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ON THE SCATTERING OF COMETS BY A PLANET

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ABSTRACT

The problems of capture and escape of comets is still a controversial problem in Celestial Mechanics. The number of short-period comets (period < 200 years) seems to be larger than it should be, based in the actual flux of near-parabolic comets entering the Solar System and the models known for the captures. In our paper, we address this problem by numerical integration of a large number of trajectories of possible comets. We used the regularized (Lamaître regularization) planar-circular-restricted-three-body-problem as our basic model. As initial conditions for our trajectories, we choose the parameters: Jacobian constant (J), the distance X_c (from the center of mass to the point where the comet crosses the Sun-Jupiter line) and the crossing angle (θ). The trajectories were integrated backward and forward in time and we calculated the energy and the angular momentum of the comet before and after the encounter. Then we can classify the orbits according to the effects of the close approach. Our results consist of letter-plots, which means that we plot a letter that describes the effect of the close encounter in a two-dimensional diagram having the parameter X_c in the horizontal axis and the parameter θ in the vertical axis. We make one plot for every value of the parameter J. With those plots we make histograms to show the distribution of the pericenter distance and the semi-major axis (equivalent to orbital period) of the captured comets. We conclude that: the probability of capture has a maximum of 3.6% at J = -1.15; more than 95% of the captured comets will have pericenter distance less than 5 AU, what makes them visible when passing near their pericenter; about 70% will have an orbital period smaller than 200 years.

1. INTRODUCTION

The comets are among the smallest bodies known in the Solar System, but they are also one of the most important and interesting topics of research in today's Celestial Mechanics. Comets are believed to carry material from the time the Solar System was formed, which means that a detailed study of that material could help us to answer many basic questions about this important process. There are also some speculations about connections between the impact of the comets and the origin of life on Earth.

The comets are so small, that they are only detected when they are passing near their pericenter. At this point, the heat coming from the Sun creates a long tail (formed by the material that leaves the weak gravitational field of the body) that makes them visible. This fact makes the estimation of their quantity a very difficult and unsolved problem. The literature almost agrees that a good estimate for a lower limit for their population is about

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10^{11} comets related to the Solar System, but several researchers believe that they are much more numerous. The actual observations show that three or four new comets are observed in the interior Solar System every year.

The origin of the comets is still not explained. They can be formed in the interstellar space and then they are captured by the Solar System (Laplace [1]; McCrea [2]; Yabushita and Hasegawa [3]) or they can be formed in the Solar System and then they are expelled to the interstellar space. One of the most popular ideas about the origin of the comets is the existence of a large cloud of comets around the Solar System, called the "Oort Cloud", with a total of 10^{11} comets at distances up to 10^5 AU (Oort [4]; Bailey [5]).

Our paper has the goal of giving a contribution to the problem of captures and escapes of comets caused by a close encounter with the planet Jupiter. The topics of escape and capture of comets have been discussed in the literature for a long time. In the majority of the papers, a close approach with a large planet, usually Jupiter, is the core of the mechanism of capture and escape. The dynamics are explained in papers as old as Russell [6] and Woerkom [7], who derived an expression to calculate the effects of a close approach between Jupiter and one comet.

On the basis of the number of comets detected up to now, the Solar System seems to have more short period comets (period less than 200 years) than it should have, based in the actual flux of near-parabolic comets coming to the Solar System and the mechanism of capture known (Bailey [8]). Several theories were developed to explain this fact. Hills [9] suggests that we can be living in a period of a large intensity of comets coming from the Oort cloud to the Solar System, that is known as "The Comet Shower Theory". Another possible explanation is the fragmentation of an original large body, that would generate a large number of smaller bodies. A third option would be to have another source of comets in the Solar System and/or another mechanism of capture, but there is no strong evidence of any of those new sources or mechanism.

2. ORGANIZATION OF OUR PAPER

Our paper is organized in seven sections, covering historical background, theoretical development and numerical results.

In the first section (Introduction) we give some background information about the problem and we list some of the most important papers related to our present research.

In the second section (the present one) we make a short abstract of each section of our paper, to allow a fast understand of the topics presented.

In section number three (A short historical review of comet research) we give a brief description of some of the most important papers about comet research in general. Our goal is to inform a reader not familiar with this topic about some of the major steps done in the history of this field and in the recent years.

In section number four (Energy change due to a close approach with a moving body) we explain in some detail the theoretical foundations of the slingshot effect. We derive the basic equations involved (the expression for the change in energy due to the close approach) and we show a summary of conclusions that comes from the equations derived.

In section number five (a short historical review of gravity assist theory for comets) we give some more historical information. In this case the information is about the use of "gravity-assist" maneuvers in cometary exploration. We again describe several papers available in the literature.

In section number six (The restricted three-body problem) we define and show the basic theory about the restricted three-body problem. We explain the systems of units and reference used and we derive the equations required for the present research.

In section number seven (The numerical integrations) we apply our equations in some practical cases. We present letter-plots that describe the effect of the slingshot as a function of the parameters involved and we do some statistics on the number of captures and escapes that resulted from the close approach.

3. A SHORT HISTORICAL REVIEW OF COMET RESEARCH

Looking back in history, the dynamics of comets were studied by many important astronomers. A special historical remark has to be made for the Halley's comet (named after Edmond Halley). Its appearance of 1758 was calculated in advance by Edmond Halley, with the use of the Newton's law of gravitation and the data that Halley observed during the passage of the same comet in 1682. It was the first time that a comet had its passage predicted in advance and a major victory for Newton's law and the science of Celestial Mechanics. In more recent years many important papers appeared in the literature, some of them are discussed in the following paragraphs.

Lyttleton and Hammersley [10] study the loss of long period comets from the Solar System. They treat the problem statistically, with successive energy-changes regarded as steps in a random-walk process, with the interval between successive returns varying inversely as the three-halves power of the energy. They use the Monte-Carlo analysis for a number of cases with energy changes selected randomly from an appropriate distribution and conclude that the percentage of comets that remains in the Solar System after M million years tends to be about $20M^{-2/3}$.

Valtonen and Innanen [11] use a regularized version of the three-body problem to integrate numerically a large number of orbits of comets. He uses his results to derive empirical velocity dependent expressions for the capture cross section of the system. He concludes that a formation of a cloud of comets, like the Oort Cloud, is only possible if the Solar System had a prolonged transit through a relatively large and dense cloud of comets.

Yabushita is one of the researchers that gave more contributions in this topic. In Yabushita [12] he studies the planetary perturbation on the orbits of long-period comets with large perihelion distance using a Monte-Carlo approach. He shows that 90% of the comets with perihelion distance less than 18 AU will be expelled from the Solar System in 4.10^9 years. In Yabushita [13] he calculates the dependence on inclination of the planetary perturbations of the orbits of long-period comets, by assuming that their orbits are parabolic and the Lyttleton-Hammersley [10] random-walk theory of cometary energy is valid. He concludes that the perturbation of the binding energy is greater for direct than retrograde orbits, so comets in direct orbits should be expelled from the Solar System earlier than those in retrograde orbits. In Yabushita [14] he uses a Gaussian distribution to represent the energy perturbations that come from the major planets and a Monte Carlo approach to follow the energy of the comets until they escape from the Solar System. He finds approximate analytical solutions for the number of comets remaining in the Solar System after a certain number of pericenter passages. He also concludes that after 3 or 6 Myears, the energy distribution of comets left in the system is almost independent of the initial distribution and there is a strong concentration in the range $0 < 1/a < 5.10^{-5}$ AU^{-1}. In Yabushita [15] he uses an analytical expression for the diffusion of comets in energy space, as a function of time and semi-major axis. Then he solves that equation to find an analytical expression that gives the number of comets as a function of time and semi-major axis. In another paper, Yabushita [16], he studies the perturbations of the major planets and the passing stars in the dynamical evolution of cometary orbits, with emphasis on escape of comets from the Solar System; evolution from nearly parabolic to short period orbits; evolution of the distribution of binding energies and dynamical perturbation by passing stars.

A. Carusi et. al. [17] uses the Öpik's [18] assumptions to describe a close encounter between a comet and a major planet in the three dimensional space. They assume that the orbital elements of the orbit after the encounter are given by a two-body scattering between the planet and the comet and that the Sun is the only body to act in the comet's trajectory before and after the encounter. This model allows prediction of the distribution of orbital elements after the encounter, that generate information for many kinds of statistical analyses.

Matese and Whitman [19] study the effects of the Galactic tidal interaction with the Oort cloud, trying to find a mechanism to explain why some comets fall down into the Solar System to make a close approach with the large planets.

Valtonen et. al. [20] describe his plan of action and initial results of the work to devise cross sections by carrying out large numbers of comet-planet encounters and by deriving approximate analytic expressions for them. The comets are supposed to start in parabolic orbits of arbitrary inclination and perihelion distance. He uses the restricted-circular-three-body model with K-S regularization.

Petrosky and Broucke [21] study the problem of captures and escapes by combining the theory of area-preserving mappings and the restricted-three-body model. They derive a long-term dynamical model from it. They show that the motion of the comet in nearly parabolic orbit is chaotic and they study it in details.

4. ENERGY CHANGE DUE TO A CLOSE APPROACH WITH A MOVING BODY

The problem deals with the flyby of a comet near a planet. The "gravitational assist" of the planet modify the orbit of the comet to give it some additional energy or to subtract energy. This is called the slingshot effect in modern space dynamics.

The present section gives a small example which illustrates the basic principle of Energy increase or decrease during a flyby of a moving body. It justifies the following fundamental conclusions: When a particle has a close approach with a receding massive body, it gains energy, while, if the massive body is approaching, the particle will lose energy.

Let us assume that Q is for a massive planet and P is the comet with negligible mass. In other words P has no effect on the motion of Q. The comet P has a close approach with the planet Q. A correct treatment of the problem requires the restricted three-body problem. The present treatment is somewhat simplified, but we will see that it illustrates the idea of energy increase or decrease during a planetary flyby, and it leads to the correct conclusions.

Q is at the location $(x_1(t), y_1(t))$. The force is the usual inverse square attraction with potential function $U=\mu/r$ or with potential energy $V=-\mu/r$. The whole system is referred to an inertial system of reference with origin O (Fig. 1).

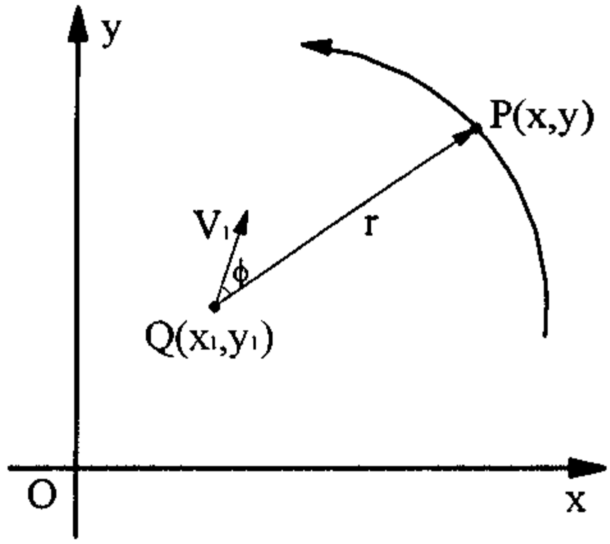


Fig.1 - A close approach between P and Q

The equations of motion of P are thus:

$$\ddot{x} = \frac{\partial U}{\partial x} = -\mu\frac{x}{r^3}; \quad \ddot{y} = \frac{\partial U}{\partial y} = -\mu\frac{y}{r^3} \tag{1}$$

where $r^2 = (x-x_1)^2 + (y-y_1)^2$

The question is now the variation of the energy

$$E = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - U\left(r(x,y,t)\right) \tag{2}$$

of the particle at P. This is obtained with the use of the energy equation

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} = \frac{\partial V}{\partial t} = \frac{\mu}{r^2}\frac{\partial r}{\partial t} \tag{3}$$

However, we have

$$r\frac{\partial r}{\partial t} = -(x - x_1)\frac{\partial x_1}{\partial t} - (y - y_1)\frac{\partial y_1}{\partial t} = -(x - x_1)\dot{x}_1 - (y - y_1)\dot{y}_1 \quad (4)$$

Therefore, the energy equation is:

$$\frac{dE}{dt} = -\frac{\mu}{r^3}\left[(x - x_1)\dot{x}_1 + (y - y_1)\dot{y}_1\right] \quad (5)$$

In the brackets we recognize the dot product of the relative position $\bar{r} = \overline{PQ}$ of **P** with the absolute velocity vector $V_1(\dot{x}_1, \dot{y}_1)$ of the perturbing body Q. The energy equation can then be written as

$$\frac{dE}{dt} = -\frac{\mu}{r^3} rV_1 Cos(\theta) = -\frac{\mu V_1}{r^2} Cos(\theta) \quad (6)$$

From this equation we now can come to an important conclusion about the increase or decrease of the energy of the particle P. This essentially depends only on the factor Cos(θ) in the above equation: when θ is below 90°, the energy decreases and when θ is above 90°, the energy increases. This result can be summarized as follows:

When Q approaches: E decreases

When Q recedes: E increases

5. A SHORT HISTORICAL REVIEW OF GRAVITY ASSIST THEORY FOR COMETS

The astronomers have known the effect that Jupiter has on cometary orbits during a close approach. The literature on cometary orbits or perturbations shows clearly that these astronomers understood the slimgshot effect very well.

Among the first mathematical papers on cometary perturbations are several articles in French by Jean le Rond d'Alembert (who lived from 1717 to 1783). One of the first articles is in his "Opuscules", Memoir 50, Vol. 6, ±1773, called "On the Orbit of the Comets," pages 304 to 311. A second article, also in his Opuscules is Memoir 58, in Vol. 8, pages 231 to 269. It is called "On the perturbations of the Comets." He examines problems such as the perturbation of the period by Jupiter, the possibility of capture by Jupiter and the probability of falling into the Sun.

Another important early contribution to Cometary orbits is by Laplace in Vol. 4 of his 5-Volume treatise: "Mécanique Céleste." This is in chapter 2, in Part 2, Vol. 4, pages 217-229, published about 1795.

Laplace lived from 1749 to 1827. The title of the chapter is: "Perturbation of the motion of Comets when they approach very close to the planets." He derives his concept of sphere of influence there. He derives formulas for the change in mean motion during the Jupiter flyby. He has numerical examples relative to the comet of 1770. He also considers a close approach with the Earth and he says that the period was decreased by two days (it was 2042 days). This is the last sentence of his chapter.

U. G. Leverrier (1847) has two ten-page [22],[23] memoirs in the "Comptes Rendus," Volume 25 (1847). Neither one of them contains any equations. However, his detailed explanations of the perturbations of Jupiter on a comet are evidence of his deep understanding of their mechanism. The second memoir contains the sentence (page 918) that "It is easy to see that if the comet follows Jupiter, the action of the planet increases its period, while, if the comet preceeds the planet, the period of the comet is decreased." Some of his more detailed publications are in the Annals of the Paris Observatory (Vol. 3, on the theory of the periodic comet of 1770).

F. Tisserand (1889) studied Jupiter's action on cometary orbits, especially the possibility of capture. It is in this paper that he gives what we now call the "Tisserand Criterion," which is just another form of the Jacobi integral, expressed in terms of three orbit elements, a, e, i. Tisserand was born in 1845 and died in 1896, shortly after the publication of the last of his four volumes on Celestial Mechanics (1889, 1891, 1894 and 1896).

Another series of remarkable articles is by M. O. Callandreau [24],[25],[26], in the Annals of the Paris Observatory, in 1892 (Memoir N° 20), and in 1902 (Memoir N° 24). The first is 63 pages long, with the title "Theory of Periodic Comets." He is especially interested in the possibility of Capture by Jupiter. On page B.38, he gives the formula for the energy change and he mentions the difference of passing in front or behind Jupiter. He also gives the formula for the Delta-V in terms of the bending angle of the hyperbola. He gives at least a dozen different formulas for the energy change during the passage through the sphere of influence.

Starting the early 19-hundreds (1914), we find a series of articles by E. Stromgren and his collaborators at the Copenhagen Observatory. Publications 19 and 20 in 1914 as well as his Copenhagen report 144 of 1947 all contain formulas for the change of the semi-major axis of the cometary orbit.

In the last 30 years we also have many articles by E. Everhart, by S. Yabushita and by M. Valtonen. An article by A. J. J. Van Woerkom [27] (1948) contains many formulas for the change of semi-major axis. However all these formulas are not easy to use because they contain definite integrals. We do not have enough space to describe these works in details here.

6. THE RESTRICTED THREE-BODY PROBLEM

The primary problem that we study in our paper is to find under what conditions a comet coming from outside the Solar System is captured. To solve this problem, we assume that the Solar System is formed only by three bodies: the Sun, a planet (Jupiter) and a third particle of negligible mass (the comet). We also assume that the total system (Sun + Jupiter + comet) obeys all the hypothesis of the planar-restrict-circular-three-body problem, which means that: all the bodies are point masses; the Sun and Jupiter are in circular orbits around their mutual center of mass; the motion of the comet is governed by the two main bodies, but the comet does not interfere with their motion; the comet moves only in the orbital plane of the two primaries.

After these assumptions, our problem becomes to be the study of the motion of the comet near the position where it crosses the line between the Sun and the planet Jupiter.

We have to study its motion only near this point, because when the comet is far from Jupiter the system is governed by a two-body (Sun + comet) problem dynamics, that does not allow any change in energy. In particular, we will be looking for the energy of the comet before and after this crossing point, to detect under what conditions a comet is captured (change its energy from positive to negative). Since our initial conditions for the comet vary in a very large range, we are also able to detect under what conditions a comet is expelled from the Solar System (change its energy from negative to positive) or have a modification in its energy without changing its type of orbit (a change in its energy that is not strong enough to modify the sign of the energy). Fig. 2 shows the geometry involved in the crossing and defines the basic variables used in our paper.

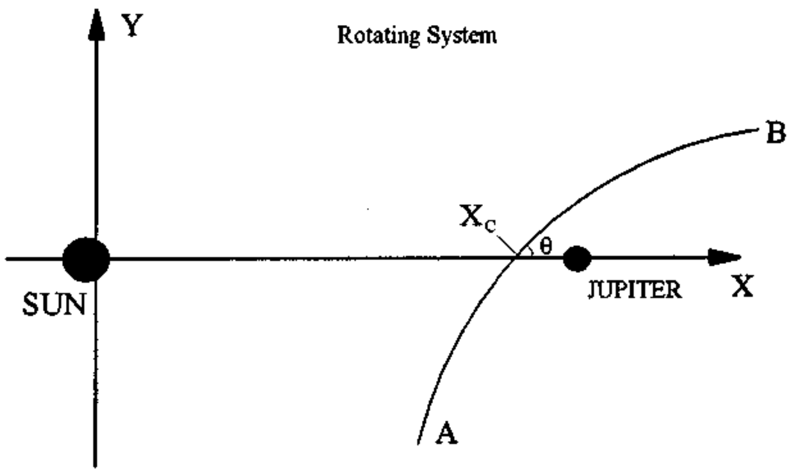


Fig. 2 - Geometry of the trajectory of the comet when it is crossing the line Sun-Jupiter

We can see that the comet leaves the point A, crosses the horizontal axis (the line between the Sun and the planet Jupiter) at the point (X_C,0), making an angle θ (measured from the horizontal axis in the counter-clock-wise direction) and goes to the point B. We choose the points A and B in a such way that we can neglect the influence of Jupiter at those points and, by consequence, we know that the energy is constant after B and before A. Under those assumptions, the procedure involved to solve this problem will be: i) To specify particular arbitrary values for the Jabobian constant, the ordinate of the crossing point (X_C) and the angle θ; ii) Starting with the comet in the line Sun-Jupiter, we integrate numerically its orbit forward in time until it reaches the point B; iii) Starting again in the crossing line (point (X_C,0)) we integrate its orbit backward in time until the comet reaches the point A; iv) At the points A and B we calculate the energy of the comet, and we can check for the effects of this close approach with Jupiter.

6.1. Mathematical Model and Algorithm

The equations of motion for the comet are assumed to be the ones valid for the well-known planar-restricted-circular-three-body problem. We also use the standard canonical system of units, what implies in the following definitions:

i) The unit of distance is the distance between M_1 (the Sun) and M_2 (Jupiter);

ii) The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one;

iii) The mass of the smaller primary (M_2) is given by $\mu = \frac{m_2}{m_1 + m_2}$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is $(1-\mu)$, to make the total mass of the system unitary;

iv) The unit of time is defined such that the period of the motion of the two primaries is 2π;

v) The gravitational constant is one.

Table 1 shows the Canonical system of units for the Sun(M_1)-Jupiter(M_2) system that is used in this paper.

Table 1 - Canonical system of units for the Sun-Jupiter system

Unit of distance	778000000 km
Unit of time	689.567 days
Unit of velocity	13.058 km/s

We use the rotating system as our reference system. It is a system with its origin in the center of mass of the two primaries and the horizontal axis lies in the line connecting the two primaries, pointing to m_2. The vertical axis is perpendicular to the horizontal axis and completes the usual right-handed-side system.

Based in those systems of units and reference, the equations of motion of the comet are:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \quad (7)$$

$$\ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \quad (8)$$

where Ω is the pseudo-potential given by:

$$\Omega = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \quad (9)$$

Since the basis of our research consists of close encounters with Jupiter, we use Lamaître's regularization [28] in those equations. The reason is to avoid numerical problems during the close encounters with Jupiter, since it is one of the singularities in the equations of motion of the comet.

Another important result that we need in this paper is the constant of motion known as the "Jacobian Integral", that is an invariant in the circular-planar-restricted-three-body problem and it is given by the equation:

$$J = E - \omega . C = \frac{\dot{x}^2 + \dot{y}^2}{2} - \frac{x^2 + y^2}{2} - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} \tag{10}$$

where E is the energy, C is the angular momentum, r_1 is the distance between the comet and the Sun, r_2 is the distance between the comet and Jupiter and ω is the angular speed of the system. The expressions for the energy and the angular momentum, that will be needed later, are given by:

$$E = \frac{(x+\dot{y})^2 + (\dot{x}-y)^2}{2} - \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{11}$$

$$C = x^2 + y^2 + x\dot{y} - y\dot{x} \tag{12}$$

where x, y and its derivatives are the coordinates of the comet in the rotating system.

With those equations, we can build a numerical algorithm to solve our problem. It has the following steps:

i) We give arbitrary values for the three parameters: J, X_C, θ;

ii) With these values we compute the initial conditions in the rotating system. The initial position will be the point (X_C, 0) and the initial velocity will be ($V\cos\theta$, $V\sin\theta$), where $V = \sqrt{\dot{x}^2 + \dot{y}^2}$ is calculated from equation (10);

iii) With these initial conditions, we integrate the equations of motion forward in time until the distance between the planet Jupiter and the comet is bigger than a specified limit d_{JC}. At this point we stop the numerical integration and we calculate the energy (E_+) and the angular momentum (C_+) after the encounter with Jupiter, from equations (11) and (12). Remember that we assumed that the energy and the angular momentum is constant after this point, due to the fact that the perturbation from Jupiter is to small to disturb significantly the two-body character of the dynamics;

iv) Then we go back to the initial conditions at the crossing point, and we integrate the equations of motion backward in time, until the distance d_{JC} is reached again. Then we use the equations (11) and (12) to calculate the energy (E_-) and the angular momentum (C_-) before the encounter with Jupiter.

v) With those results, we have all the information required to calculate the change in energy (E_+-E_-) and angular momentum (C_+-C_-) due to the close approach with Jupiter.

With this algorithm available, we can vary the given initial conditions (values for J, X_C and θ) in any desired range and study the effects of the close approach with Jupiter in the orbit of the comet.

7. THE NUMERICAL INTEGRATIONS

Our results consist of plots that show what happens to the comet after the close encounter with the planet for a large range of given initial conditions. First of all we have to classify all the close encounters between the planet and the comet, according to the change obtained in the orbit of the comet. We use the letters H, B, E, h, b, c and O for this classification. They are assigned to the orbits according to the rules showed in Table 2.

Table 2 - Rules for the assignment of letters to orbits

Letter	Energy	Orbit before	Orbit after
h	decrease	hyperbolic	hyperbolic
b	decrease	elliptic	elliptic
c	decrease	hyperbolic	elliptic
H	increase	hyperbolic	hyperbolic
B	increase	elliptic	elliptic
E	increase	elliptic	hyperbolic

We also use the letter O for an orbit that stays around Jupiter for a long time. In this case the comet became a temporary satellite of Jupiter. This is a case with little interest in our present research.

With those rules defined, our results consist of assigning one of those letters to a position in a two-dimensional diagram that has the parameter X_C in the horizontal axis and the parameter θ in the vertical axis. We make one plot for each desired value of the Jacobian constant.

To decide the best range of values for these three parameters we made several exploratory simulations. We noticed that for values of the Jacobian constant greater than 0.8, the number of "captures" is very small, and for values below -1.45 a large number of orbits remain around Jupiter for a long time. So, we decided to use the range $-1.45 < J < 1.10$ for this parameter, and we make 18 plots in steps of 0.15 for J.

For X_C, we noticed that the values of interest (large number of captures) are between 0.8 to 1.2. Outside this range, the effect of the close approach is too small to be considered, due to the large distance from the comet to Jupiter. Taking that in consideration, we decided in favor of a little wider range, and we confined the ordinate of the crossing point to the internal $0.7 < X_C < 1.3$.

For the angle θ, we noticed that there is a symmetry with respect to the line $\theta = 90$ °. This symmetry comes from the fact that a trajectory with an angle θ is physically the same as a trajectory with an angle $\theta+180°$ and the direction of time reversed. It means that the change in energy and angular momentum for these two trajectories has the same magnitude, and they differ only by the sign. Fig. 3 shows this symmetry for the case $J = -1.3$.

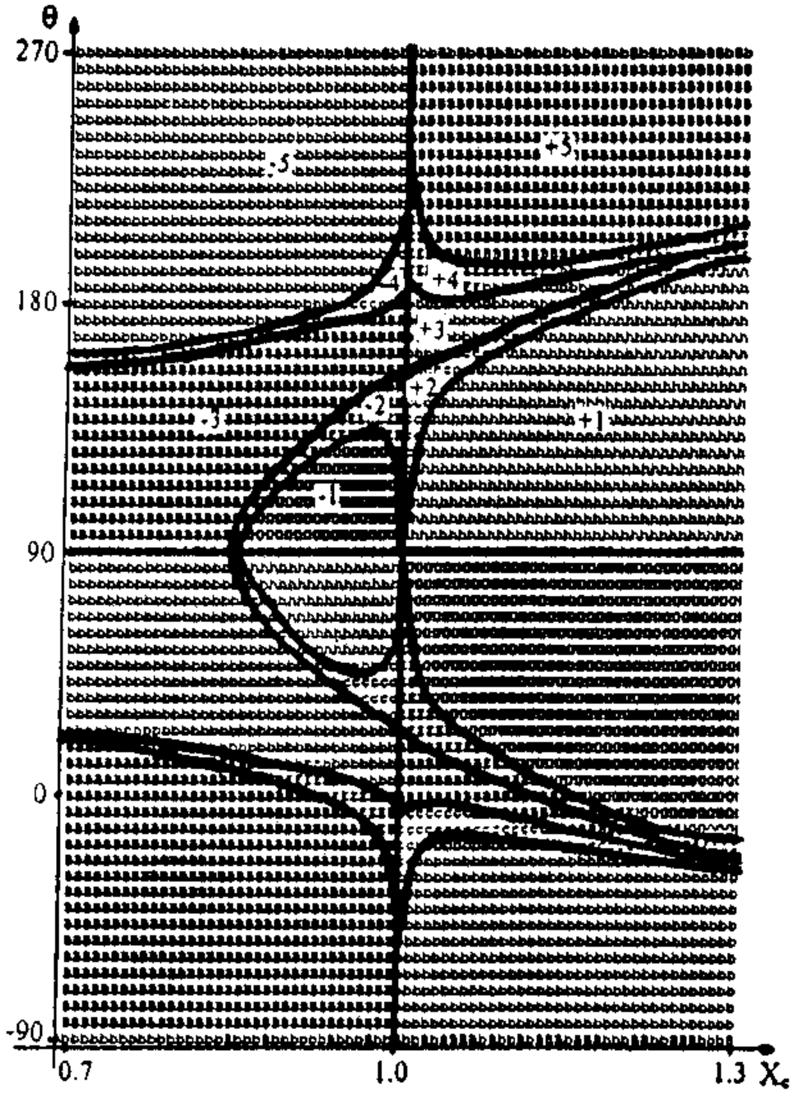


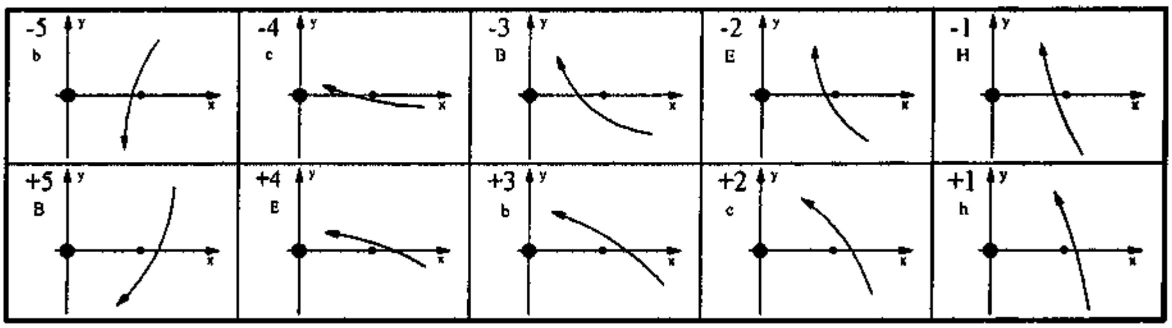
Fig. 3 - Results for J = -1.30

We can see clearly that the rules for the symmetry between the upper side of the diagram (90° < θ < 270°) and the bottom side (-90° < θ < 90°) is: **H** ⇔ h, **B** ⇔ b and **E** ⇔ c.

We decide to keep θ in the interval 90° < θ < 270° (the upper side of the diagram) and the rest (-90° < θ < 90°) can be derived by the symmetry rules.

From Fig. 3 we can also see the existence of several distinct regions. We name them with the numbers 1 to 5, starting from the line θ = 90° and going in the sense of increasing θ. We also use the arithmetic symbols "-" and "+" to indicate if the orbit crosses the Sun-Jupiter line in a point located between the two main bodies or outside Jupiter's orbit around the Sun. For example, orbit named "-1" stands for an orbit that crosses the Sun-Jupiter line almost perpendicular to this line (θ is a little more then 90°) in a point located between the Sun and Jupiter. For higher numbers the angle θ increases for each location of the crossing point. For orbits named with positive signs, the crossing points go to the other side of Jupiter. Table 3 shows a sketch of each type of orbit.

Table 3 - A sketch of each type of orbit that we found



A brief description of each region and border follows.

Region -1: It is formed by trajectories of type H. This is a type of orbit that is hyperbolic before and after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes the comet to leave the Solar System faster than it approaches it.

Region -2: It is formed by trajectories of type E. This is a type of orbit that is elliptic before and hyperbolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that belongs to the Solar System to escape from it.

The border between the regions -1 and -2: This border is formed by orbits that are parabolic before and hyperbolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that is in the limit of belonging or not to the Solar System to escape faster from it.

Region -3: It is formed by trajectories of type B. This is a type of orbit that is elliptic before and after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes the comet to increase its semi-major axis and to go to the direction of the outer Solar System.

The border between the regions -2 and -3: This border is formed by orbits that are elliptic before and parabolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that belongs to the Solar System to go to the limit of belonging or not to it and to slowly escape to the interstellar space. This makes this case to be considered a maneuver of escape.

Region -4: It is formed by trajectories of type c. This is a type of orbit that is hyperbolic before and elliptic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that does not belong to the Solar System to be captured and become part of it.

The border between the regions -3 and -4: This border is formed by orbits that are parabolic before and elliptic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that is in the limit of belonging or not to the Solar System to be completely captured and become part of it. This makes this case to be considered a maneuver of capture. This is also the line that divides gains and losses in energy. Below this line the comet always gains energy and above this line it always losses energy.

Region -5: It is formed by trajectories of type b. This is a type of orbit that is elliptic before and after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet to decrease its semi-major axis and go to the direction of the interior of the Solar System.

The border between the regions -4 and -5: This border is formed by orbits that are parabolic before and elliptic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that is in the limit of belonging or not to the Solar System to be completely captured and become part of it. This makes this case to be considered a maneuver of capture.

Region +1: It is formed by trajectories of type h. This is a type of orbit that is hyperbolic before and after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes the comet to leave the Solar System slower than it approaches it.

Region +2: It is formed by trajectories of type c. This is a type of orbit that is hyperbolic before and elliptic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that does not belong to the Solar System to be captured and become part of it.

Region +3: It is formed by trajectories of type b. This is a type of orbit that is elliptic before and after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet to decrease its semi-major axis and go to the direction of the interior of the Solar System.

Region +4: It is formed by trajectories of type E. This is a type of orbit that is elliptic before and hyperbolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that belongs to the Solar System to escape from it.

Region +5: It is formed by trajectories of type B. This is a type of orbit that is elliptic before and after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes the comet to increase its semi-major axis and to go to the direction of the outer Solar System.

The border between the regions +1 and +2: This border is formed by orbits that are hyperbolic before and parabolic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that does not belong to the Solar System to go to the limit of belonging or not to it.

The border between the regions +2 and +3: This border is formed by orbits that are parabolic before and elliptic after the close encounter with Jupiter. There is a loss in energy in that maneuver, what makes a comet that is in the limit of belonging or not to the Solar System to be captured and become part of it. This makes this case to be considered a maneuver of capture.

The border between the regions +3 and +4: This border is formed by orbits that are elliptic before and parabolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that belongs to the Solar System to go to the limit of belonging or not to it. This makes this case to be considered a maneuver of escape. This is also the line that divides gains and losses in energy. Below this line the comet always losses energy and above this line it always gains energy.

The border between the regions +4 and +5: This border is formed by orbits that are elliptic before and parabolic after the close encounter with Jupiter. There is a gain in energy in that maneuver, what makes a comet that belongs to the Solar System to go to the limit of belonging or not to it. This makes this case to be considered a maneuver of escape.

The horizontal line $\theta = 90°$ is formed by orbits that have no change in energy. The close encounter with Jupiter has no effect in the trajectory of the comet. This is also the line of symmetry of the figure. The orbits below this line are the same ones above it with a reverse of time.

The vertical line $X_C = 1.0$ is the line that reverses gains and losses in energy. For a given θ, the left side has orbits with gain in energy, the right side has orbits with loss in energy and vice-versa. This line is close to the position of Jupiter, what means that crossing it is equivalent to move from one side to another of Jupiter with respect to the Sun.

Fig. 4 shows a series of diagrams covering the desired range for all the three variables. Plots for some values of J are omitted, since they are very similar to each other.

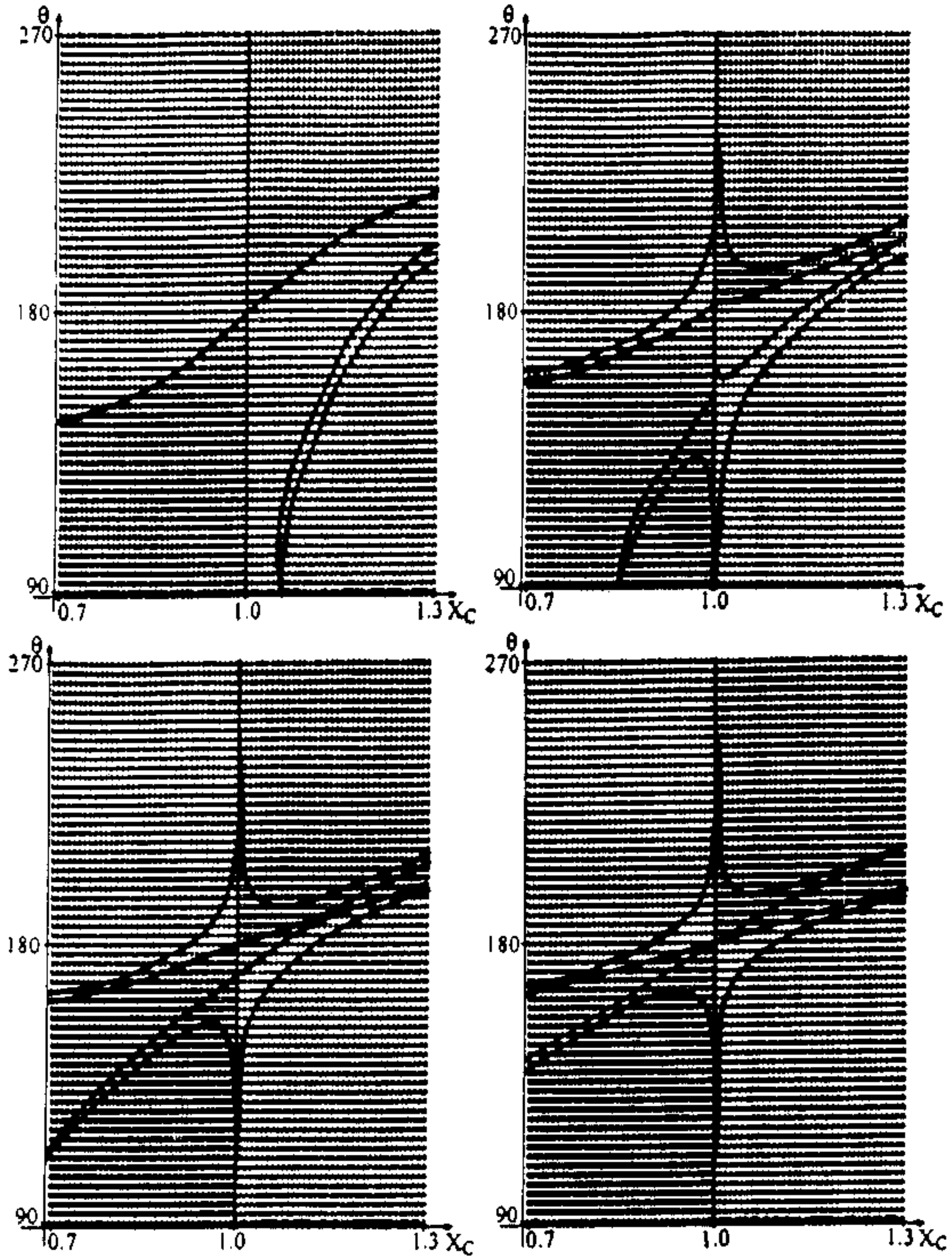


Fig. 4 - Results of the Close Approach Between Jupiter and the Comet (J = -1.45, -1.30, -1.15, -1.00, Starting at the Top-Left).

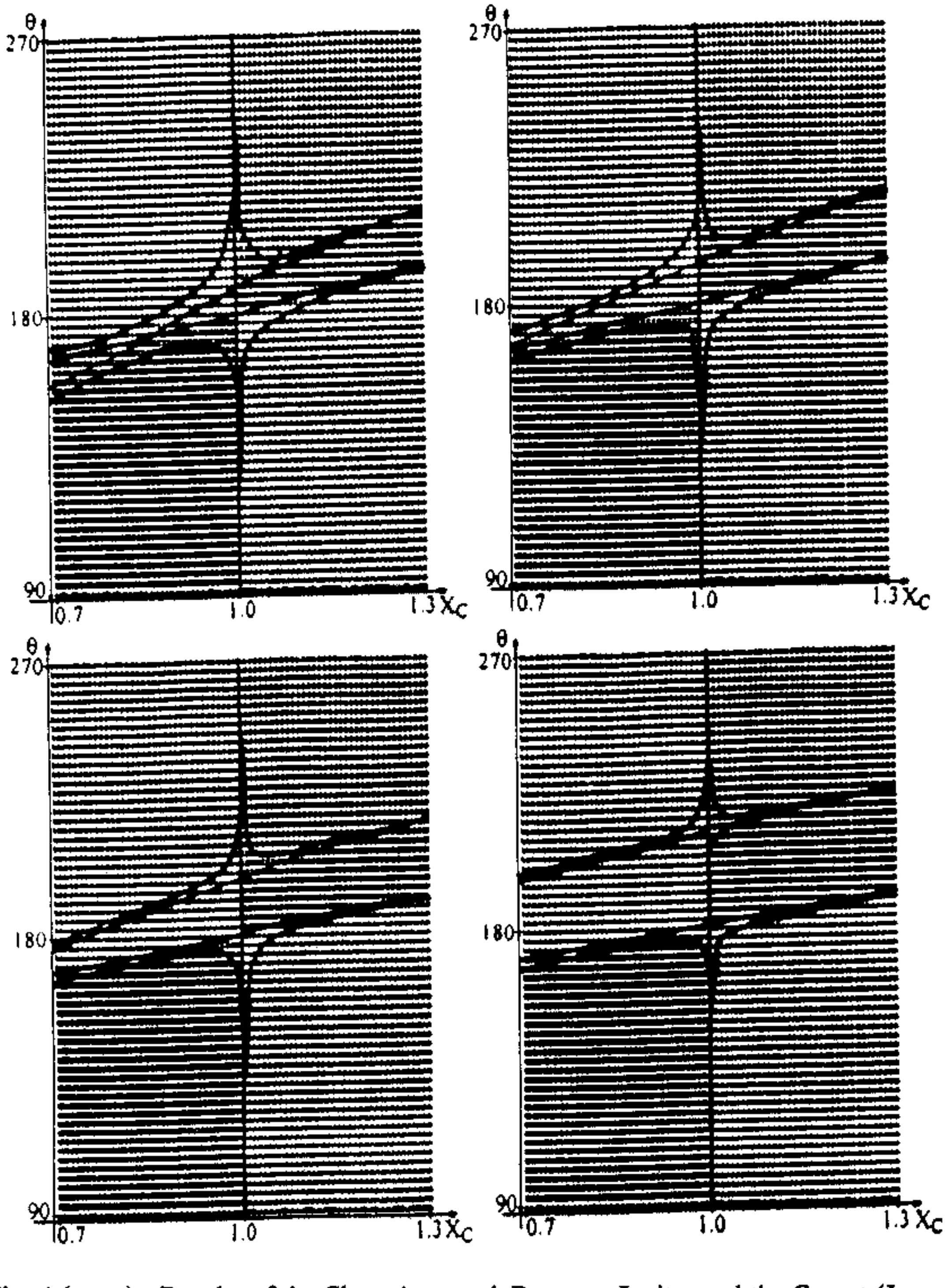


Fig. 4 (cont.) - Results of the Close Approach Between Jupiter and the Comet (J = -0.85, -0.70, -0.55, -0.10, Starting at the Top-Left).

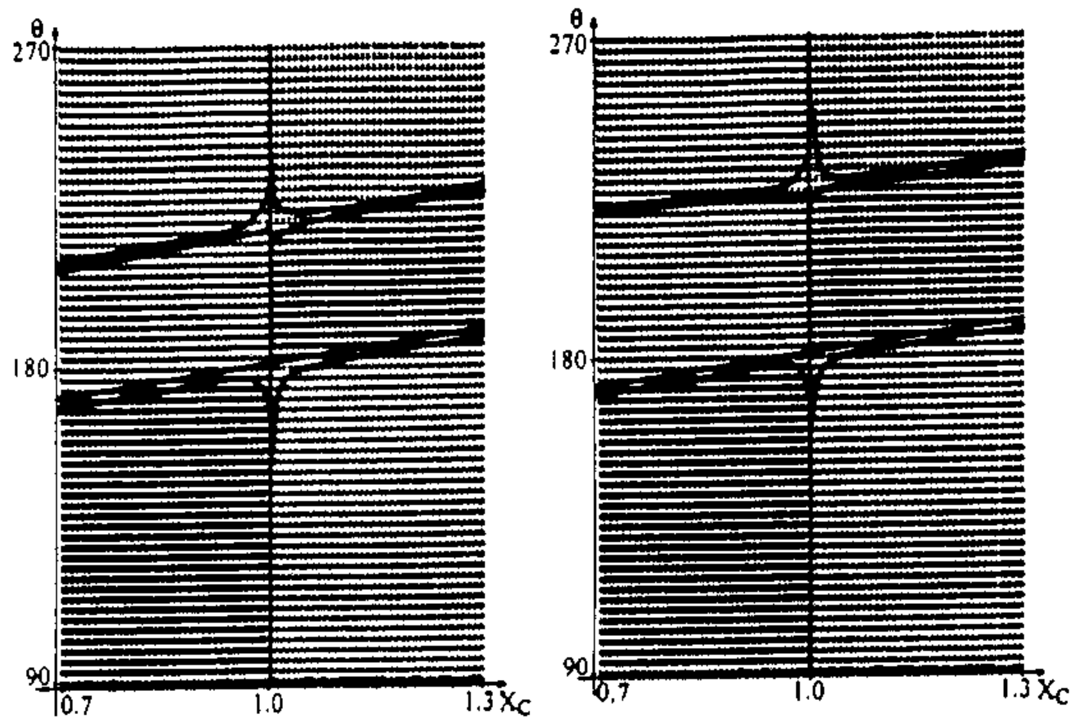


Fig. 4 (cont.) - Results of the Close Approach Between Jupiter and the Comet (J = 0.20, 0.50, Starting at Left).

To have a better understanding of the process, we plotted some of the trajectories in the rotating and fixed frame in Fig. 5. Table 4 gives some of the numerical data for those trajectories, including the initial conditions.

We can see that the number of captures and escapes (letters "c" and "E") has its maximum at J = -1.15 and it decreases fast when increasing or decreasing the value of the Jacobian constant. Assuming that comets traveling in this plane will cross the horizontal axis with a homogeneous distribution of crossing angles θ and ordinates of crossing points X_C, we can estimate the probability of capture by counting the number of capture and escape orbits in this diagram (remember that an escape in the upper part of the diagram has a counterpart capture in the bottom part). Fig. 6 shows the occurrence of each type of orbit as a function of the Jacobian constant and the probability of capture (total number of orbits type "c" and "E" divided by the total number of orbits). Remember that the above called "probability of capture" is the chance that a comet travelling in the orbital plane of the primaries and crossing the Sun-Jupiter line with $0.7 < X_C < 1.3$ is captured. It is not a chance that a generic comet coming to the Solar System is captured.

With the data available, we can make several kinds of statistical analyses. In our paper we decide to make histograms that show the distribution of pericenter distance and the semi-major axis (both in Astronomical Units - AU) of the captured comets. They are shown in Fig. 7. We can see that more than 95% of the comets have a pericenter distance less than 5 AU, what makes them visible when passing by the pericenter of their orbits. We can also see that about 70% of those comets have a semi-major axis less than 34.2 AU, which implies in an orbital period less than 200 years, and it makes them short-period comets. We also include, in Fig. 8, a detailed statistic of the percentage of short-period comets for each value of the Jacobian constant used. We can see that there is a tendency in decreasing the percentage of short-period comets when we increase the Jacobian constant.

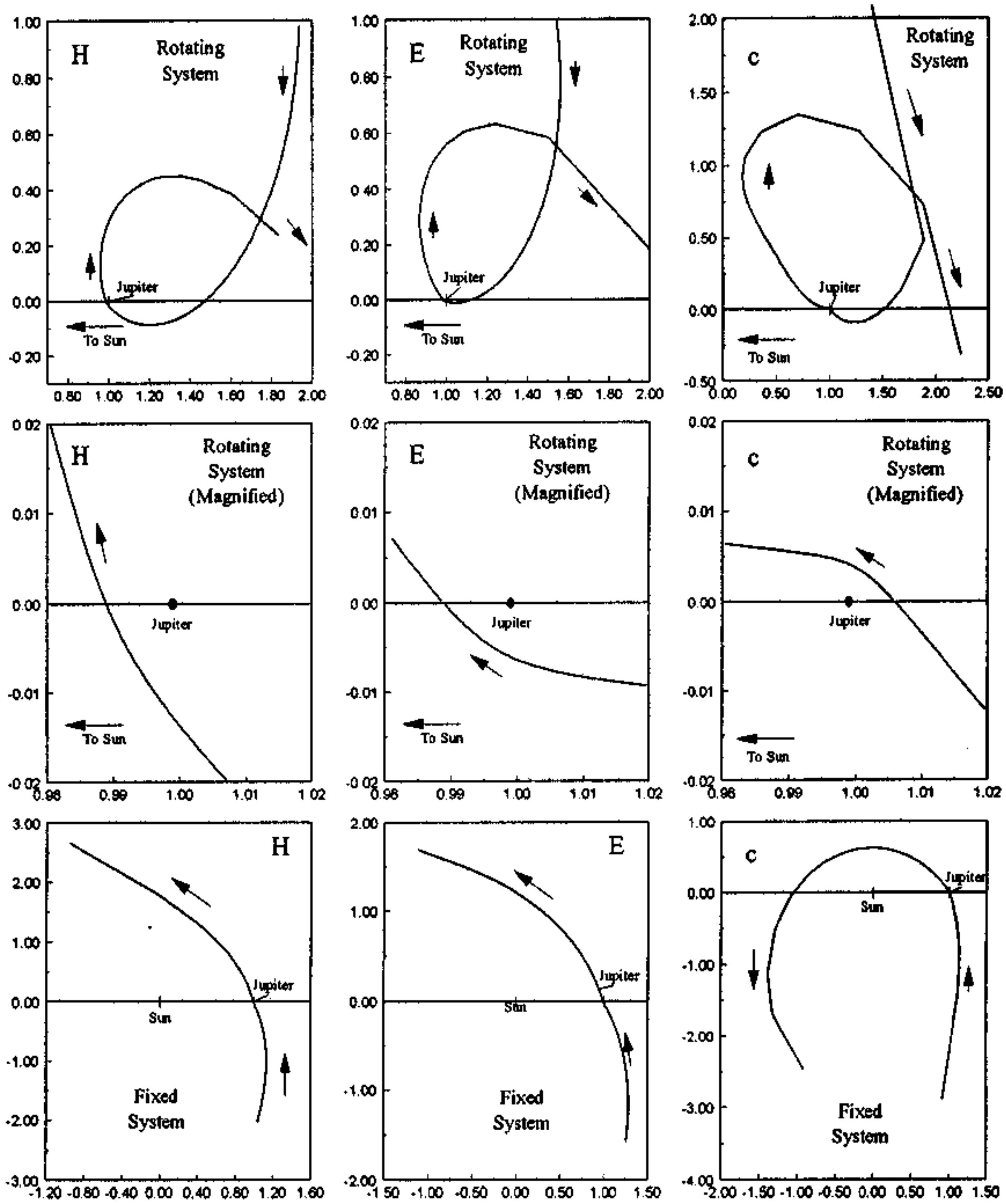


Fig. 5 - Some examples of trajectories (Orbit H is of type "-1", E is "-2" and c is "+2")

Table 3 - Numerical data for the orbits plotted in Fig. 5

Orbit	J	X_C	θ	E_-	E_+	ΔE	C_-	C_+	ΔC
c	-1.30	1.006	140.4	0.1690	-0.2876	-0.4566	1.4690	1.0124	-0.4566
H	-1.30	0.989	118.8	0.0987	0.2512	0.1525	1.3987	1.5512	0.1525
E	-1.30	0.989	140.4	-0.2232	0.1030	0.3262	1.0768	1.4030	0.3262

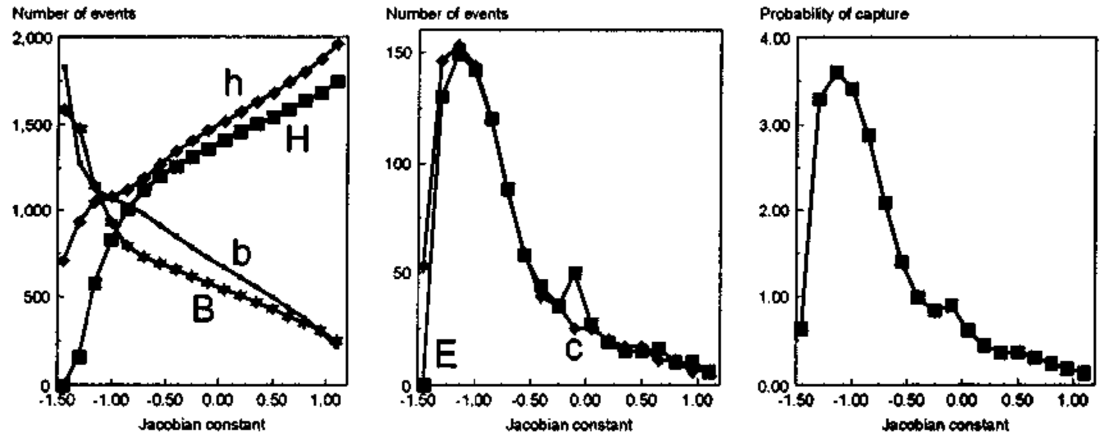


Fig. 6 - Occurrence of each type of orbit and probability of capture as a function of J

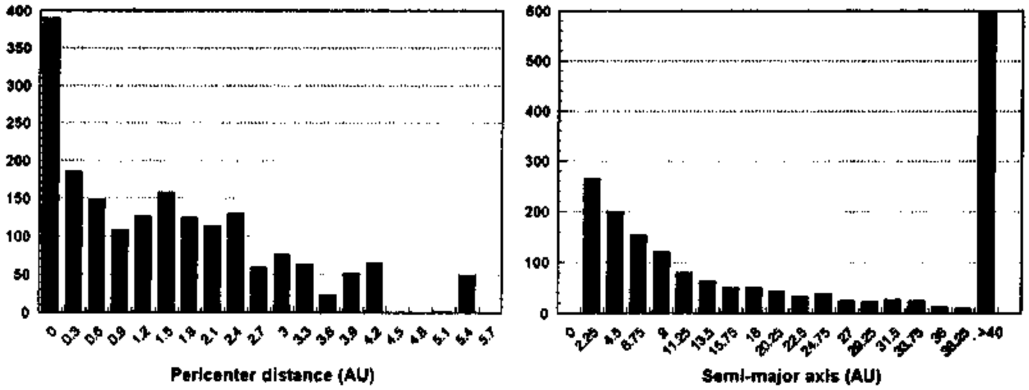


Fig. 7 - Distribution of pericenter distance and semi-major axis for the captured comets

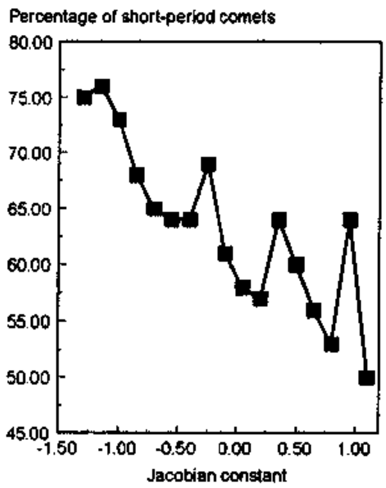


Fig . 8 - Percentage of short-period comets after capture

8. CONCLUSIONS

A numerical algorithm based in the circular-planar-restricted-three-body problem is developed to study the motion of a comet in a Solar System formed by the Sun, a planet (Jupiter) and one comet. The effect of the close approach between the planet and the comet is studied for several values of the initial conditions for the comet. We conclude that a comet can be captured by the Solar System due to this close approach, and the probability of capture goes from 0.2% to 3.6%, depending on the value of Jacobian constant. The details of this mechanism and some trajectories are shown. We also make some statistical analyses with the data available. We conclude that more than 95% of the captured comets have pericenter distance less than 5 AU (what makes them visible) and that about 70% of them are short-period comets.

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