differential Doppler data (Eq. 21).

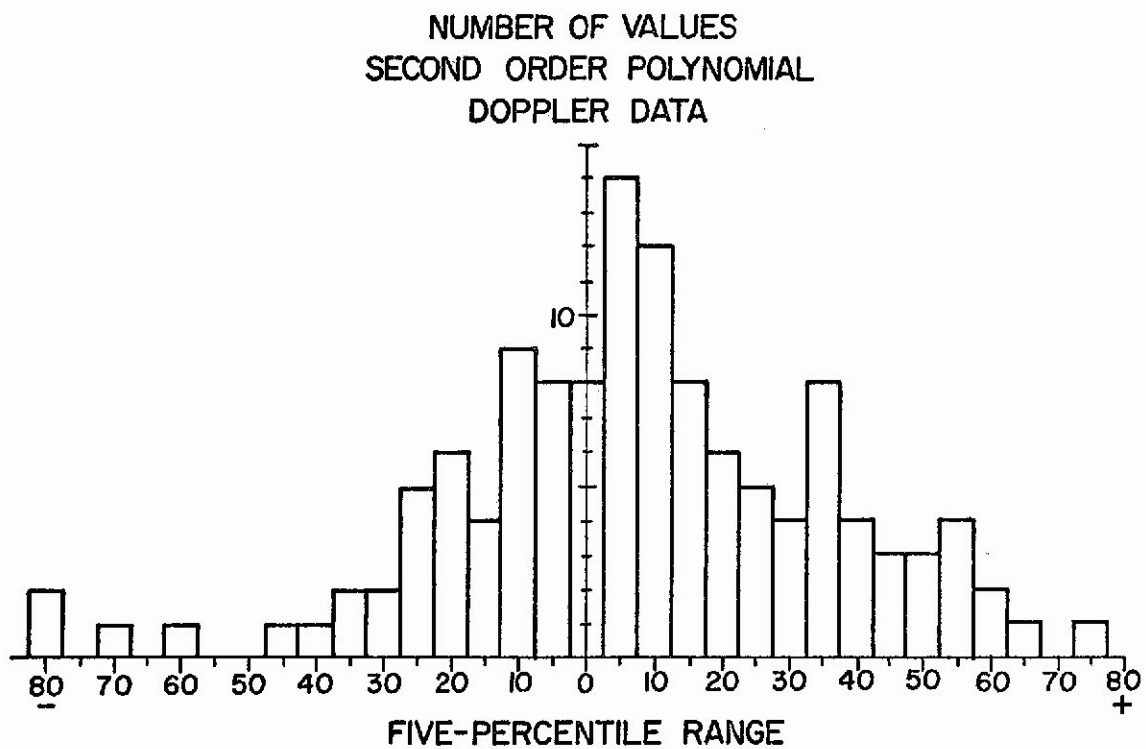


Figure 5b Similar to the previous Figure, except that a polynomial of the form $I = b_0 + b_1 t + b_2 t^2$ was used.

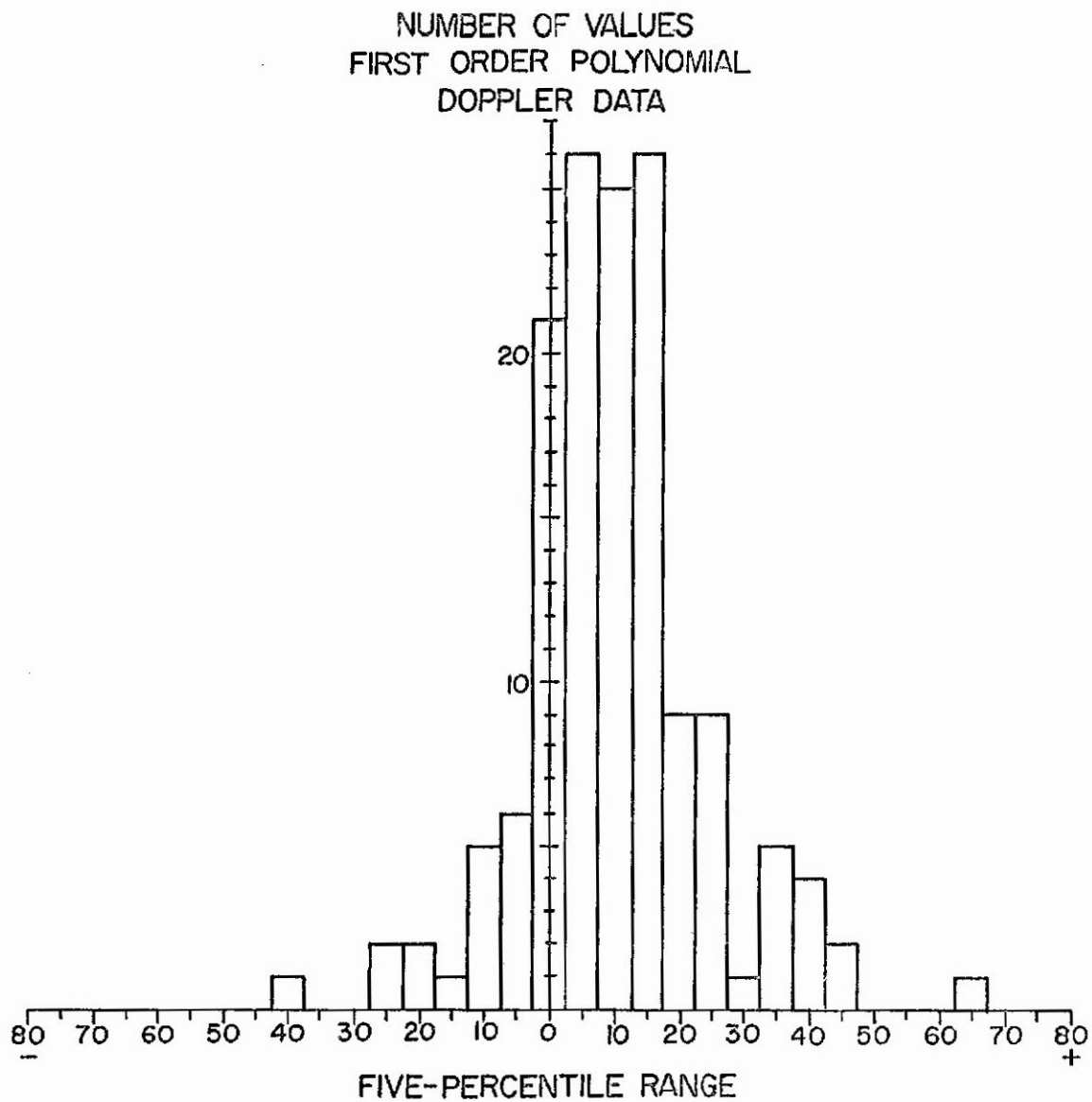


Figure 5c Similar to the previous Figure, except that a polynomial of the form $I = b_0 + b_1 t$ was used.

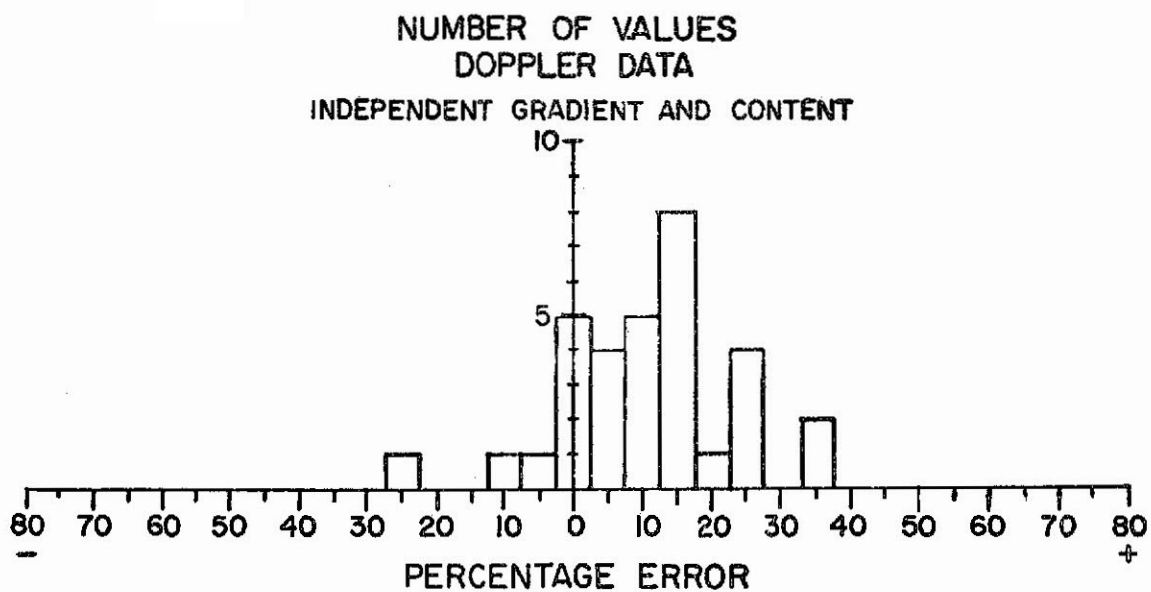


Figure 6 Similar to the previous Figure, except that the linear gradient was calculated from Eq. 22 and the value of electron content obtained from Eq. 24.

satisfactory as long as first degree polynomials only are used. Burgess also considered a method in which (dI/dt) was estimated independently, analogous to the method we have described in Sec. IID. Although he found a relatively large error in one case, we have obtained rather small average errors for our complete data set. It is rather difficult to understand the reasons for these differences, but they may be attributable to the selection of one unfortunate satellite passage for his analysis. His results may also have been obtained with polynomials of too high a degree, since this is found to lead to erratic values of electron content.

There is still another method which holds considerable promise, although there has been no opportunity to employ it as yet except with moon echoes. This is a "differential Faraday" method (Daniels, 1957; Evans, 1957) in which the angular difference in the planes of polarization at two closely spaced frequencies is measured. Forthcoming radio beacon satellites are expected to provide the necessary frequencies and relatively good accuracy should be obtained.

V. Acknowledgement

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