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<td>This paper presents some results in controllability of nonlinear systems of the type ( \dot{z} = Az + Nz + Bu ), where ( A ) is linear, ( N ) is nonlinear, ( B ) is linear, ( z ) is the state and ( u ) is the control. It is assumed that the dynamics of the linear part (i.e., ( \dot{z} = Az )) can be described in terms of a strongly continuous semigroup on an appropriate Banach space (the state space), so that this formulation includes distributed parameter systems and delay systems, as well as lumped parameter systems. Theoretical results in controllability are obtained by using fixed point theorems. It is considered the space ( X ) of functions from the interval ([0,7]) to ( Z ) (the possible trajectories of the system). In this setting, some mappings ( F: X \rightarrow X ) are constructed and the solution of the problem of nonlinear controllability is obtained via the fixed points of such ( F )'s.</td>
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CONTROL OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS

J.A.M. Felippe de Souza

Dept. Mecânicas Espaciais e Controle
Instituto de Pesquisas Espaciais - INPE/CNPq
Caixa Postal 515
12200 - São José dos Campos - SP - BRASIL

ABSTRACT

This paper presents some results in controllability of nonlinear systems of the type $\dot{z} = Az + Nz + Bu$, where $A$ is linear, $N$ is nonlinear, $B$ is linear, $z$ is the state and $u$ is the control. It is assumed that the dynamics of the linear part (i.e., $\dot{z} = Az$) can be described in terms of a strongly continuous semigroup on an appropriate Banach space (the state space), so that this formulation includes distributed parameter systems and delay systems, as well as lumped parameter systems.

Theoretical results in controllability are obtained by using fixed point theorems. It is considered the space $X$ of functions from the interval $[0, T]$ to $Z$ (the possible trajectories of the system). In this setting, some mappings $F: X \rightarrow X$ are constructed and the solution of the problem of nonlinear controllability is obtained via the fixed points of such $F$'s.
1. FORMULATION OF THE PROBLEM

In this paper we consider nonlinear systems of the type

\[ \dot{z} = Az + Nz + Bu, \quad z(0) = z_0, \]  \hspace{1cm} (1)

where \( A \) is a linear operator on a Banach space \( Z \) (the state space), \( N \) a nonlinear operator, \( B \) a linear operator from an input space \( U \) to \( Z \), and \( u \) the control. It is assumed that the dynamics of the linearized system

\[ \dot{z} = Az, \quad z(0) = z_0, \]  \hspace{1cm} (2)

can be described in terms of a strongly continuous semigroup \( S(t) \) on \( Z \). The problem of controllability is to find a control \( u(\cdot) \in U \) which drives system (1) from \( z_0 \) at \( t=0 \) to a given desired state \( z_d \in Z \) at \( t=T \), where \( U \) is a space of functions from \([0,T]\) to the input space \( U \) of the system. The problem has been studied by the author in [3,4,5,6, 7,8,9 and 10] and other authors in [2,11,12 and 13]. Here we present the basic ideas of the results obtained when \( Z \) is a Hilbert space.

Note that the above formulation includes distributed parameter systems and delay systems, as well as lamped parameter system.

2. EXAMPLE

An example of a problem of control of a nonlinear distributed parameter system we consider the following diffusion process on \( Z = L^2(0,1) \)

\[ \frac{\partial z}{\partial t} = \alpha \frac{\partial^2 z}{\partial x^2} - \beta z + Nz + Bu, \]

with boundary conditions

\[ z_x(0,t) = z_x(1,t) = 0 \]

and initial condition
\[ z(x,0) = z_0(x), \]

where \( \alpha, \beta \in \mathbb{R}, \ b(\cdot) \in L^2(0,1) \) and \( N \) is a nonlinear operador on \( L^2(0,1) \) such as

\[ N_z = z^4, \ z_x^2, \ z z_x, \ z^2 z_x, \ \text{etc.} \]

This system can be expressed in the form (1) by setting \( U = \mathbb{R}, \ B : R \rightarrow L^2(0,1) \) given by

\[ Bu = b(x)u \]

and \( A \) the linear operator on \( L^2(0,1) \)

\[ Az = \alpha z_{xx} - \beta I , \]

\[ D(A) = \left\{ z \in L^2(0,1) : \frac{\partial^2 z}{\partial x^2} \in L^2(0,1), \ \frac{\partial z}{\partial x} = 0 \text{ at } x = 0,1 \right\} . \]

It can be shown (see p. 46 of [1]) that \( A \) generates a strongly continuous semigroup \( S(t) \) on \( Z \).

3. RESULTS

In order to solve the controllability problem posed above, we first consider a space \( X \) of trajectories (i.e., a space of functions from \([0,T]\) to \( Z \)), e.g., \( X = C(0,t;Z) \) or \( X = L^p(0,T;Z) \) for some \( p > 1 \), etc., and the expression

\[ z(t) = S(t)z_0 + \int_0^t S(t-s)Nz(s) \, ds + \int_0^t S(t-s)Bu(s) \, ds, \quad (3) \]

which is the mild form of system (1). Now, defining the family of linear operators \( L_t \) on \( X = X \) for each \( t \in [0,T] \)

\[ L_t z(\cdot) = \int_0^t S(t-s)z(s) \, dx , \]
we can write (3) as
\[ z(t) = S(t)z_0 + L_t^*Nz(\cdot) + L_tBu(\cdot). \] (4)

Let \( G: U \rightarrow Z \) be the linear operator
\[ Gu(\cdot) = \int_0^T S(t-s)Bu(s)ds = L_tBu(\cdot) \]
and \( G^\dagger \) denotes the pseudoinverse of \( G \).

Now we state the following result which is very easy to verify:

**Theorem A:** If \( z^*(\cdot) \) is the actual trajectory of the system and the input control applied to the system is \( u^*(\cdot) \) given by
\[ u^*(\cdot) = G^\dagger(z_d - S(T)z_0 - L_TNz^*(\cdot)), \] (5)
then \( u^*(\cdot) \) drives the system from the initial state \( z_0 \) at \( t=0 \) to the desired state \( z_d \) at \( t=T \).

In other words, \( u^*(\cdot) \) is the desired control.

The above result says that the existence of such control \( u^*(\cdot) \) given by (5) is guaranteed by the existence of a trajectory \( z^*(\cdot) \) which satisfies the dynamics of the system. On the other hand, the existence of such trajectory \( z^*(\cdot) \) is equivalent to the existence of a fixed point of the mapping \( F: X \rightarrow X \) defined in the theorem which follows.

**Theorem B:** If \( z^*(\cdot) \) is a fixed point of the mapping \( F: F \rightarrow X \) given by
\[ (Fz(\cdot))(t) = S(t)z_0 + L_tBG^\dagger(z_d - S(T)z_0 - L_TNz^*(\cdot)) + L_tNz(\cdot), \] (6)
then \( z^*(\cdot) \) is a trajectory of the system which satisfies
\[ z^*(T) = z_d. \]

**Proof -** If \( z^*(\cdot) = F(z^*(\cdot)) \), then
\[ z^*(t) = S(t)z_0 + L^+_tBG^+(z_d - S(Tz_0 - L_tNz^*(\cdot)) + L_tNz^*(\cdot) . \] (7)

Hence,

\[ z^*(t) = S(t)z_0 + L_tNz^*(\cdot) + L_tBu^*(\cdot) \]

and therefore \( z^*(\cdot) \) satisfies (4) (that is, \( z^*(\cdot) \) is a possible trajectory of the system). Now, since

\[ L^+_tBG = GG^+ , \]

we have that by using (7)

\[ z^*(T) = S(T)z_0 + z_d - S(T)z_0 - L_TNz^*(\cdot) + L_TNz^*(\cdot) = z_d . \]

Q.E.D.

So, the problem of controllability of nonlinear systems of type (1) has been converted to finding a fixed point for \( F \). Several fixed point theorems, such as the Contraction Mapping Principle, has been used in \([2,3,4,5,6,7,8,9,10,11,12]\) to determine the existence of fixed points of mappings which are similar to the mapping \( F \) given in (6).

REFERENCES


