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14. Abstract/Notes

This paper describes the synthesis and analysis of a control law for a flexible spacecraft. The control law is considered a simple proportional, integral, and derivative law together with a second order structural filter. Parameter optimization is applied for finding the controller parameters, so as to have an optimized behavior when applied to the high order model. Frequency and Laplace domain analysis are shown, which indicate the satisfactory behavior of the proposed controller.

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PARAMETER OPTIMIZATION AND ATTITUDE STABILIZATION OF A FLEXIBLE SPACECRAFT

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ABSTRACT

This paper describes the synthesis and analysis of a control law for a flexible spacecraft. The control law is considered a simple proportional, integral, and derivative law together with a second order structural filter. Parameter optimization is applied for finding the controller parameters, so as to have an optimized behavior when applied to the high order model. Frequency and Laplace domain analysis are shown, which indicate the satisfactory behavior of the proposed controller.

1. INTRODUCTION

A three-axis stabilized spacecraft frequently has large flexible solar panel arrays which interact with the control system[1-4]. A second order structural filter together with a simple proportional, integral, and derivative law have been considered[1] to reduce this interaction. Parameter optimization methods have also been used for finding low order controllers in design of control systems in the time domain. These low order controllers are usually derived from optimal control structures, and have been found to be less sensitive to modelling errors and vibration effects. The idea here is to apply the parameter optimization procedure, which is used in Ref. 4, to find the parameters for the classical structure presented in Fig. 1, in order to obtain an optimized behavior for the high-order model. A performance index (PI) is defined as a function of the steady-state covariance of the control and the state vectors. Gradients of the PI with respect to the parameters of the control structure are determined by the procedure presented in Ref. 5. The algorithm is initialized by a solution corresponding to a stable controller. The gain determination procedure is applied to find the controller parameters for a three-axis stabilized spacecraft with large flexible solar panels. Satisfactory results are obtained, as shown by Laplace and frequency domain analysis. The root-locus and magnitude-phase Bode diagrams indicate large stability margin. The magnitude Bode diagram shows a good attenuation factor in the range of the structural vibration mode frequencies.

2. CONTROL STRUCTURE AND MODELLING

The flexible dynamical linear equations can be placed in state variable form as (Refs. 2-4):

$$\dot{x} = Fx + Gu + Lw, \tag{1}$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \qquad q = \begin{bmatrix} \theta_x \\ s \end{bmatrix}, \tag{2}$$

$$G = L = \begin{bmatrix} 0 \\ J^{-1} \; b_r \end{bmatrix}, \quad b_r = \begin{bmatrix} b \\ 0 \end{bmatrix}, \tag{3}$$

$$F = \begin{bmatrix} 0 & I \\ J^{-1}V_2 & J^{-1}V_1 \end{bmatrix}, \quad J = \begin{bmatrix} J & -g^T \\ -g & I \end{bmatrix}, \tag{4}$$

$$V_1 = \begin{bmatrix} 0 & 0 \\ 0 & -2C_a f \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 & 0 \\ 0 & -f^2 \end{bmatrix} \tag{5}$$

$$f = \mathrm{diag} \left[f_1, f_2, \ldots, f_r \right], \tag{6}$$

$$s^T = \left[s_1, s_2, \ldots, s_r \right], \tag{7}$$

$$g^T = \left[g_1, g_2, \ldots, g_r \right], \tag{8}$$

$$Q = E\,(w.w^T), \tag{9}$$

Where Ca is the damping coefficient, b is a torque scale parameter, Jxx is the rotational inertia, Q is the noise variance, fi is a frequency of structural vibration, and gi is a coupling term.

The control is supposed to be based on the loop structure given in Fig. 1. The control logic is implemented in a microprocessor and the necessary torques are provided by a reaction jet system. In spite of digital implementation, continuous time design procedures are used because the microprocessor sampling period is supposed to be relatively fast compared to the dominant system frequencies. The control law is a simple proportional, integral, and derivative law together with a second order structural filter.

Regarding the state variable representation of the controlled systems, the following augmented state vector may be define as

$$x_a^T = \left[\theta_x \quad q \quad \dot{\theta}_x \quad \dot{q} \quad z_1 \; z_2 \; z_3 \right], \tag{10}$$

where z_1 is the integral and z_2 and z_3 are the coordinates which result from the phase variable representation of the structural filter.

The control u(t) can be obtained from Fig. 1, and if the fourth order model (Eqs. 1-9) is considered it is given by results:

$$u = K*x_{a1} + K*K_V*x_{a3} + K*K_I*x_{a4} + (K*K_B - K*K_D)*x_{a5} +$$

$$+ (K*K_A - K*K_C)*x_{a6} \tag{11}$$

and using this value of u, one obtains

$$\dot{x}_a = F_a x_a + L_a w \tag{12}$$

$$F_a = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ D_7 & D_1 & D_9 & D_3 & D_{11} & D_{13} & D_{15} \\ D_8 & D_2 & D_{10} & D_4 & D_{12} & D_{14} & D_{16} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & K_V & 0 & K_I & -K_D & -K_C \end{bmatrix}, \tag{13}$$

$$L_a^T = [\,0 \quad 0 \quad D_5 \quad D_6 \quad 0 \quad 0 \quad 0\,], \tag{14}$$

where

$D_0 = J_{xx} - g^2$; $D_1 = gf^2/D_0$;

$D_2 = -J_{xx}f^2/D_0$; $D_3 = -2gCaf/D_0$;

$D_4 = -2J_{xx}Ca\ f/D_0$; $D_5 = -\ b/D_0$;

$D_6 = -bg/D_0$;

$D_7 = D_5*K$; $D_8 = D_6*K$;

$D_9 = D_5*K*K_V$; $D_{10} = D_6*K*K_V$;

$D_{11} = D_5*K*K_I$; $D_{12} = D_6*K*K_I$;

$D_{13} = D_5*(K*K_B - K*K_D)$; $D_{14} = D_6*(K*K_B - K*K_D)$;

$D_{15} = D_5*(K*K_A - K*K_C)$; $D_{16} = D_6*(K*K_A - K*K_C)$.

The control gains K_I, K_v, K_A, K_B, K_C, K_D, K can be obtained so as to minimize the performance index

$$PI = tr\ [A\ X_a\ (\infty)] + tr\ [BU\ (\infty)], \tag{15}$$

where X_a (∞) is the steady state covariance of the state vector X_a ; U (∞) is the steady state covariance of the control u; A is a symmetrical positive semidefinite matrix; B is a symmetrical positive definite matrix.

If u (Eq. 11) is introduced in the Eq. 15, PI is given by

$$PI = tr\ [\overline{A}\ X_a\ (\infty)], \tag{16}$$

where

$\overline{A}_{11} = A_{11} + K^2*B_{11}$; $\overline{A}_{13} = \overline{A}_{31} = A_{31} + K^2*K_V*B_{11}$;

$\overline{A}_{33} = \overline{A}_{33} + K^2*K_V{}^2*B_{11}$; $\overline{A}_{51} = \overline{A}_{15} = A_{15} + K^2*K_I*B_{11}$;

$\overline{A}_{53} = \overline{A}_{35} = A_{53} + K^2*K_I*K_V B_{11}$; $\overline{A}_{55} = A_{55} + K^2*K_I{}^2*B_{11}$;

$\overline{A}_{61} = \overline{A}_{16} = A_{16} + K^2*(K_B - K_D)*B_{11}$; $\overline{A}_{63} = \overline{A}_{36} = A_{36}$ +

$K^2(K_B K_v - K_D K_v)*B_{11}$;

$\overline{A}_{65} = \overline{A}_{56} = A_{65} + K^2*K_I*(K_B - K_D)*B_{11}$; $A_{66} = A_{66} + K^2*$

$(K_B - K_D)^2*B_{11}$;

$\overline{A}_{71} = \overline{A}_{17} = A_{71} + K^2*(K_A - K_C)*B_{11}$; $\overline{A}_{73} = \overline{A}_{37} = A_{73} * K^2*$

$K_v*(K_A - K_C)*B_{11}$;

$\overline{A}_{75} = \overline{A}_{57} = A_{57} + K^2*K_I*(K_A - K_C)*B_{11}$; $\overline{A}_{67} = \overline{A}_{76} = K^2*$

$(K_A*K_B - K_B*K_C - K_D*K_A + K_D*K_C)*B_{11} + A_{76}$;

$\overline{A}_{77} = A_{77} + K^2*(K_A - K_C)^2*B_{11}$.

Since by assumption the closed-loop control system is asymptotically stable, X_a (∞) satisfies the Lyapunov equation:

$$F_a X_a + X_a F_a^T + G_a Q_a G_a^T = 0 \tag{17}$$

Let P be the vector whose elements are the controller parameters to be determined. Kwakernaak and Sivan[5] show that the partial derivatives of PI with respect to P_i can be computed by using Eq. 17 and the Lyapunov equation adjoint to Eq.17.

A direct gradient procedure can be applied to calculate the vector of parameters P. To start the process an initial vector P corresponding to a stable control has to be adopted. In each interation a correction in P is sought to satisfy the objective of reducing PI.

The solution of the two Lyapunov equations involves most of the computation required to compute the gradient of the performance index. These equations can be solved using the concepts of Hamiltonian matrix and matrix sign function[6].

3. NUMERICAL APPLICATIONS

The algorithm described above is used to find the control gains whose basic parameters for a fourth-order design are J_{xx} = 48800 Kg*m²; g_1 = 181 Kg*m; C_a = .003; f_1 = .36 rad/sec; b = kg*m; Q = 1.E-4. The weight parameters for PI (Eq. 16) are chosen to minimize the square of Θx; $\dot{\Theta} x$, integral of Θx, and u. These parameters are respectively assumed to be $A_{11} = 10^4$; $A_{33} = 10^4$; $A_{55} = 10^2$, and $B_{11} = 1$.

To apply the parameter optimization algorithm it is necessary to have an initial stable solution. If a stable solution is not known yet, it can be found as follows. In one first phase the gains K and K_v are evaluated from the Riccati equations where the design model is supposed to be of the order two, deterministic, and the structural filter and the gain K_I are not considered in the control scheme. By this simplification, K and K_v are give by

$$K = A_{11}^{1/2} = 100,$$

$$K*K_v = (2K/D_5 + A_{33})^{1/2} = 205, \text{ or}$$

$$K = 100 \text{ and } K_v = 2.05.$$

In one second phase, by considering the control scheme without the structural filter (Fig. 1), the algorithm developed can be applied to find an optimal preliminary solution for K, K_v and K_I. The weight parameters are regarded as defined before ($A_{11} = 10^4$, $A_{33} = 10^4$, $A_{55} = 10^2$). The inital solution for this phase assumed to be K = 100, K_v = 2.05 and K_I = .01. By inspection it is easy to see that K_I should be positive and the initial magnitude can be chosen by trial and error. The final values obtained are K = 155, K_I = .021 and K_v = 3.55.

Finally the procedures can be used to obtain the optimal solution for K_T, K_v, K_A, K_B, K_C and K . The initial stable solution is composed of the final solution of second phase (K = 155, K_I = .021, K_v = 3.55) and of $K_A = K_B = K_C = K_D$ = .3 chosen to have a neutral filter in the initial condition. The final optimal parameters are K = 192, K_I = .0203, K_v = 3.47, K_A = .281, K_B = .346, K_C = .342 and K_D = .260.

The transfer function correspond with Θ and a demande Θd is given by

$$T(s) = \frac{\Theta(s)}{\Theta_d(s)} = \frac{T_c T_m}{1+T_c T_m}, \tag{18}$$

where

$$T_m = b \frac{s^2 + Caf\ s + f^2}{J_{xx}\ s^2[(1-g^2/J_{xx})s^2 + Caf\ s + f^2]} \tag{19}$$

$$T_c = K(1 + K_v s + 1/s + 1/s\ K_I)\ (s^2 + K_A s + K_B)/(s^2 + K_C s + K_D).$$

The loop gain determines how fast the response of the system is and the pointing accuracy for disturbance torques. The optimal K is strongly associated with A and B weight parameters. Smaller values of B elements as compared to A elements mean less importance to power consummation tha fastness and accurancy. A loop gain K sensibility analysis can be performed by using root locus. The Fig. 2 presents the locus for a gain variation from .5K to 2.0K in .1K intervals.

The Fig. 2 shows that the optimal loop gain K has a good margin for variation (.5 to 1.5K), keeping good stability characteristics in terms of locus position.

Nonlineatities, truncated vibration modes and modelling errors are always present in flexible spacecraft. The Bode diagram may be used to do the evaluation of the control system robustness for the flexible effects. Figs. 3 and 4 present respectively the magnitude and phase Bode diagram for the closed loop system.

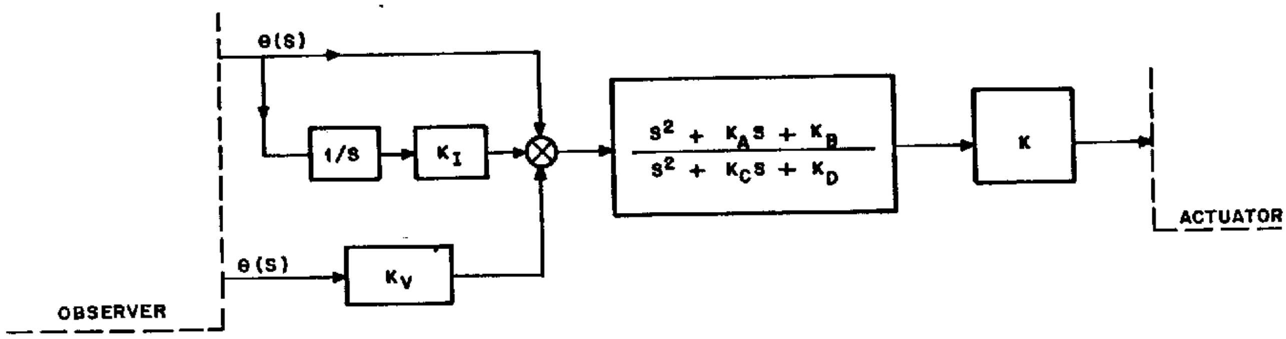

Figure 1 - Basic control structure.

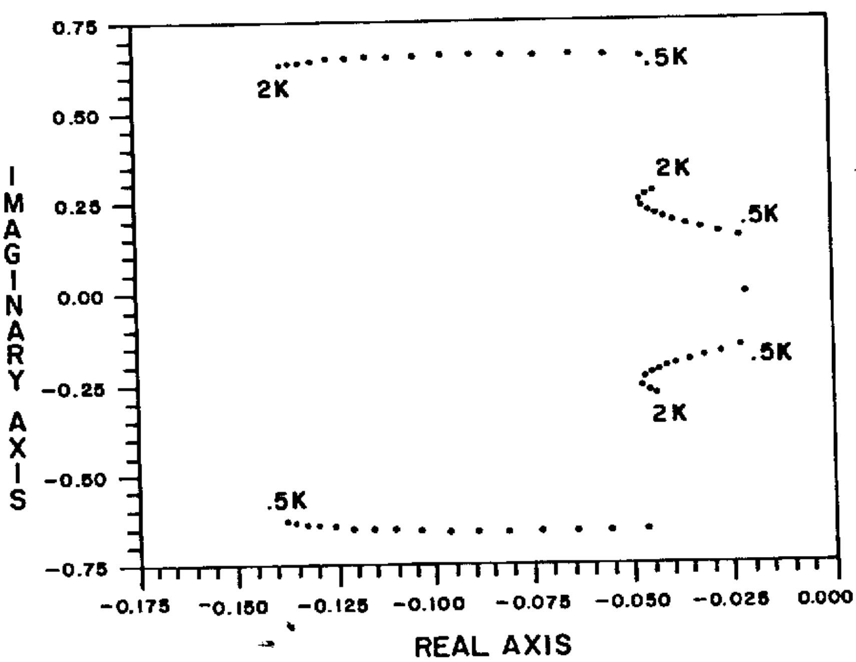

Figure 2 - Root locus: K variation from .5K to 2K (.1K interval).

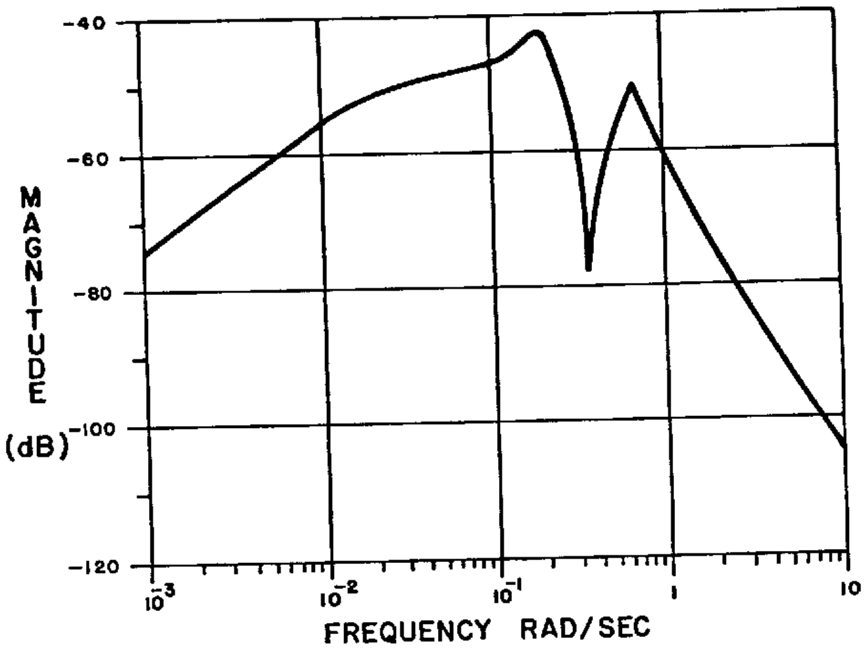

Figure 3 - Bode magnitude diagram.

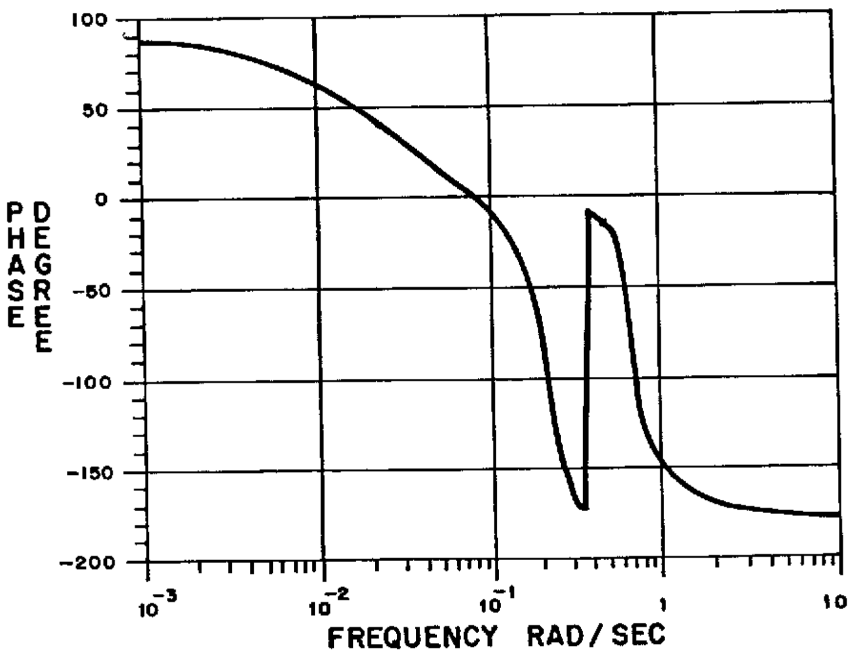

Figure 4 - Bode phase diagram.

The magnitude bode diagram indicates the steady state response of the system to a synusoidal input signal. The figure shows a good attenuation factor in the range of the structural vibration mode frequencies. The phase and magnitude bode diagrams together may be used as a measure of the system stability. The critical stability point is the 0 db, -180 Deg. point. The nearest points to reach this condition are in .2, .36, and 10 rad/seg. At frequencies equal to .2 rad/seg the gain margin in phase is round 90 deg. At .36 rad/seg the magnitude gain margin is near 80 db. At 10 rad/seg or higher frequencies the gain margin in magnitude is very high (100 db).

4. CONCLUSIONS

The design of an attitude control system for a spacecraft having large flexible appendages can be complex. The appendage flexibility interacts with the controller specially in presence of high loop gain. A compensating structure has to be used and frequently a large number of parameters need to be selected. This paper shows that parameter optimization can be applied as a fast way to do the controller synthesis. The controller analysis and gain adjustment can be performed by classical techniques as root locus and Bode plot.

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