THE DYNAMICS OF THE GRAVITY-ASSISTED MANEUVER

Antonio Fernando Bertachini de Almeida Prado
Research Engineer, Depto. of Space Mechanics and Control
Instituto Nacional de Pesquisas Espaciais - INPE - Brazil
São José dos Campos - SP - 12227-010 - Brazil

ABSTRACT

In the present paper we study the dynamics and classify the gravity-assisted (swing-by) maneuvers that use the Moon as the body for the close approach. The goal is to simulate a large variety of initial conditions for these orbits and classify them according to the effects caused by the close approach in the orbit of the spacecraft. The well-known planar restricted circular three-body problem is used as the mathematical model. The equations are regularized (using Lamaître's regularization), so it is possible to avoid the numerical problems that come from the close approach with the Moon.

Introduction

The importance of the gravity-assisted (or swing-by) trajectories can be very well understood by the number of missions that flew or are scheduled to fly using this technique. A very successful example is the Voyager missions that flew to the outer planets of the Solar System with the use of successive swing-bys in the planets visited to gain energy. In this paper, the same study done for the maneuvers involving the planet Jupiter in Broucke and Prado[1] is repeated for maneuvers involving the Moon. The maneuvers involving the Moon have many important practical applications. Those applications have potential for utilization in a shorter time than those involving the planet Jupiter.

Among the several sets of initial conditions that can be used to identify uniquely one trajectory, the same one used in the papers written by Broucke[2] and Broucke and Prado[1] is used here. It is composed by the following variables: 1) J, the Jacobian constant of the spacecraft (an integral of the restricted three-body problem); 2) The angle ψ, that is defined as the angle between the line M_1-M_2 (Earth-Moon) and the direction of the periapse of the trajectory of the spacecraft around the Moon; 3) r_p, the distance from the spacecraft to the center of the Moon in the moment of the closest approach to the Moon (periapse distance). Note that the Jacobi constant is essentially equivalent to the hyperbolic excess velocity V_∞, since they can be related by one single expression.

For a large number of values of these three variables, the equations of motion are integrated numerically forward and backward in time, until the spacecraft is at a distance that can be considered far enough from the Moon, such that the Moon's effect is neglected and the system formed by the Earth and the spacecraft can be considered a two-body system. At these two points, two-body celestial mechanics formulas are valid to compute the energy and the angular momentum before and after the close approach. Those quantities are used to identify up to sixteen classes of orbits, accordingly to the changes in the energy and angular momentum caused by the close encounter. They are named with the first sixteen letters of the alphabet.

The results are shown in letter-plots, where one letter describing the effects of the swing-by is plotted in a two-dimensional graph that has in the horizontal axis the angle ψ (the angle between the periapse vector and the Earth-Moon line) and in the vertical axis the Jacobian constant of the spacecraft. There is one plot for each value of the parameter r_p.

Definition of the Problem

The primary problem that is studied in this paper is to simulate and classify swing-by trajectories passing near the Moon. To solve this problem, it is assumed the existence of three bodies: the Earth, the Moon and a third particle of negligible mass (the spacecraft). It is also assumed that the total system (Earth + Moon + spacecraft) satisfies the hypothesis of the planar restricted circular three-body problem: all the bodies are point masses; the Earth and the Moon are in circular orbits around their mutual center of mass.

With these assumptions, the problem consists in studying the motion of the spacecraft near the close encounter with the Moon. It is necessary to study its motion only near this point, because when the spacecraft is far from the Moon the system is governed by two-body (Earth + spacecraft) dynamics, that has no change in energy or angular momentum. In particular, the energy and the angular momentum of the spacecraft before and after this close encounter are calculated, to detect the changes in the trajectory during the close approach. The orbits are classified in four categories: elliptic direct (negative energy and positive angular momentum), elliptic retrograde (negative energy and angular momentum), hyperbolic direct (positive energy and angular momentum) and hyperbolic retrograde (positive energy and negative angular momentum). The problem now is to identify the category of the orbit of the spacecraft before and after the close encounter with the Moon. Fig. 1 explains the geometry involved in the close encounter.

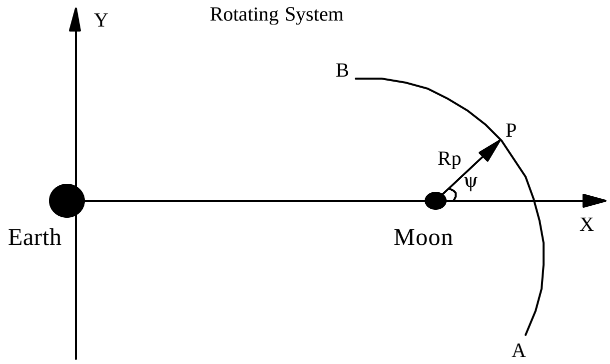

Fig. 1 - Geometry of the Close Encounter.

The spacecraft leaves the point A, crosses the horizontal axis (the line between the Earth and the Moon), passes by the point P (the periapse of the trajectory of the spacecraft around the Moon) and goes to the point B. Points A and B are chosen in a such way that the influence of the Moon at those points are neglected and, consequently, the energy is constant after B and before A. Two of the initial conditions are clearly identified in this figure: the periapse distance r_p (distance measured between the point P and the center of the Moon) and the angle ψ, measured from the horizontal axis in the counter-clock-wise direction. The distance r_p is not to scale, to make the figure easier to understand. The third initial condition is the Jacobian constant J of the spacecraft (also called Jacobi Energy - see equation 4).

Under those assumptions, the procedure involved to solve this problem is: i) To specify particular arbitrary values for the Jacobian constant, the periapse distance (r_p) and the angle ψ; ii) Starting with the spacecraft in the periapse (P), integrate numerically its orbit forward in time until it reaches the point B; iii) Starting again in the periapse (P), integrate its orbit backward in time until the spacecraft reaches the point A; iv) At the points A and B the energy and the angular momentum of the spacecraft are calculated, and the classification of both segments in the above defined categories is made.

Dynamics and Algorithm

The equations of motion for the spacecraft are assumed to be the ones valid for the well-known planar restricted circular three-body problem. They are:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \qquad \ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \qquad \Omega = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$$

(1)-(2)-(3)

This system of equations is non-integrable, and numerical integration is required to solve any problem that involves this model. The standard canonical system of units is used in this paper.

One of the most important reasons why the rotating frame is more suitable to describe the motion of M_3 in the three-body problem is the existence of an invariant, that is called Jacobi integral (or energy integral). There are many ways to define the Jacobi integral and the reference system used to describe this problem (see Szebehely[3], pg. 449). In this paper the definitions used in Broucke[4] are followed. Under this version, the Jacobi integral is given by:

$$J = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \Omega(x, y) = \text{Const} \qquad (4)$$

It is also necessary to have equations to calculate the energy and the angular momentum of the spacecraft. It can be done with the formulas:

$$E = \frac{(x+\dot{y})^2 + (\dot{x}-y)^2}{2} - \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \qquad C = x^2 + y^2 + x\dot{y} - y\dot{x} \qquad (5)\text{-}(6)$$

Results

The results consist of plots that show the change of the orbit of the spacecraft due to the close encounter with the Moon, for a large range of given initial conditions. First of all it is necessary to classify all the close encounters between the Moon and the spacecraft, according to the change obtained in the orbit of the spacecraft.

The letters A, B, C, D, E, F, G, H, I, J, K, L, M, N, O and P are used for this classification. They are assigned to the orbits according to the rules showed in Table 1.

Table 1 - Rules for the assignment of letters to orbits

After: / Before:	Direct Ellipse	Retrograde Ellipse	Direct Hyperbola	Retrograde Hyperbola
Direct Ellipse	A	E	I	M
Retrograde Ellipse	B	F	J	N
Direct Hyperbola	C	G	K	O
Retrog. Hyperbola	D	H	L	P

With those rules defined, the results consist of assigning one of those letters to a position in a two-dimensional diagram that has the parameter ψ in the horizontal axis and the parameter J in the vertical axis. There is one plot for each desired value of the periapse distance. The same range for the variables ψ ($180^\circ \leq \psi \leq 360^\circ$) and J ($-1.45 \leq J \leq 1.55$) used in Broucke[2] and Broucke and Prado[1] are used here. They are very adequate in showing the main characteristic of the plots. The interval $180^\circ \leq \psi \leq 360^\circ$ is used, and not the full range ($0^\circ \leq \psi \leq 360^\circ$), because there is a symmetry between the chosen interval and the complementary interval $0^\circ \leq \psi \leq 180^\circ$. This symmetry comes from the fact that an orbit with an angle $\psi = \theta$ is different from an orbit with an angle $\psi = \theta + 180^\circ$ only by a time reversal. It means that there is a correspondence between these two intervals. This correspondence is: I⇔C, J⇔G, L ⇔O, B⇔E, N⇔H, M⇔D. The orbits A, F, K and P are unchanged.

To decide the best range of values for the third parameter (periapse distance) several exploratory simulations have to be made. It is noticed that, for values greater than 50 Moon's radius, the effects of the swing-by are very small, with the exception of very few special cases. Then, it is decided to make plots for the values: 1.1, 2.0, 5.0 and 50.0 Moon's radius. They span a useful range of values and they are able to show very well the evolution of the effects. Fig. 2 shows two of those diagrams. From a detailed study of those plots and the numerical values not showed here, it is possible to confirm the theoretical predictions[2] that:

i) When the fly-by is in front of the Moon ($0^\circ \leq \psi \leq 180^\circ$), there is a loss of energy. This loss is maximum when $\psi = 90^\circ$;

ii) When the fly-by is behind the Moon ($180^\circ \leq \psi \leq 360^\circ$), there is an increase of energy. This increase is maximum when $\psi = 270^\circ$.

A possible application is to obtain some saving in ΔV for missions involving an escape from the Earth. A more interesting application is to obtain some saving for missions to the Moon itself, using the basic principles used by the Belbruno-Miller trajectories[5,6,7]. In this type of maneuver, the spacecraft is launched from the Earth in a trajectory that makes it to pass close to the Moon (to make a swing-by to achieve a near-parabolic escape trajectory from the Earth), travel a long distance from this point and then return to be captured by the Moon (after using a three-dimensional fourth-body iteration with the Sun). This maneuver has a longer transfer time (several months) but it has a lower ΔV (about 200 m/s in saving) than a standard Hohmann transfer.

This is a very modern approach to send a spacecraft to the Moon that already proved its value in the Japanese mission Muses-A/Hiten. For this particular application, the trajectories that have more potential for use are the ones that transform an elliptic into a near-parabolic orbit. Those orbits are close to the frontier (in the letter-plots shown in this research) between the ones that transform an elliptic into another elliptic orbit (A, B, E, F) and those that transform an elliptic into a hyperbolic orbit (I, M, J, N). In the present research, the detected frontiers between those regions are A-I, B-J and F-N.

Conclusions

A numerical algorithm to calculate the effect of a close approach with the Moon in the trajectory of a spacecraft is developed. Many trajectories are classified. The theoretical prediction that for $0^\circ \leq \psi \leq 180^\circ$ the spacecraft losses energy and for $180^\circ \leq \psi \leq 360^\circ$ the spacecraft gains energy is confirmed. Regions containing trajectories that are candidates to generate Belbruno-Miller trajectories are identified.

Acknowledgment

The author is grateful to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (Post-Graduate Education Federal Agency) - CAPES - Brazil and INPE (National Institute for Space Research - Brazil) for funding this research.

References

1. Broucke, R.A. and Prado, A.F.B.A., 1993, "Jupiter Swing-By Trajectories Passing Near the Earth," **Advances in the Astronautical Sciences**, Vol. 82: Space Flight Mechanics, Part II, Editors: Robert G. Melton, Lincoln J. Wood, Roger C. Thompson, Stuart J. Kerridge, pp. 1159-1176.
2. Broucke, R.A., 1988, "The Celestial Mechanics of Gravity Assist," AIAA paper 88-4220. In: **AIAA/AAS Astrodynamics Conference**, Minneapolis, MN, 15-17 Aug..
3. Szebehely, V.G., 1967, **Theory of Orbits**, Academic Press, New York.
4. Broucke, R.A., 1979, "Traveling Between the Lagrange Points and the Moon," **Journal of Guidance, Control, and Dynamics**, Vol. 2, No. 4, pp. 257-263.
5. Belbruno, E.A., 1987, "Lunar Capture Orbits, a Method of Constructing Earth Moon Trajectories and the Lunar Gas Mission," AIAA-87-1054. In: 19th **AIAA/DGLR/JSASS International Eletric Propulsion Conference**, Colorado Springs, Colorado.
6. Belbruno, E.A., 1990, "Examples of the Nonlinear Dynamics of Ballistic Capture and Escape in the Earth-Moon System," AIAA-90-2896. In: **AIAA Astrodynamics Conference**, Portland, Oregon.
7. Miller, J.K. and Belbruno, E.A., 1991, "A Method for the Construction of a Lunar Transfer Trajectory Using Ballistic Capture," AAS-91-100. In: **AAS/AIAA Space Flight Mechanics Meeting**, Houston, Texas.

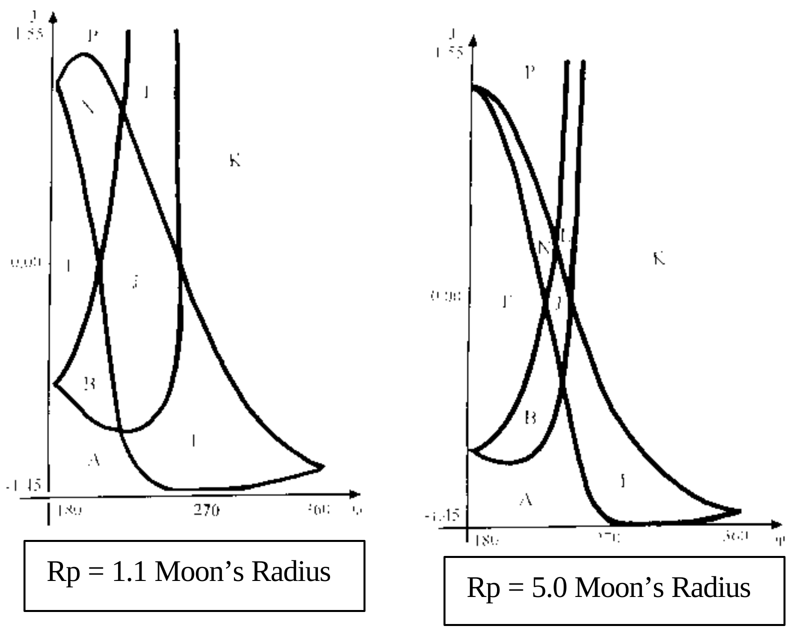

Fig. 2 - Results for r_p = 1.1, 5.0.