On the basic trends of the upper atmosphere modeling - a review

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An overview is presented on modeling alternatives. The work is devoted to provide a general orientation on the subject to novices and non-specialists in the field. The problem of upper atmosphere modeling is presented as well as the basic trends followed to solve the equations employed for this purpose. It is shown that according to these trends the methods can be classified as analytical-empirical, integral transform and numerical. The essentials of these three categories are discussed and commented upon. A brief account of their historical evolution is also included. It allows the reader to observe that, in spite of the present sophistication, the foundations on which the methods are grounded remain unchanged.

Key words: Atmospheric modeling; Thermospheric modeling; Ionospheric modeling.

Sobre as tendências básicas para modelagem atmosférica - uma revisão - Uma visão geral sobre alternativas para modelagem é apresentada. O trabalho é destinado a prover uma orientação geral sobre o assunto para iniciantes e usuários deste campo. O problema de modelagem da atmosfera superior é apresentado bem como as tendências básicas seguidas para resolver as equações empregadas para este propósito. Mostra-se que, de acordo com estas tendências, os métodos podem ser classificados em analíticos-empíricos, de transformadas integrais e numéricos. O essencial dessas três categorias
é discutido e comentado. Uma consideração breve da evolução histórica delas foi também incluída. Esta permite ao leitor observar que, a despeito da presente sofisticação, os fundamentos sobre os quais os métodos baseiam-se, permanecem inalterados.

*Palavras-chave*: Modelagem atmosférica; Modelagem termosférica; Modelagem ionosférica.

**INTRODUCTION**

In the last four decades extensive research effort was done to improve our knowledge about the earth’s upper atmosphere environment. As a result, our understanding of the physical processes governing its behavior now seems reasonably complete. At this time it is adequate to look at the modeling alternatives to reproduce its behavior.

The upper atmosphere is a medium composed of neutral and charged particles which interact mutually and are subjected to the action of external forcing of solar origin and constrained by the action of gravitational, electric and magnetic fields. To model this medium is to find a satisfactory time description of the behavior of its constituent particles in a chosen space range. Modeling is necessary because it is not feasible to observe the medium structure and dynamics at all places and at all times.

Upper atmosphere modeling is by no means an easy task. The involved aspects are so many that we just mention a few which are sufficient to give to the problem enough complexity.

a) The external forcing is variable in time (solar cycle variation, solar activity variation etc.).

b) The neutral and charged particles behave differently but interact mutually.

c) The local action of the external forcing may be well understood (heating and excitation of neutral particles and ionization) but its subsequent consequences are manifested elsewhere in space (because of photoelectron transport) or in time (due to recombination of charged particles).

d) The earth’s curvature and the tilting of the earth’s rotational axis relative to the normal to the ecliptic plane introduce peculiar space and time characteristics on the atmospheric behavior (difference between polar and seasonal asymmetries).
The main geomagnetic dipole is tilted relative to the earth’s rotational axis.

The behavior of the upper atmosphere considered as its response to the driving mechanisms presents an average or regular behavior, which corresponds to the climatology of the medium. However, the system of neutral and charged particles has also an irregular feature due to either localized or short-time actions of the external forcing, which is referred to as its weather (see Schunk & Sojka, 1996a). We will be concerned here with the climatology of the upper atmosphere.

The space-time behavior of the upper atmosphere particles is properly described by the Boltzmann equation (see Schunk, 1977; Barakat & Schunk, 1982; Zamlutti, 1994) which is written as:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \mathbf{a} \cdot \nabla_{\mathbf{v}} f = (\mathbf{\tau}_{\text{col}}) + (\mathbf{\tau}_{\text{p,l}}) \tag{1}
\]

where \( f(\mathbf{r}, \mathbf{v}, t) \) is the distribution function which at a given instant of time \( t \), expresses the density of particles occupying an average position \( \mathbf{r} \) having an average velocity \( \mathbf{v} \). Here \( \nabla \) stands for the gradient operator in the space coordinates, \( \nabla_{\mathbf{v}} \) for the gradient operator in velocity coordinates, \( \mathbf{a} \) is the acceleration imposed by the external forces, \( (\mathbf{\tau}_{\text{col}}) \) is the rate of change of \( f \) due to collision of particles and \( (\mathbf{\tau}_{\text{p,l}}) \) is the rate of change of \( f \) by external driving mechanisms. The acceleration \( \mathbf{a} \) is given by:

\[
\mathbf{a} = \mathbf{G} + (Q/m)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{2}
\]

where \( \mathbf{G} \) is the resultant of the mechanical forces acting on the considered particle, \( Q \) and \( m \) are the particle charge and mass respectively, \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields. The non-specified quantities will be detailed later on in this work.

The modeling problem can be characterized as that of finding the parameters, which specify the functional relation:

\[
f = F(f_o) \tag{3}
\]

connecting a prospective parameter-dependent distribution function, \( f \), for the actual non equilibrium state, to a distribution function, \( f_o \), for the equilibrium situation of (1). Two alternatives were considered theoretically:

a) The kinetic approach;

b) The fluid approach.
The first method is much more general and is used in some localized phenomena (e.g., equatorial electrojet, polar wind). No constraint is imposed as to the form of $f$ which is chosen to match the observed physical characteristics. The second method called also multimoment method applies to regular behavior of most of the upper atmosphere. It consists in the use of an orthogonal polynomial expansion about the equilibrium distribution function (see Zamlutti, 1994 and references therein for a review). A set of hydrodynamic equations is generated from (1) which allow us to determine the parameters of the orthogonal expansion. We will be concerned with this method here.

Three main trends prevailed in upper atmosphere modeling, according to the approach for the solution of the hydrodynamic equations:

a) Analytical-empirical modeling, which combines analytical expressions with data in an attempt to reproduce the observed behavior of the parameters representative of the fluid characteristics of the medium.

b) Integral transform approach, which resorts to space-time periodic functions to simulate the wavelike dynamical behavior of its representative parameters.

c) Numerical modeling, which uses numerical methods to compute the space-time dynamical behavior of the characteristic parameters.

In this work, the fundamentals of the trends followed in upper atmosphere modeling are briefly reviewed. Emphasis is given to the mathematical aspects of each method. This approach provides the reader with a better tool to properly choose a model appropriate to his needs.

The review begins with a section on the division of upper atmosphere regions for modeling purposes. In section 3 the basic equations of fluid dynamics applicable to the problem are presented. The next three sections present the three main trends mentioned above. In each section a proper note is made of the grounds and essentials of the method under consideration. These sections are composed of three parts: one describes the essences of the methodology, the other gives a brief mathematical account of the development of the method and the last is concerned with a short historical retrospective of the evolution of the considered approach. Our presentation of the matter is closed with an outlook on the general aspects of the three alternatives and comments on the advantages and shortcoming of each one of the three techniques described.

THE UPPER ATMOSPHERE REGIONS FOR MODELING
With regard to modeling, the main division of the upper atmosphere can be traced to the characteristics of the magnetic field within the region. Thus we will be distinguishing two regions:

a) Low and middle latitude (L < 6) – including all the plasma contained within the plasmasphere domain above 100 km.

b) High latitude (L > 6) – including the upper atmosphere plasma contained outside the plasmasphere but in the nearby earth’s environment above 100 km.

The first of these regions comprises a toroids-shaped volume surrounding the Earth containing a cool (T<10^4 K), high density (n > 10^2 cm^3) plasma. The charged particles can flow along the closed geomagnetic field lines from one hemisphere to the other. The fluid contained in this region corotates with the Earth.

In the second region the plasma does not corotate with the Earth and its motion is controlled by electric fields of magnetospheric origin. The geomagnetic field lines are open which allows the thermal plasma to escape along them. Of course, solar particles can also penetrate along these lines. The magnetospheric electric field that affects this region is prevented to influence the plasmasphere by the effective shielding provided by the ring current (see Earle & Kelley, 1987).

Under climatology conditions the driving mechanism that affects the upper atmosphere, regardless the considered region, is the solar radiation (solar EUV and UV radiation). This radiation in incident at the top of the upper atmosphere in the range 50Å-1500Å which correspond roughly to the energy range 250-8 eV. A recent work on the solar EUV flux was presented by Richards et al. (1994). The incident energy acts on the upper atmosphere particles through wave-particles interactions producing the ionized species as well high energy electrons (photoelectrons). A recent work on this matter is due to Zamlutti (1997). The photoelectrons transfer their energy to the neutral particles locally producing further ionization or excitation of electronic states (Zamlutti, 1997; Schunk & Nagy, 1978) or non locally through an inter-hemispheric two-stream transport model (Stamnes, 1980, 1981). The ultimate fate of these photoelectrons was modeled by Lilensten et al. (1989). The excited neutral and charged species undergo subsequent chemical reactions before an equilibrium state be reached. This aspect was studied by Eccles & Raitt (1992) and Zamlutti (1997).

A large portion of the solar energy (in the range 2000Å– 3000Å) reaches the lower atmosphere where it excites the O₃ constituent. Part of this energy is transferred upwards by means of thermally driven atmospheric waves (gravity and tidal modes). Theoretical accounts of this effect were proposed (e.g. Teitelbaum & Vial, 1981; Yamanaka & Fukao, 1994), numerical models carried out (e.g.
Miyahara et al., 1993) and experimental data base analysed (Forbes et al., 1994). Estimates exist that the contribution of the lower atmosphere is less than 10% of the local upper atmosphere energy budget (Killeen, 1987).

At high latitudes, mainly during enhanced solar activity, two additional driving mechanisms become effective:

a) Plasma precipitation along magnetic field lines causing aurora.

b) A dawn to dusk electric potential drop across magnetic conjugate polar caps (see Roble et al., 1987).

The precipitation of energetic particles is an energy driving mechanism which excites the upper atmosphere through collisional interactions. A theoretical account for this effect can be done using the approaches proposed by Zamlutti (1997), Stamnes (1980) or other equivalent.

The electric field of magnetospheric origin is a momentum driving mechanism. It results from the interplanetary magnetic field (IMF) which is mapped down along the geomagnetic field lines to the ionosphere (100-500 km). It is perpendicular to the geomagnetic field and drives the charged particles plasma into horizontal motion (see Schunk, 1983).

These high latitude mechanisms are responsible for the weather effects on the behavior of the upper atmosphere. Although they do not affect the climatology of the region proviso is necessary to account for them for a satisfactory mathematical modeling.

Under regular conditions the two considered regions can be assumed to be independent of one another. Thus, a separate mathematical model can be developed for each region in accordance with its physical characteristics. Space weather disturbances act to couple the two models transferring electric field variations instantaneously from the polar region to the Equator by means of a zero-order transverse magnetic waveguide mode with attenuation factor of 1/10 (see Kikuchi et al., 1978). Energy transfer from high to low latitudes, however, is a much slow process by means of gravity waves propagation and meridional circulation (see Richmond, 1979 and references therein).

According to the altitude range the above regions are subdivided in:

a) Thermosphere and ionosphere (100 km < z < 800 km), denoting neutral and charged species respectively.

b) Exosphere and protonosphere (z > 800 km) denoting neutral and charged species.
THE BASIC EQUATIONS

Low and middle latitudes

It has been well established in the literature (Schunk, 1975, 1977; Zamlutti, 1994) that the average behavior of each upper atmosphere constituent within the plasmasphere domain above 100 km of altitude can be properly described by the Navier-Stokes system of equations:

\[
d\rho/dt + \rho \nabla \cdot \mathbf{u} = \delta \rho
\]

\[
\rho \mathbf{d} \mathbf{u}/dt + \nabla p - \rho \mathbf{a} + \nabla \cdot \mathbf{\tau} = \delta \mathbf{\mathcal{M}}
\]

\[
(3/2)\rho d\mathbf{e}/dt + (3/2)\rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla \cdot \mathbf{q} + \mathbf{\tau} \cdot \nabla \mathbf{u} = \delta \mathbf{\varepsilon}
\]

where \(\rho\), \(\mathbf{u}\) and \(p\) denote respectively the mass density, flow velocity and pressure, \(\mathbf{\tau}\) is the stress tensor and \(\mathbf{q}\) the heat flow vector. The right hand side (RHS) of the above equations expresses the perturbations in the parameters produced by mutual interaction (elastic collisions) and external forcing (production and loss mechanisms). The particle motion is constrained by the action of the fields represented by the acceleration \(\mathbf{a}\). The double dot in (6) stands for the scalar product of two tensors or double contraction or double scalar product (see Chapman & Cowling, 1970). The other symbols have their usual meaning, thus \(\nabla\) is the space gradient and \(d/dt = \partial/\partial t + \mathbf{u} \cdot \nabla\). the convective derivative.

The stress tensor can be computed by:

\[
\mathbf{\tau} = -\eta \left[ \nabla \mathbf{u} + \nabla \mathbf{u}^\intercal - 2 \mathbf{\mathcal{J}} \right] \mathbf{\mathcal{J}}^{-1} \mathbf{\mathcal{J}} \left[ \nabla \mathbf{u} + \nabla \mathbf{u}^\intercal - 2 \mathbf{\mathcal{J}} \right]
\]

in the case of neutral particles. Subscript \(t\) denotes transpose and \(\mathbf{\mathcal{I}}\) is the identity matrix. Here \(\eta\) in the viscosity coefficient given by:

\[
\eta = \frac{\zeta}{6} \rho \mathbf{u}
\]

and \(\mathbf{u}\) is the collision frequency (Schunk, 1975). For the charged particles we have:

\[
\mathbf{\tau} + (3/2) (\zeta/m) \mathbf{u}^{-1} \left( \mathbf{E} \times \mathbf{\tau} - \mathbf{\tau} \times \mathbf{B} \right) = \mathbf{\tau}_0
\]

where \(e\) is the particle charge and \(\mathbf{\tau}_0\) stands for the value computed by (7). Here \(\mathbf{B}\) represents the magnetic field intensity.
The heat flow vector for neutral particles is given by:

\[ \underline{q} = k \nabla \left( \frac{\rho}{\rho} \right) \]  \hspace{1cm} (10)

where \( k \) is the thermal conductivity computed by:

\[ k = \left( \frac{25}{9} \right) m^{-1} \frac{\mu}{\nu} \]  \hspace{1cm} (11)

according to Schunk (1975).

For the charged particles the heat flow is computed by:

\[ \underline{q} - \left( \frac{5}{4} \right) \frac{e}{m} \nu^{-1} \underline{q} \times \underline{B} = \underline{q}_e \]  \hspace{1cm} (12)

where \( \underline{q}_e \) is the value given by (10).

The particles acceleration is determined by (2) with \( \underline{G} \) given by:

\[ \underline{G} = \underline{g} + 2 \underline{\Omega} \times \underline{u} + \underline{\Omega} \times \underline{\Omega} \times \underline{r} \]  \hspace{1cm} (13)

where \( \underline{g} \) is the acceleration of gravity, \( \underline{\Omega} \) is the earth's angular velocity and \( \underline{r} \) is the radius vector from the center of the earth to the point where the equations are applied. In the case of neutral particles only (13) holds in (2) since they are insensitive to the Lorentz force. The correct way to treat the electric and magnetic fields is to incorporate the two Maxwell’s equations:

\[ \frac{\partial (\varepsilon \underline{E})}{\partial t} = \nabla \times \left( \mu^{-1} \underline{B} \right) - \underline{J} \]  \hspace{1cm} (14)

\[ \frac{\partial \underline{B}}{\partial t} = -\nabla \times \underline{E} \]  \hspace{1cm} (15)

to the system of fluid equations and consider \( \underline{E} \) and \( \underline{B} \) as parameters to be determined. Here \( \varepsilon \) is the permittivity, \( \mu \) the permeability of the medium and \( \underline{J} \) is the total current density given by:

\[ \underline{J} = \sigma (\nabla \times \underline{u} - \nabla \times \underline{u}_e) \]  \hspace{1cm} (16)

As for the RHS of Eqs (4)-(6) the contribution due to elastic collisions is given by:

\[ \delta \rho = 0 \]  \hspace{1cm} (17a)

\[ \delta M_j = \sum_j C_j \left[ u_j - u \right] \]  \hspace{1cm} (17b)
\[ \delta E = \sum_j C_j \left( \frac{m_j + m}{v_j} \right)^{\frac{1}{2}} \left( \frac{p_j}{\eta_j} - \frac{p}{\eta} \right) \]  

(17c)

where \( C_j = \mu_j \gamma_j \kappa_j \left( S_j / S \right) \), with \( \mu_j \) being the reduced mass, \( v_j \) the collision frequency, \( n \) the number density and \( S_j / S \) the transfer rate. The subscript \( j \) denotes the colliding species. The transfer rate is unit for neutral-neutral interactions, is 0.57 for charged-neutral encounters and given by:

\[ S_j / S = 0.5 \left( U^2 / W^2 \right) \ln \left( 1 + 4W^2 / U^2 \right), \]

where \( U \) is the potential and \( W \) the kinetic energies, for Coulomb collisions (Zamiutti, 1994). The square of the ratio between these energies is:

\[ U^2 / W^2 = \frac{1}{16\pi \alpha \left( 9K^2T^3 \right)^{\frac{1}{4}}} \]

where \( K \) is the Boltzmann constant and \( T \) the temperature.

The perturbation resulting from the production and loss mechanisms due to chemical reactions was computed by Eccles & Raitt (1992) to be:

\[ \delta \rho = m \left( \sum_{a,b} k_{ab} \rho_a \rho_b - \sum_k \rho_k \right) \]

(18a)

\[ \delta \rho = m \left[ \sum_{a,b} k_{a\alpha} v_a v_b \left( u_{\alpha} - u \right) \right] - \sum_k k_{p\alpha} \left( u_{\alpha} - u \right) \]

(18b)

\[ \delta E = \sum_{a,b} \rho_{ab} \rho_a \rho_b \left[ 1.5 K T_{ab} + 0.5m \left( u_{ab} - u \right) \right] - \sum_k \rho_{k\alpha} \left[ 1.5 K T_{k\alpha} + 0.5m \left( u_k - u \right) \right] \]

(18c)

where \( k \) represents reaction rates, subscripts \( a \) and \( b \) denote reactant species \( a \) and \( b \) whose product is the desired species \( s \) and subscript \( l \) denotes the species which react with \( s \) to destroy it.

The affected parameters during the transformation are determined by:

\[ u_{s\beta} = u \]

\[ T_{s\beta} = T \]

\[ u_{ab} = m^{-1} \left( \rho_{a\alpha} u_a + \rho_{b\alpha} u_b \right) \]
where $\mu_{ab}$ is the reduced mass of particles $a$ and $b$, $m_v = m_a + m_b$ and $E$ is the energy absorbed or released during the production reaction. The reaction rates and energy requirements for the neutral and charged particle chemical interactions in the upper atmosphere were compiled by Torr & Torr (1979).

The primary photoelectron production function can be described by:

$$(\delta \rho)_\gamma = m \sum n_b \int_\gamma^{\infty} \sigma_b \psi(w_a) dw_a$$

(19)

(see Schunk, 1983; Lilensten et al., 1989; Zamlutti, 1997).

Here $\psi$ represents the solar radiation flux at the considered altitude with the energy $w_a$ and $\sigma_b$ is the cross section of particle species $b$.

The radiation flux $\psi$ is attenuated by absorption as it penetrates the atmosphere according to:

$$\psi = \psi_\infty \exp(-\zeta)$$

(20)

where $\zeta$ is the optical depth given by:

$$\zeta = \sum n_b \sigma_b H_b CH(\chi)$$

(21)

and $\psi_\infty$ represents the flux at the top of the atmosphere (usually assumed above 1000 km). The function $CH(\chi)$ is the Chapman function for solar zenith angle $\chi$, $H_b$ is the scale height expressed by:

$$H_b = KT_b/m_g$$

(22)

The primary photoelectrons travel along magnetic field lines (Stamnes, 1980, 1981) and produce secondary ionization at the magnetic conjugate location. This requires lengthy computations (see Lilensten et al., 1989 and references therein) but as shown by Roble et al., 1987) can be simply accounted for assuming it to be 30% of the primary ionization.

The primary photoelectrons receive energy from the incident radiation according to:

$$\langle \delta E_b \rangle_\gamma = (2\pi\delta\lambda) - \langle \mathcal{J} \rangle_b$$

(23a)
where \( h \) is the Planck’s constant, \( \lambda \) the wavelength of the incident radiation and \( (U_i)_b \) is the ionization energy of particle specie \( b \) (neutral target). Higher energy photoelectrons travel along magnetic field lines to the conjugate location where secondary electrons are produced receiving an energy:

\[
(\delta E)_p = E_\lambda - (U_i)_b
\]  

(23b)

Lower energy photoelectrons exchange their excess energy locally through inelastic collisions with ambient electrons, ions and neutrons according to (18c). Medium energy photoelectrons excite metastable species (rotational, vibrational and electronic excitation) with energy variations of:

\[
(\delta E)_f = E_\lambda - U_{\text{ex}}
\]  

(23c)

where \( U_{\text{ex}} \) is the exchange potential as in Zamlutti (1997). This last energy variation is incorporated, in thermodynamics, in the energy equation by increasing the coefficients of the pressure terms in the LHS of it (see Richmond, 1983; Meador et al. 1996) for the case of diatomic targets.

### High latitudes

Although the standard set of equations, (4)-(6), at the level of the Navier Stokes approximation, with the stress tensor, (7) and (9), at the level of Rayleigh friction and heat flow vector, (10) and (12), at the level of Newtonian cooling can be used for high latitude modeling under restricted conditions (Blelly & Schunk, 1993; Schunk, 1975) other systems of equations are more suitable for this purpose. This matter was undertaken by Blelly & Schunk (1993) who concluded that the simplest of these systems is the 8-moment isotropic system (Schunk, 1977), which is now currently used in high latitude modeling (Robineau et al., 1996; Diloy et al., 1996; Cordier & Girard, 1996). It is obtained if we add to the Navier-Stokes equations a heat flow expressed as:

\[
\frac{d q}{dt} + 1.4 \left[ \nabla \cdot q + \frac{1}{2} (\nabla \cdot v) - 0.4 (\nabla \cdot v) \cdot q + 2 \sqrt{v} \left( \rho^2 \cdot q^2 \right)^{1/2} + 1.4 \\
1.4 \sqrt{v} \left( \rho^2 \cdot q \right) \left( \rho \cdot q \right) \left( \rho / (\rho + 1) \right) \nabla \cdot q \delta_2
\]  

(24)

which puts the heat flow on an equal footing with density, velocity and temperature. This equation replaces (10) and (12). Its corresponding elastic collision term is:

\[
\delta_2 = \frac{1}{7} \mu \rho \left[ 4 \mu \rho \left( \rho \cdot q \right) q + \left( 4 \rho \cdot q - z \right) q \right], ...
\]  

(25a)
where we displayed only the first component. A more complete collision term is given by Schunk (1977) or Zamlutti (1994).

To date no attempt was made to incorporate the effect of production-loss mechanisms on (24). These effects can be computed with the theory presented by Zamlutti (1997). Fortunately the expressions provided by Eccles & Raitt (1992) fills up this gap. It is

$$\delta q = \sum_{j=0}^{\infty} k_{ji} q_{ji} - \sum_{i=0}^{\infty} k_{ij} q_{ij} + \cdots$$  \hspace{1cm} (25b)

Where again we displayed only the first component. The complete term is given by Eccles & Raitt (1992). The parameter affected during the chemical reaction is determined by:

$$q_{ab} = n_{ab}^{-1} \left[ m_{ab}^{2} n_{ab} q_{ab} + \left( \frac{1}{2} \right) m_{ab} n_{ab} \left( q_{ab} + n_{a} q_{b} \right) \right]$$  \hspace{1cm} (25c)

**ANALYTICAL AND EMPIRICAL MODELS**

Experimental data bases for the study of the upper atmosphere are constructed based on data sets obtained by probing it (locally or remotely) using electronic, optical or magnetic devices.

Empirical models range from simple data organization of data bases according to space, time or level of the driving mechanisms, to plots and simple curve fitting to the data.

Analytical models are obtained from the solution of one or more of the basic equations for over simplified physical concepts and conditions. The data bases for obtaining coefficients of these analytical functions are the actual results of the complete set of basic equations got for some specified conditions.

Semi-empirical models describe the upper atmosphere parameters using analytical expressions combined with observational data bases. This type is preferred because combines theory and experimental results.

The upper atmosphere parameters (density, temperature and velocities) are functions of space \((r)\) and time \((t)\). It became common practice to split the space presentation of models in: local or vertical parameter profiles and global or horizontal parameter variations. The combined results of local and global modeling must conform to the basic equations describing the behavior of the medium. The usually employed systems of coordinates are the standard geographic spherical system (altitude, latitude, longitude) and the tilted spherical system aligned with the direction of the geomagnetic dipole axis (altitude, geomagnetic latitude, geomagnetic longitude). The first system is appropriate to the study of
the neutral particle behavior whereas the latter is preferable for the charged particles.

The simplified expressions used to produce semi-empirical and analytical models are obtained assuming steady state conditions \(d/dt=0\) and a static situation \((\mu=0)\). Under these conditions the fundamental equations become:

\[
\frac{\delta p}{\delta t} = 0 \tag{26}
\]

\[
\nabla p - \rho \frac{\partial z}{\partial t} = \delta M \tag{27}
\]

\[
\nabla \cdot \zeta = \delta B \tag{28}
\]

whose validity has been verified for the upper atmosphere (e.g. Rishbeth 
& Garriott, 1969; Nagy & Schunk, 1987). In the steady state one can consider the longitude-local time equivalence under which conditions we have:

\[
\frac{\partial}{\partial t} = \Gamma \cos \theta \frac{\partial}{\partial \phi} \tag{29}
\]

where \(\theta\) stands for latitude and \(\phi\) for longitude. These equations are complemented with the equation of state.

\[
p = n_k T \tag{30}
\]

Hereafter the subscript 0 will be used to represent fixed reference values. Charged particle local models (see Dudeney & Kressman, 1986) are derived from (26) and (27) which provide the necessary physical grounds for them (see Schunk, 1983). Neutral particle local models on the other hand employ (27) and (30) to provide the fundamental expression for them.

Below and around the \(F_2\) region peak of the ionosphere the charged particle densities are modeled by the idealized Chapman layer

\[
n = (p_e \cdot \alpha \cdot z) \frac{1}{\pi} \left\{ 1 + \frac{z - z_0}{H} - \exp \left( \frac{z_0 - z}{H} \alpha \right) \right\} \tag{31}
\]

where \(p_e^\ast\) is the peak production function, \(\alpha\) the recombination rate, \(z\) the altitude, \(\chi\) is the solar zenith angle and \(H\) the neutral scale height. Around the peak of the layers (31) reduces to a parabolic shape and this is the basis for the so-called parabolic models (Dudeney, 1978). Otherwise the exponential forms predominate (see Booker, 1977; Rawer, 1984; Rawer & Bilitza, 1989). Expression (31) is easily derived from (26), (19) and (18a).
Above the $F_2$ region of the ionosphere ambipolar plasma diffusion predominates (see Rishbeth & Garriott, 1969) and the charged particles reach diffusive equilibrium with the densities expressed by:

$$n = n_o \exp \left[ - (z - z_o) / H_T \right]$$  \hspace{1cm} (32)

where $n_o$ and $z_o$ are the density and altitude at a reference height and $H_T = K (n_o + T_o) / mg$ is the scale height (e.g. Benkova et al., 1984). Subscripts e and i denote electrons and ions respectively. Expression (32) is obtained from (27) for an isothermal ionosphere (see Schunk, 1983).

Ionospheric semi-empirical models are all grounded on the results of (31) and (32) as far as densities of charged particles are concerned.

As for neutral particles Eqs. (27) and (30) produce the fundamental expression:

$$p = p_o \exp \left( - K^{-1} \int_{z_o}^{z} g n T^{-3} ds \right)$$  \hspace{1cm} (33)

which represent the hydrostatic equilibrium valid for all local type thermospheric models if $\delta \mathbf{M} = 0$. This equation establishes the behavior of the $nT$ product in the thermosphere. To determine $n$ and $T$ from (33) Bates (1959) proposed the relevant analytical expression:

$$T_z = T_o - (T_o - T_i) \exp \left( - s \zeta \right)$$  \hspace{1cm} (34)

where $T_o$ is the asymptotic temperature upper limit, which constitutes to date the basic relation for all sorts of thermospheric temperature models. In (34) $s$ is an adjustable parameter and $\zeta$ is given by:

$$\zeta = \int_{z_o}^{z} \left[ g (z) / g(z_o) \right] dx$$

depending only on the variation of gravity with altitude. Relation (34) is obtained from (28), (10) and (18c). Expression (33) holds for the major neutral atmosphere constituent. To determine the diffusion velocity of a minor constituent we apply (27) to it and to the major species and get by difference:

$$u_j - u = \left( \rho_j \gamma_j \rho_j \right)^{-1} \left[ \nabla p - \rho \nabla \gamma - \nabla \rho_j + \rho_j \nabla \gamma \right]$$

when association, dissociation, ionization and recombination effects are neglected and provided that the density of the minor constituent is
known. Density variation in this case may be determined using (26) and
the diffusion velocity (see Rishbeth & Garriott, 1969).

Regarding charged particles, model (32) can also be used if appropriate
temperature models are known. So far it is necessary to mention the
relevant contribution of Brace & Theis (1978, 1984) who provided the
following very useful expression to compute the electron temperature:

\[ T_e = a_1 + (a_2 z + a_3) \exp (a_4 z + a_5 n_i + a_6 n_i) \]  

where:

\[ a_1 = 1.051 \cdot 10^3 \, K; \quad a_2 = 1.707 \cdot 10^{10} \, K \, km^{-1}; \]
\[ a_3 = 2.746 \cdot 10^{-3} \, K; \quad a_4 = 5.122 \cdot 10^{-4} \, km^{-1}; \]
\[ a_5 = 6.094 \cdot 10^{-12} \, m^3; \quad a_6 = 3.353 \cdot 10^{-14} \, km^{-1} \, m^3; \]

This expression was obtained by empirical methods but it can be
justified by theory if we combine (28) with (10) and (18c) imposing the
ionospheric constraints that electrons are cooled by collisions with ions
above 200 km and given by (32) above 300 km. In it the ion density is
made equal to the electron density for simplicity. It is valid for day-time
hours, since during nighttime hours the electron temperature must relax
to the neutral temperature. A comparison by Pandey & Mahajan (1984)
confirmed the importance of (35) for upper atmosphere studies. An
alternative theoretical expression for (35) is presented in Schunk
(1983).

For the ion temperature far fewer modeling attempts have been
made. So far all we know is that up to 400 km we can use for it a
Bates temperature profile (34). Above 450 km \( T_i \) grows to \( T_e \) up to
4000 km where \( T_i = T_e \). The growth is nearly linear in logarithmic
scale (Kohnlein, 1986). An alternative theoretical computation
using \( \nabla T_i = 0 \) above 600 km was suggested by Schunk (1983).

As far as upper atmosphere composition is concerned all that is known is
by means of probing the medium using mass spectrometers. Neutral
particle composition is described in Hedin (1983) and charged particle
composition is found in Philbrick & Rawer (1984) and Kutiev et al.

The global description of the parameter variations is obtained employing
a space-time function which accounts for latitudinal and time variation of
the vertical profile of the considered parameters. It is important to know
that longitude-time equivalence is usually assumed unless higher
accuracy is necessary (see Hedin, 1983; Kohnlein, 1986). The global
function has normally the following form:

\[ G = 1 + \sum \frac{c_j}{l} P_j(\sin \theta) \rightarrow \text{time dependent term} \]
\[ + \sum P_i (\sin \theta) \sum_j (a_{ij} \cos \omega_j t + b_{ij} \sin \omega_j t) \rightarrow \text{time dependent term} \quad (36) \]

where \( P \) are the Legendre polynomials and \( \omega_j \) are the angular frequencies of the constituent periodicities observed in the time behavior of the parameter. Here \( c_i, a_{ij}, \) and \( b_{ij} \) are the coefficients of the spherical harmonic expansion, regularly determined using the least squares fitting procedure applied to the available data bases.

Besides global variations the profiles are weighted by a function which accounts for the regular known variation of the driving sources (solar radiation and magnetic activity). The corresponding function has the form:

\[ C_i = 1 + C_s \Delta F + C_m \Delta A \quad (37) \]

where \( C_s \) and \( C_m \) are the coefficients for the solar and magnetic activity terms, \( \Delta \) denotes variation relative to the mean value (e.g. \( \Delta F = \bar{F}_{10.7} - \bar{F}_{10.7} \) with \( \bar{F}_{10.7} \) denoting average value).

So far we were concerned mainly with low and middle latitude, low altitude (\( z < 800 \text{ km} \)) conditions. Density models can be extended to high latitudes, but temperature models must be looked with some caution because relation (10) may not be valid (except as a crude approximation).

The next step of complexity implies the release of the static constraint still preserving the steady state equilibrium restriction. Under these conditions we have:

\[ \nabla \cdot (\rho \mathbf{u}) = \delta \rho \quad (38) \]

\[ \nabla \mathbf{p} - \rho \mathbf{a} + \nabla \cdot \mathbf{z} = \delta \mathbf{M} \quad (39) \]

\[ \nabla \cdot \left( 5 \rho \mathbf{u}^2 / 2 + q \right) - \mu \cdot \nabla \rho = \delta E \quad (40) \]

Expressions (38-40) were used to predict steady state velocities for both neutral particles (Geisler, 1967; Titheridge, 1995) and charged particles (e.g. Banks & Holzer, 1969; Sanatani & Hanson, 1970; Lockwood & Titheridge, 1982; Anderson, 1973; Anderson et al., 1987). They were important for semi-empirical models of the protonosphere (plasmasphere), of the polar wind and also of the low latitude ionosphere (equatorial anomaly).

Under the assumptions of ambipolar plasma diffusion (see Rishbeth & Garriott, 1969; Schunk, 1983) the electric field is determined using (39) for the electrons which yields:
\[ e \varepsilon = -u_e^{-1} \frac{\partial p_e}{\partial z} \]  

(41)

and the vertical component of the electron and ion velocity is computed using (39) for the ions to get:

\[ u_i = u_{n\theta} \sin I \cos I - D_p \sin^2 I \left( u_{i\theta}^{-1} \frac{\partial n_i}{\partial z} + T_p^{-1} \frac{\partial T_p}{\partial z} + H_p^{-1} \right) \]  

(42)

where \( u_{i\theta} \) is the latitudinal component of the neutral velocity, \( I \) is the dip angle, \( D_p \) the diffusion coefficient and \( T_p \) the plasma temperature defined as:

\[ T_p = \frac{1}{2} (T_e + T_i) \]

\[ D_p = 2kT_p \left/ (\ln m_e m_n) \right. \]

To determine neutral particle velocities from (39) we assume that above 160 km the charged particles are constrained to move along the magnetic field lines and the \( u_\phi = 0 \) and \( u_{i\phi} = u_{i\theta} \cos I \) (see Roble & Dickinson, 1974). Moreover to a first order approximation viscosity is neglected. Therefore from (13), (17b) and (39) results:

\[ u_{so} = \frac{C_i \sin^3 i \nabla_{\phi} p + 2T \sin \theta \nabla_{\phi} \sin^2 \theta}{C_i^2 \sin^3 i + 4I^2 \sin^2 \theta} \]  

(43)

\[ u_{s\phi} = \frac{C_i \nabla_{\phi} p - 2T \sin \theta \nabla_{\phi} \sin^2 \theta}{C_i^2 \sin^3 i + 4I^2 \sin^2 \theta} \]  

(44)

which constitute Geisler (1967) approach to the computation of neutral particle bulk velocities. Here \( \nabla_i \) and \( \nabla_{\phi} \) represent meridional and zonal gradients respectively. Expressions (43) and (44) can be refined using their first order vertical profile to introduce the effects of viscosity and nonlinear inertia pointed out as shortcomings of this approach (see Titheridge, 1995). Other limitations concern the validity of the assumption of steady state during dawn and dusk periods and the quality of the results during nighttime when \( C_i \) decreases considerably. Since \( p \) depends on \( G \), of Eq. (36), it seems reasonable to determine velocities by least square fitting of data using tidal periods. This is the base of now used methods (see Hedin et al., 1988; Titheridge, 1995).

Thermosphere empirical models date from the early 50’s when Bates (1951) established the shape of the thermospheric temperature profile above 200 km. The basic ionospheric Chapman function (21), (31) has
been known since 1931 (Chapman, 1931). Ionospheric models were originally devoted to radio wave propagation purposes (see Ratcliffe, 1962 and references therein), with the construction of world-wide maps of critical frequencies.

However, it was during the 60’s that thermospheric models received considerable emphasis with the work by Jacchia (1964) who standardized the types of atmospheric variations and ionospheric model development took place (see Nisbet, 1971 for past references) with geophysical research purposes. During the 70’s upper atmosphere modeling got a definite place in space research with thermosphere modeling (Jacchia, 1970, 1977; Hedin et al., 1974, 1977) based on a spherical harmonics development for each neutral constituent parameter, and ionosphere modeling (Nisbet, 1971; Bradley & Dudeney, 1973; Ching & Chiu, 1973; Chiu, 1975; Booker, 1977; Dudeney, 1978) which was in part dedicated to communication objectives but started looking at the aim of properly reproducing the behavior of charged particles. In the 80’s the present ideas of modeling were set down (Hedin, 1983, 1987, 1988; Rawer, 1984; Rawer & Bilitza, 1989).

Theoretical models of wind velocities date the late 60’s (Geisler, 1967). These models were steady state. During the 70’s velocity models were described in terms of spherical harmonics (e.g. Volland & Mayr, 1972; Straus et al., 1975). Blum & Harris (1975) solved the full system of Navier Stokes equations including winds. Spherical harmonics are now used in recent semi-empirical models (Hedin et al., 1988).

The mathematical development of ambipolar plasma velocities is attributed to Ferraro (1945). In the late 60’s it received considerable attention for protonospheric models (Banks & Holzer, 1969; Rishbeth & Garriott, 1969). Its mathematical theory is found in Chapman & Cowling (1970). It is still used to determine the electron velocity in high latitude modeling (e.g. Belfry & Schunk, 1993).

Protonospheric (plasmaspheric) and high latitude modeling received more attention after Johnson (1960) suggestion that the reversible charge exchange reaction $O+H\rightarrow H+O$ was the dominant chemical mechanism above the altitude 800 km. They received considerable improvement with the works by Banks & Holzer (1968, 1969) who used the ambipolar plasma model. Although representative of the climatology of these regions steady state models lost their importance after 1976 (see Young et al., 1980 for a brief past historical note).

At equatorial and low-latitudes the geomagnetic field lines are nearly horizontal introducing important transport effect at an above the F2-peak density. Therefore its modeling followed Eqs. (38-40). It started around the 60’s (Hanson & Moffett, 1966), but the highlights of semi-empirical versions occurred with the SLIM and FAIM models.
(Anderson et al., 1987; 1989). Problems and details of this sort of modeling are given in the review by Stenning (1992).

**INTEGRAL TRANSFORM METHODS**

Integral transforms are used to convert a differential equation into an algebraic equation or at least to reduce the number of parameters on which the variables of a differential equation depend. With these purposes Fourier transform methods were employed originally and expansions in orthogonal polynomials have been used more recently in the study of thermospheric and ionospheric dynamics.

The goal in this section is to describe the dynamical behavior of the upper atmosphere by means of analytical expressions necessary to express the global space time behavior of the parameters of local empirical models.

In this work we will be concerned with the tidal equations of the thermosphere because of their relevance to upper atmosphere studies (see Volland & Mayr, 1977; Lindzen, 1979; Forbes & Garrett, 1979). The tidal studies require spherical geometry and produced the shape of the now used global function $G$ of (36). To evaluate their importance on the behavior of charged particles one just needs to recall that the ionization process occurs on an oscillatory background of neutral particles. A rough idea on this can be got combining (18a-c) with (26-28).

The routine in the theoretical derivation of tidal equations uses the following assumptions:

a) The earth is assumed to be a smooth sphere.

b) The atmosphere is assumed to be a compressible, hydrostatic, shallow, perfect gas.

c) The wave fields are assumed to be small perturbations on a basic state.

We will be more interested in the thermally driven perturbations of migrating type. Under these circumstances linearization holds and the longitude-time variations go like $\exp^{-i\omega(t+\phi)}$ where $\phi$ is the longitude. Furthermore we denote with subscript 0 the mean value of the parameter and subscript 1 its linear perturbation.

Following Forbes & Garrett (1976, 1979) the continuity equation can be used beforehand when the only component of the mean velocity is in the direction of the known space variations (i.e. the longitudinal direction). Under these circumstances the density perturbation becomes:
where \( r \) is the radial distance from the earth’s center to the considered volume element and \( \theta \) is the latitude.

Temperature variations can also be expressed in terms of pressure and velocity variations using (30) and (45) from which:

\[
\frac{T_i}{T_0} = \frac{\rho_i}{\rho_0} - \frac{p_1}{p_0}
\]  

(46)

We are now left with a system of four differential equations depending on \( r \) and \( \theta \). The equations are essentially non separable but approximating the effects of ion drag and viscous force by an effective collision frequency they can be made separable (see Volland & Mayr, 1977). The heat conduction is also approximated by a Newtonian cooling term. With these simplifications the equations depending on \( r \) are:

\[
\frac{\partial p_i}{\partial r} = -g \rho_i \]  

(47a)

\[
-\left(\frac{3}{2}\right)\left[ i\pi \left( \omega + \frac{u_{\phi}}{r \cos \theta} \right) \right] p_i + \left(\frac{3}{2}\right) p_0 \nabla \cdot \mathbf{u}_1 - u_{\phi} H^T p_0 +

+ \left(\frac{3}{2}\right) \frac{u_{\phi}}{r \cos \theta} \rho_i - \infty \left( \rho_0 \nabla \cdot \mathbf{u}_1 - \rho_0 H^{-1} \rho_i \right) = \delta E
\]  

(47b)

where \( \infty \) stands for the Newtonian cooling coefficient and we assumed that the horizontal scale lengths are much larger than the vertical scale height. The effect of stresses in (47b) was also discarded. The uncomfortable presence of the divergence of horizontal velocities will be removed from (45) and (47b) with the introduction of the concept of equivalent depth as done by Volland & Mayr (1977). The presence of zonal winds do not significantly affect the separability since they can be discretized in \( \theta \) anyway.

The equations depending on \( \theta \) are:

\[
- i \pi \left( \omega + \frac{u_{\phi}}{r \cos \theta} \right) u_{1\theta} - \bar{u} u_{1\theta} + 2 \Gamma \times u_{1\theta} \sin \theta + \rho_0 H^{-1} \nabla \cdot \mathbf{u}_1 = 0
\]  

(48a)

\[
K \rho_0 \nabla \cdot \mathbf{u}_1 - i \omega p_1 = 0
\]  

(48b)
where subscript \( h \) was used to denote horizontal entities, \( \Gamma \) stands for the earth’s angular frequency and \( h \) is the separation constant called equivalent depth. Here \( \nu \) is an effective collision frequency which simulates the ion drag and viscous force effects (see Volland & Mayr, 1977).

After elimination of \( \nabla_h \cdot \mathbf{u}_h \) in (45) and (47b) we get a system of linear inhomogeneous ordinary differential equations of first order in \( r \) comprising (45), (46) and (47a,b). Conversely (48a,b) constitute a system of homogeneous differential equations of first order in \( \theta \) know as Laplace equation. The solutions of this last system are expressed in terms of the Legendre polynomials (eigenfunctions) which determine the actual structure of all upper atmosphere parameters and is expressed by the function \( G \) presented in (36). The standard procedure consist in writing \( \mathbf{u}_h = -\nabla \Phi(\theta) + \nabla \times \Psi(\theta) r^2 \epsilon \) and then solve (48a and b) to determine \( \Phi(\theta), \Psi(\theta) \) and \( h \) (eigenvalue of the Laplace equation).

The details about the solution of the tidal equation will not be presented here since they can be found elsewhere (e.g. Volland & Mayr, 1977; Forbes & Garrett, 1979 and references therein). Instead, we call attention to the fact that the importance of the study of tidal waves comes from three characteristics of them:

1) Diurnal and semidiurnal tidal oscillations are excited by \( \text{H}_2\text{O} \) and \( \text{O}_3 \) absorption in the troposphere, stratosphere and lower mesosphere.

2) Upward propagating tides are not attenuated below 90 km.

3) Tidal oscillations are subject to significant dissipation by ion drag, eddy and molecular diffusion of heat and momentum above 90 km.

Therefore, tidal oscillations constitute an important transport mechanism of energy from the lower to the upper atmosphere.

The upper atmosphere wavelike phenomena start being systematically studied after the landmark work of Hines (1960), who established the bases for the study of gravity waves. Tidal atmospheric phenomena received special attention after the pioneering works of Lindzen (1966) and Volland (1966); in these earlier days the trend can be understood this way: the tidal perturbation in the thermospheric parameters is a function of time, longitude, latitude and altitude, \( f(t, \lambda, \theta, \phi) \), and approaches were attempted in order that an approximation of the type:

\[
f(t, \phi, \theta, r) \approx f(t) f_\phi(\phi) f_\theta(\theta) f_\lambda(\lambda)
\]  

(49)
could be used. This method certainly holds for planar geometry (gravity waves theory) and can be extended for spherical geometry if the equations are oversimplified (classical tidal theory). A review on this matter was presented by Lindzen & Chapman (1969).

The late 60’s and earlier 70’s were characterized by attempts to conform tidal and gravity waves theory into an unified approach. Along this line we have the works by Volland (1969); Spizzichino (1969a,b,c, 1970a,b) and Lindzen (1970).

Hong & Lindzen (1976) showed that the joint presence of Coriolis and viscous forces prevents the separation of the vertical from the latitudinal dependence of the thermospheric parameters in spherical geometry. One must then resort to the solution of the full system of four differential equations comprising the three momentum equations and the energy equation by numerical methods (see Forbes & Garrett, 1979). Part of the advantage of the spectral models is then lost by the non separability of the equations.

The approach presented here is close to that reviewed by Volland & Mayr (1977) which reduces the quality of the numerical results in favor of separability and analytical solutions, more appropriated to be used in empirical methods. A general review commenting on the shortcomings of each methodology was presented by Lindzen (1979).

During the 80’s the efforts were concentrated to conform theoretical simulations with actual observations of tidal oscillations (e.g. Forbes, 1982a,b), as well as to determine the extent of influence of the energy brought by these oscillations to the lower and upper thermosphere and ionosphere (e.g. Groves & Forbes, 1985).

Recent efforts are concentrated on the amount of momentum and energy transferred by tidal oscillations from lower to the upper atmosphere (e.g. Forbes et al., 1993; Miyahara & Forbes, 1994).

Tidal studies are appropriate to account for the thermosphere dynamics at low and middle latitudes. Tidal oscillations also modulate the behavior of the ionized particles, up to altitudes of 600 km. The technique is not used for the protonosphere. At high latitudes impulsive driving sources of weather phenomena can trigger gravity waves.

### NUMERICAL MODELS

These models are otherwise called theoretical models or physical models. They are characterized for the allowance for an arbitrary time dependence for the parameters. To this extent they are suitable to more
properly model the actual time behavior of the upper atmosphere than the preceding approaches. Conversely they require much more computational work which make them feasible if huge computer facilities are available.

The methodology consists in transforming the differential equations into difference equations which are solved using finite difference techniques. Finite element techniques, which to a first order reduce to the finite differences, can be looked as a possible step towards improving numerical computations. The system of equations is divided into two sets: the diagnostic and the prognostic equations. The first set includes the continuity, the hydrostatic (vertical momentum) and the equation of state (30) which hold no matter the instant of time. The second set comprises the energy and the horizontal momentum equations, which forecast the future state of the model. This division is very similar to the one adopted in Section 4. The vertical variable is replaced according to the relation:

\[ z = \ln \left( \frac{p_0}{p} \right) \]  

which is based on (33) and therefore the horizontal equations are solved on an isobaric surface. This surface is covered with a grid that has latitudinal and longitudinal step sizes which depend on the computer capacity. Anyway, the separation of the grid knots cannot be larger than the horizontal scale length. Also, the largest vertical separation allowed between isobaric surfaces is the scale height. The radial range is then covered by unequally spaced computational results. Finally, the time resolution is constrained on the one hand by the scale time of the phenomenon modeled and on the other hand by the computer limitation. Therefore, this method may not conform to all the available computer facilities.

To date two groups could satisfactorily model the upper atmosphere using this method: the NCAR (National Center for Atmospheric Research) in the U.S.A. and the U.C.L. (University College, London) in England. They use CRAY type computers and are now able to handle coupled thermospheric-ionospheric models with this approach. Electrodynamic coupling and feedback was also introduced in the NCAR model (Richmond et al., 1992), using steady state versions of (14) and (15). They established the resolution limits as: 5° for latitude, 20° for longitude, 20 pressure levels and 5 min. time steps.

The numerical procedure consists in writing the horizontal differential operators in terms of finite difference operators and then getting at each grid element a differential equation depending on time and altitude as:

\[ \frac{\partial^{2} y}{\partial t} = A \frac{\partial^{2} y}{\partial z^{2}} + B \frac{\partial y}{\partial z} + C y + \phi^* \]  

(51)
where underlined symbols denote vectors and symbols underlined twice represent matrices. For thermospheric models (see Fuller-Rowell & Rees, 1980; Washington & Williamson, 1977; Schunk, 1988a; Schunk & Sojka, 1996a), the solution vectors, $\mathbf{s}$ at each mesh point has the form:

$$
\mathbf{s} = \left( T \mathbf{u}_0, \mathbf{u}_0 \right)^t
$$

(52)

where the superscript $t$ stands for transpose. The vector $\mathbf{a}^*$ is the forcing vector per unit mass which is equivalent to an acceleration. For ionospheric models the density $\rho$ replaces $T$ (see Schunk, 1988a).

The nonlinear terms present in (51) prevent analytical solutions of it to be obtained. Therefore one must rely on numerical techniques to solve (51). The use of Rayleigh friction and Newtonian cooling can reduce (51) to a first order equation but still with nonlinear characteristics.

In order to solve the above differential equation upper and lower boundary conditions must be specified. The constraint at the top is that $\partial \mathbf{s} / \partial z = 0$ and the restriction at the lower boundary is that $u_\phi = u_\theta = 0$ in the lower limit and the pressure surface be isotermal. More recently the lower boundary conditions for the horizontal velocities and pressure surface were modified to account for the tidal energy transferred from the lower atmosphere (Forbes et al., 1993). Initial conditions are imposed to integrate (51).

The influence of composition changes in global thermospheric models was assessed by the NCAR (Dickinson et al., 1984) and by the UCL (Fuller-Rowell & Rees, 1983) groups using different schemes. Regardless the undertaken approach the method consists in assuming thermal equilibrium among the different species and then handling each constituent set of continuity and momentum equations to conform with the average determined density and velocity for the whole mixture. Density composition and diffusion velocities are computed at each step of integration; or, as in Fuller-Rowell & Rees (1983) use is made of the mean molecular mass for a binary mixture and its continuity equation of the form:

$$
\frac{\partial m}{\partial t} + u_0 \cdot \nabla m + u_\phi \partial m / \partial p - \\
- D \nabla^2 m - (\mathbf{H}_n)^{-1} \rho \partial (\mathbf{H}^{-1} D m \partial m / \partial p) / \partial p - \\
- (\mathbf{H}_n)^{-1} \rho \partial \left[ \mathbf{H}^{-1} D m (m - m_0) (m - m_2) \right] / \partial p = 0
$$

(53)

where $m_1$ and $m_2$ are respectively the masses of the major and minor constituents.

In spite of the considerable efforts done so far (see Rasmussen & Schunk, 1990) the incorporation of a plasmaspheric model in conjunction with the already existing thermospheric-ionospheric coupled
models has not being done yet. Among the problems encountered were the questions about the adequacy of the existing plasmaspheric models, to properly quantify the actual phenomena occurring under quiet and disturbed conditions. More recent effort (Wilson et al., 1992), however, indicate that regardless of the type of the model (kinetic or fluid) the effective restriction is connected with the RHS of the modeling equations. To this extent the latest works on elastic collisions and production-loss mechanisms (Wilson et al., 1992; Zamlutti, 1994, 1998a,b) are promising.

Bailey & Sellek (1990) presented a comprehensive plasmasphere numerical model in agreement with the current trend for numerical processing. Solving the fluid equations along a magnetic field line they obtained a diffusion equation of the form (51) where the components for the vector \( \mathbf{\gamma} \) are the densities of \( \rm O^+, H^+ \) and \( \rm He^+ \) and another identical equation for the ion temperatures. The electron temperature is also computed with an equation of the form (51). Since the ion motion is organized along the magnetic-field lines and decoupled from the perpendicular drift, it can be computed with a modified version of (42) and the problem reduced to one dimension. Its extension to three dimensions is done using the horizontal velocity:

\[
\mathbf{\nu}_k = \mathbf{B} \times \mathbf{r} + \mathbf{E} \times \mathbf{D} / D^2
\]  

(54)

The additional assumptions of plasma neutrality and a basic ambipolar flow allow the determination of the electron parameters from the ion ones.

High latitude coupled ionosphere-protonospheric models are based on the same grounds as the plasmaspheric models, namely:

a) Charged particles motion is organized along magnetic field lines (one dimensional problem).

b) Field lines and perpendicular drifts are decoupled.

c) Charge neutrality and ambipolar flow characteristics holds.

Also the use of the eight-moment approximation for the distribution function discards the effect of stresses and the heat flux is computed with appropriate equations. Under these circumstances (51) can be reduced to a first order differential equation (see Cordier & Girard, 1996) and becomes the basic equation to determine the field lines characteristics of the parameters. Since \( \mathbf{B} \times \mathbf{r} \) decreases considerably the horizontal velocities will be due to the external electric fields according to (54). This results was pointed out by Schunk (1983).
Considering the time dependent equations for the equatorial and low latitude ionosphere it is simple to show that the density variation follows the diffusive equation which is essentially (51).

Thermospheric numerical models started to be developed as early as 1962 with the work by Harris & Priester (1962). It was a one dimensional model which presented theoretical inconsistencies but contributed to show that the solar EUV energy could not be the only heating source of the thermosphere. Soon Dickinson & Geisler (1968) proposed the first model with three-dimensional characteristics.

During the 70’s the model by Stubbe (1970) considered the solution of the full coupled system of hydrodynamic equations for neutral and charged particles. He considered ten particle species with a total of 50 equations which were reduced to a system of 10 equations. The results did not improve significantly the quality of thermosphere modeling but contributed significantly to the understanding of the coupling mechanisms between neutral and charged particles. Dickinson et al. (1975, 1977) finally set down the guidelines for the present three-dimensional global models of the thermosphere and confirmed the need of two energy sources: solar heating and joule heating to properly reproduce experimental measurements. In the 80’s numerical models were considerably improved with the development of huge computer facilities. Initially the bulk thermospheric characteristics were determined (Fuller-Rowell & Rees, 1980; Dickinson et al., 1981). Next an account for two or more neutral species was considered (Fuller-Rowell & Rees, 1983; Dickinson et al., 1984). This was followed by an extension to coupled thermosphere-ionosphere models (Fuller-Rowell et al., 1987; Roble et al., 1988). Finally electrodynamical coupling was incorporated in the combined thermosphere-ionosphere models (Richmond et al., 1992).

Eq. (51) to determine the ion densities was used by Stubbe (1970). However, the methodology of using the Lagrangian framework for the solution of the transport equations in ionospheric models was employed by Schunk & Walker (1973) and adopted since then (e.g. Fuller-Rowell, 1987; Sojka, 1989).

During the 70’s and earlier 80’s regional models were developed for middle latitude (Stubbe, 1970; Schunk & Walker, 1973; Roble, 1975; Young et al., 1980) and high latitudes (Schunk et al., 1975; Schunk & Raitt, 1980). Combined global models appeared in the late 80’s (Schunk, 1988a; Sojka, 1989).

Protonospheric numerical models started in the earlier 70’s (e.g. Schunk & Walker, 1973; Moffett & Murphy, 1973; Banks et al., 1974) but it was only in the late 70’s that plasmaspheric models with interhemispheric fluxes of plasma, which use (51) to determine the ion densities appeared (see Bailey et al., 1978). During the 80’s various groups dedicated their efforts to plasmaspheric modeling (e.g. Young et al., 1980; Bailey et al.,
and several questions were raised concerning the influence of chemistry, collisions, kinetic or fluid approaches, existence of thermal anisotropics. More recent models use the fluid approach (Rasmussen & Schunk, 1990; Bailey & Sellek, 1990).

High latitude, charged particles numerical models were developed since the late 60’s using hydrodynamic approaches (Banks & Holzer, 1969; Schunk et al., 1975). During the 70’s kinetic models received considerable attention (Lemaire, 1972; Lemaire & Scherer, 1973). In the 80’s semi-kinetic models were focussed (Barakat & Schunk, 1983). More recent efforts use the eight-moment approximation of Blelly & Schunk (1993) with the advantage of reducing the problem to one dimension along the magnetic field line (Robineau et al., 1996; Cordier & Girard, 1996; Diloy et al., 1996). Comparisons between semi-kinetic and hydrodynamic models (Demars & Schunk, 1991) showed close agreement which led us to conclude that regardless of the type of the model the effective constraint in the quality of the results is connected with the RHS approximations.

Although the essentials for numerical modeling of the equatorial and low-latitude ionosphere were reviewed by Moffett (1979) effective implementation occurred since then. Ingenious working versions of numerical models were developed by Bailey et al. (1993) and Bittencourt (1996).

OUTLOOK

The three fundamental trends of upper atmosphere modeling present distinct aspects and each one fits specific needs. The semi empirical models conform better with reality since their data bases are experimental measurements. Spectral models describe reasonably the regular space-time behavior of the upper atmosphere and are based on spectral analysis of observed data. Numerical models are more representative of the physical mechanisms involved in the upper atmosphere response to the solar radiation excitation. Conversely, each alternative to modeling presents inherent shortcomings resulting from their form to deal with the modeled phenomena. Semi-empirical models exclude transient responses during the curve fitting procedure and are dependent on representative indices to account for the driving energy mechanisms. Spectral methods require a large number of harmonics to model transient responses and are dependent on the effectiveness in the description of the energy sources. Numerical models require also a large amount of computational work and are presently restricted to macrocomputers.

Three elements have been considered by Zamlutti (1990) as relevant in atmospheric modeling:
a) The way to handle the system of fundamental equations.

b) The choice of the boundary conditions.

c) The effectiveness of the considered models of the energy driving mechanisms.

Regarding item a) the three alternatives to upper atmosphere modeling present strict similarity. All the trends tend to separate the vertical (or equivalently the field aligned) equations from the horizontal (or perpendicular) ones. Besides the computations are carried out to account for the bulk particles (major species) characteristics. Particular species parameters are derived based on diffusive equilibrium.

As for item b) all modeling alternatives essentially agree since the preferred choice goes to time instants and places when and where the driving mechanisms amplitudes are minima.

As far as item c) is concerned the different trends are alike. The terms modeling the various driving mechanisms can all be added in the RHS of the basic equations. This is equivalent to a multiplicative weighting amplitude of the upper atmosphere response. The use of energy indices (solar and magnetic) in the semi-empirical models has long been pointed out as a restrictive aspect of this option but studies carried out were inconclusive (see Roemer et al., 1983; Killeen, 1987; Joselyn, 1995). Conversely, the storm time models of high latitude sources are usually considered as limiting elements of the quality of spectral and numerical models (see review by Rees, 1995).

In a balance it results that all the modeling approaches end up been equivalent. The spectral or spherical harmonics models can be considered as the space time integral transform of the numerical methods. The semi-empirical models discard some less significant terms of the vertical profile equations but compensate this by conforming the simple analytical profile to the actual observed shape function. When extending them with the global function one is making a formal equivalence with the space time behavior of spectral models.

Detailed comparisons showing collected data on density, temperature and velocity and their respective models have been carried out by Schunk & Szuszczewicz (1988); Rees & Fuller-Rowell (1992); Schunk & Sojka (1992, 1996b); Sojka et al. (1992); Rees (1995). It was realized that regardless the type of modeling fine-structured space features and rapid temporal variations still challenge upper atmosphere modellers Considering their results and what was presented in this work one can summarize for users:

a) Semi-empirical models provide a simple bases for comparisons with data. They account for the climatological component of the results.
Discrepancies must be analysed as weather aspects and eventual measurements errors.

b) High quality numerical models are able to produce weather information if their driving terms refer specifically to the considered time interval. Deviations in this case are most probably due to inaccuracies in the data taking and processing procedure. Otherwise only the climatological component is reliable.

CONCLUSIONS

In this work a brief overview of some of the essential aspects of the three basic trends of the upper atmosphere modeling was made. It could be concluded that concerning the average response of the upper atmosphere to forcing mechanisms all models are equivalent. Transient responses are, however, better modeled by numerical methods.

From what has been shown in this work we can suggest a few guidelines for future research:

a) Search for effective simple analytical expression for density and temperatures of charged particles.

b) Make improvements on the model to represent the effect of molecular viscosity and ion drag in the simplified tidal theory to still keep the equation separable.

c) Research more thorough going forms to solve the vertical differential equation of the numerical models.

Action on this last suggestion direction started with Schunk (1988b) but is still in progress (Lu et al., 1995; Millward et al., 1996).

ACKNOWLEDGEMENTS

Support for this work came through Instituto Nacional de Pesquisas Espaciais (INPE), from Financiadora de Estudos e Projetos (FINEP) under contract 537/CT. The author received also a complementary fellowship from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under the process 300901/90 (RN). The computer facility that helped the preparation of the manuscript was provided by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).
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SOBRE AS TENDÊNCIAS BÁSICAS PARA MODELAGEM ATMOSFÉRICA - UMA REVISÃO -

A atmosfera superior é um meio composto por partículas neutras e carregadas, que interagem entre si, estão sujeitas a ação de excitação externa de origem solar e restritas pela ação de campos: gravitacional, elétricos e magnético. Modelar este...
meio é encontrar uma descrição matemática satisfatória sobre seus comportamentos temporal e espacial.

O comportamento médio de cada constituinte da atmosfera superior pode ser descrito pelas equações de Navier Stokes:

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{u} = \delta \rho
\]

\[
\rho \frac{d\mathbf{u}}{dt} + \nabla \mathbf{p} - \rho \mathbf{g} - \mathbf{v} = \delta \mathbf{M}
\]

\[
(3/2)\frac{d\mathbf{u}}{dt} + (3/2)\rho \nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{q} + \mathbf{v} = \delta \mathbf{q}
\]

onde, \(\rho\), \(\mathbf{u}\) e \(p\) representam respectivamente densidade de massa, velocidade de fluxo e pressão; \(\mathbf{v}\) é o tensor de esforços e \(\mathbf{g}\) o vetor de fluxo de calor. O lado direito das equações acima expressam as perturbações produzidas nesses parâmetros pelas ações de interações mútuas (colisões elásticas) e excitação externa (produção e perda de partículas). O vetor \(\mathbf{a}\) exprems a força por unidade de massa que atua sobre a partícula considerada.

Os métodos analíticos e empíricos baseiam-se em soluções matemáticas para um sistema reduzido das equações básicas. Podem representar o comportamento local (perfis verticais) dos parâmetros ou global (variações horizontais). As expressões usadas para produzi-los consideram nulo o tensor de esforços, condições de estabilidade (\(d/dt=0\)) e situação estática (\(\mathbf{u} = 0\)). Neste caso:

a) A densidade, para partículas neutras, permanece constante para cada altura, no decorrer do tempo. Para partículas carregadas, ela é determinada pelo equilíbrio entre produção e perda das mesmas.

b) A equação dos momentos complementada pela restrição \(\delta \mathbf{M} = 0\) e pela equação de estado, \(p=nKT\) onde \(n\) é densidade numérica de partículas, \(K\) a constante de Boltzmann e \(T\) a temperatura absoluta, determina o comportamento da pressão com a altura.

c) A equação da energia pode ser aproximada pela fórmula de Bates:

\[
T_{0} = T_{0} - (T_{0} - T_{0}) \left[ \exp(-sz) - z_{0} \right]
\]

onde \(z\) é altura, \(s\) um parâmetro ajustável os índices 0 e infinito indicam extremos inferior e superior do modelo.

Os modelos globais utilizam expressões obtidas pelos métodos de transformadas integrais.

Os métodos de transformadas integrais são usados para converter uma equação diferencial em equação algébrica. No caso são utilizadas variações do tipo \(\exp\left\{\omega (t+\phi)\right\}\) onde \(\omega\) é a velocidade angular, \(t\) o tempo e \(\phi\) a longitude (em unidades de tempo). Adota-se o método de perturbações para resolver as equações. Assim obtém-se:

a) A perturbação linear da densidade depende do divergente da perturbação linear de velocidade.
b) A equação horizontal dos momentos estabelece a dependência latitudinal das perturbações.

c) A dependência vertical das variações lineares é obtida usando-se as equações vertical dos momentos e da energia.

Os comportamentos latitudinal e vertical das perturbações lineares são interligados pelo divergente horizontal de velocidades. Utiliza-se uma constante de separação para usá-los. Obtém-se a equação de Laplace cuja solução é expressa em polinômios de Legendre, para dependência latitudinal.

Os métodos numéricos permitem aos parâmetros uma dependência temporal arbitrária. Consistem em transformar as equações diferenciais em equações de diferenças finitas. O sistema é dividido em 2 conjuntos de equações: diagnósticas e prognósticas.

a) O primeiro conjunto inclui a equação da continuidade, a hidrostática e a equação de estado cuja solução independe do tempo.

b) O segundo conjunto compreende as equações da energia e dos momentos horizontais que prevêem o comportamento futuro do modelo.

Para a variável vertical usa-se a relação

A divisão para diferenças finitas implica numa cobertura horizontal em forma de grade. Para cada elemento da grade obtém-se nova equação diferencial dependente de tempo e da altura na forma:

$$\frac{\partial \xi}{\partial t} = A\xi \frac{\partial^2 \xi}{\partial z^2} + B\xi \frac{\partial \xi}{\partial z} + C\xi + a^t$$

(5)

onde $\xi$ é o vetor das variáveis prognósticas e $a^t$ o vetor das excitações.

Cada um dos métodos preenchem necessidades específicas. O semi-empírico fornece uma informação média sobre a atmosfera superior. O de transformadas integrais faz uma análise espectral do comportamento esperado da atmosfera para uma dada excitação. Os métodos numéricos descrevem mais adequadamente a evolução temporal dos fenômenos.

NOTES ABOUT THE AUTHORS
NOTAS SOBRE OS AUTORES

ON THE BASIC TRENDS OF THE UPPER ATMOSPHERE MODELING - A REVIEW

Carlos José Zamlutti


Alguns artigos atuais são:


