Consistent On-Board Multipath Calibration for GPS-Based Spacecraft Attitude Determination

Roberto V. F. Lopes and Paulo G. Milani
National Institute for Space Research – INPE, Brazil

BIOGRAPHY

Roberto Lopes was born in Rio de Janeiro, Brazil, in 1955. B.S. in Aeronautical Engineering in 1977 from the Aeronautical Institute of Technology and M.Sc. and Dr. degrees in Space Sciences in 1982 and 1898 from the National Institute for Space Research, INPE, Brazil. Post-doctoral fellow at The University of Maryland in 1994. Senior Staff at INPE where he works since 1978.

Paulo Milani was born in 1955, received the Dr. degree in Space Sciences since 1997 and the M. Sc. in the same area in 1988, both from INPE. He is the current manager of the Sub-orbital Platform (PSO) at INPE, a micro-gravity platform launched on sounding rockets. His interests are in real time simulation, guidance, control and GPS.

ABSTRACT

Aspects of the on-board calibration of spacecraft three-axis attitude determination systems employing the GPS phase-observable signal are examined. More specifically, a new consistent on-board estimation algorithm is proposed that uses a special set of calibration coefficients for multipath mitigation. A two-dimensional Taylor series in a convenient planar-projection coordinate system is used to model the global distortion effect of the GPS line of sight due to multipath. This selected projection avoids distortion and singularities close to the antenna zenith. Based on a previous solution to the similar problem of on-board star-sensor calibration, the model is purged of all misalignment-like components to cope with the inherent lack of observability. The algorithm was tested in the presence of a strong multipath effect using an empirical model based on a ground experiment data set. The test considered an inertially-stabilized satellite in a low-Earth orbit with low inclination. The data span covered 24 hours of continuous simulation with no *a priori* attitude information. The results were compared with those obtained when exact *a priori* attitude knowledge was included using a ground calibration algorithm. Both algorithms present the same significant gain in accuracy. The simulation results show that this methodology should be employed in order to assure precise autonomous on-board attitude determination using the GPS.

INTRODUCTION

The research on autonomous navigation, guidance and control of low Earth orbiting satellites has been greatly improved with the advent of the Global Positioning System (GPS). Several GPS-based schemes for autonomous navigation solution, attitude determination and other related subjects have been developed[1-11]. In particular, the important aspect of multipath interference on GPS observable has been analyzed and ground calibration algorithms proposed[12-14].

Multipath delay is the most relevant error source in attitude measurements from GPS interferometry. Baseline and antenna-phase-center uncertainties are also among the factors limiting attitude-determination accuracy. In addition, the space environment reduces the effectiveness of pre-launch calibrations. On the other hand, the problem of on-board calibration of spacecraft attitude sensors presents observability aspects that has not yet been carefully considered. Specifically, it is shown that in space those errors are not completely separable from the attitude itself or from three-axis misalignment of the antenna frame. Furthermore, they are not separable from one another. Therefore, in such scenarios ground calibration schemes may even lead to meaningless, inconsistent estimates.

The aim of this paper is to present a consistent on board calibration algorithm for multipath mitigation in order to allow a precise autonomous attitude determination for a spacecraft using the GPS. The paper states the problem similarity with the on board calibration problem for star sensors and proposes a constrained calibration function based on the ideas presented in a previous work[15].

In the following, basic concepts on GPS observables and their main error sources are overviewed as an introduction to the well-known technique of GPS interferometry for attitude determination. Afterwards the problem of multipath ground calibration is reviewed and differences with respect to the on board calibration problem are highlighted. A new algorithm to on board autonomous multipath calibration is then proposed in section 6 and its mathematical development presented in section 7. Finally, the performance of the new algorithm is compared with that obtained by a ground calibration algorithm that uses a priori attitude knowledge. The multipath simulation model to test the algorithms is based on experimental results[14].

THE GPS OBSERVABLES

According to the specialized literature (see Leick[16], 1995, for instance), all GPS satellites broadcast their messages in two frequencies, namely L1 and L2, which are respectively 154 and 120 times the fundamental frequency of 10.23 MHz. Both L1 and L2 carriers are phase modulated by the GPS navigation message at 50 bps. The navigation message contains ephemeris data of the GPS constellation and clock correction parameters. The carriers are also phase modulated by the pseudo-random noise codes. Those codes provide range information. Because each GPS satellite transmit a different segment of the pseudo-random noise (PRN) code at a time, GPS receivers can lock each GPS satellite separately. GPS satellites are so labeled by the respective week reference number of the PRN code they use. The encrypted P(Y) precision code is available for military navigation only and modulates both L1 and L2. The coarse acquisition code C/A is available for civilian users and modulates L1.

GPS receivers use the same pseudo-random noise codes to lock the satellites by means of cross-correlation techniques. Once a GPS satellite is locked its navigation message is demodulated and the code delay (the so called pseudo-range) is measured.

Code observable is suitable for orbit determination, but is affected by error sources like ionosphere and troposphere delays and the intentional degradation of the C/A code, namely the selected availability (SA). A position fix and the receiver clock bias can be evaluated from the C/A code observable if at least four GPS satellites are simultaneously locked. For high accuracy (sub-meter level) orbit determination, the carrier phase observable becomes necessary.

Also for attitude determination the code observable is not acceptable, especially in spacecraft applications where typical baselines are too short in face of the pseudo-range accuracy. In this case, the phase observable is rather appropriate, as described in the next section. This paper considers the carrier phase observable L1 with no loss of generality.

Although phase observable is more accurate than code observable it is still affected by clock instability and by ionosphere and troposphere delays, as well as by hardware and multi-path delays.

GPS INTERFEROMETRY

Let $\wp_k$ be the set of PRN of all GPS locked satellites at a given sample time t_k; T the sample size; $p \in \wp_k$ the upper script referring to the p-th GPS satellite; u the line of sight unit vector of a GPS satellite from the body position in the reference frame; w the same unit vector in the body frame and A_k the spacecraft attitude matrix at t_k. The attitude matrix is such that:

$$w_k^p = A_k u_k^p \ , \ \forall p \in \wp_k \ , \ k \in [1, T] \tag{1}$$

Attitude determination can be performed if the line of sight of at least two GPS locked satellites are known in both reference and body frames at every sampling time. Since the spacecraft position can be determined from the pseudo-range data set and the GPS satellite ephemeris are provided by the navigation message, their line of sight vectors in the reference frame u_k^p can be straightforwardly computed.

As for w_k^p, they are usually obtained from the carrier phase observable by means of the interferometry technique, as follows. A set of N antennas (usually 3 or 4) linked to a common GPS receiver are placed on a suitable satellite surface, one of them arbitrarily chosen as the master antenna (labeled by the under script 0) and the others taken as slave ones (labeled by the under script i). Thus, a set of N-1 non co-linear master-slave baselines are formed. The single difference of carrier phase for every antenna baseline allow the evaluation of w_k^p as better explained in the sequence.

Let $\varphi_{i,k}^p$ be the carrier phase L1 of the p-th GPS satellite received by the i-th antenna at the k-th sample time. Let $\Delta\varphi_{i,k}^p$ be the between-antenna single-difference of the carrier phase observable L1:

$$\Delta\varphi_{i,k}^p \equiv \varphi_{i,k}^p - \varphi_{0,k}^p \ , \forall i \in [1, N-1], \ p \in \wp_k, \ k \in [1, T], \tag{2}$$

Let r_i be the position vector of the i-th antenna in the body frame and Δr_i the relative position of the i-th slave antenna with respect to the master one:

$$\Delta r_i \equiv r_i - r_0 \ , \ \forall i \in [1, N-1]. \qquad (3)$$

Ideally, the single difference of carrier phase is a linear observation of the respective line of sight unit vector (see Figure 1):

$$\Delta\bar{\varphi}_{i,k}^p = \frac{2\pi}{\lambda}\Delta r_i{}^T w_k^p \ , \forall i \in [1, N-1], p \in \wp_k \ , k \in [1, T]], \qquad (4)$$

where the over bar indicates ideal value; the upper symbol T indicates transpose; and λ is the L1 wavelength. In principle, given a set of single differences from at least two non co-linear baselines, it would be straightforward to solve equation (4) for w_k^p.

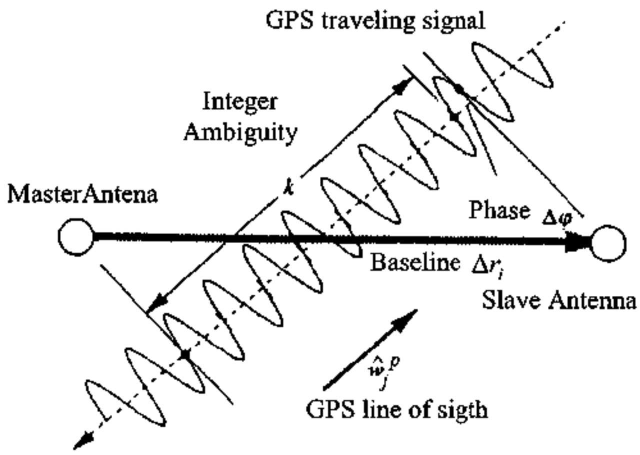

Fig. 1 – Geometry of the carrier phase as an attitude observation.

Once one has a set of w_k^p and u_k^p Eq. 1 can be solved for the attitude matrix A as usual in most attitude determination problems. When using GPS observations a dual problem is often considered. It comes from the fact that the scalar product is invariant to rotations. So the term $\Delta r_i{}^T w_k^p$ may be replaced by its counterpart $\Delta s_i{}^T u_k^p$, where Δs_i is the antenna baseline vector in the reference frame. If two or more GPS satellites are locked, Eq. 4 may be solved for Δs_i and the attitude matrix determined from the fact that $\Delta r_i = A_k \Delta s_{i,k}$. The first approach is followed in this paper.

In any case the carrier phase observable presents a number of drawbacks that ask for different treatments. First, there is an integer ambiguity due the fact that there may be a limited but unknown integer number $K_{i,k}^p \in \mathbf{Z}$ of whole wavelengths between the antennas. They are due to the receiver arbitrary initialization of an internal counter and are also subject to unpredicted cycle slips.

Second, the attitude observation technique provided by GPS interferometry is affected by the following internal error sources: receiver clock instability; antenna phase center displacements (mechanical or electromagnetic); hardware (cables and receiver electronics) delay; and thermal random noise.

Finally, the carrier phase observable is corrupted by external error sources like ionosphere, troposphere and multi-path delays.

Actually, some of the above mentioned error sources affect the carrier phase evenly over all the antennas and therefore are canceled out when the single difference is performed. They are the atmosphere related delays (which are actually not relevant to space applications) and the receiver clock instability (assuming that the antennas are linked to a common receiver since otherwise they would present independent clock instability). Therefore the phase single-difference may be more realistically represented by:

$$\begin{aligned}\Delta\varphi_{i,k}^p = {} & \Delta\bar{\varphi}_{i,k}^p + 2\pi K_{i,k}^p + \frac{2\pi}{\lambda}\delta r_i{}^T w_k^p + d_i \\ & + \delta\varphi_{i,k}(w)\big|_{w=w_k^p} + \varepsilon_{i,k}^p\end{aligned} \qquad (5)$$

where δr_i represents the error on the i-th baseline due to phase center variations; d_i represents the hardware delay; $\delta\varphi_{i,k}$ the multi-path delay and $\varepsilon_{i,k}^p$ the random noise.
Integer ambiguity resolution is a subject out of the scope of this article, but a reliable algorithm for it is given in[17].

Antenna phase center variation is a constant bias plus a repeatable function of the line of sight unit vector, while baseline misalignment δr_i is constant. In principle, their effect on $\Delta\varphi_{i,k}^p$ could be ground calibrated. The hardware delay is a scalar, constant bias and could be ground calibrated as well. Indeed, some GPS receivers offer a self-positioning feature that allows to determine the baseline length and the hardware delay during the equipment setup section that may last for tens of minutes. Within the least squares approach the effect of random noise is not critical as well, for it decreases when the sample size increases.

Multipath is in general more difficult to deal with and has been considered the accuracy limiting factor in current algorithms for GPS based attitude determination. Carrier diffraction on the dynamically changing geometry of the surrounding environment causes multipath. So, $\delta\varphi_{i,k}$ is a deterministic, temporal function of the GPS line of sight in the body frame. Multipath mitigation techniques usually recommended include a careful choice of the antennas placement and the use of ground planes and chokers. Calibration algorithms have also been proposed and claimed to be the key point for a precise attitude determination using GPS.

In space applications the multipath becomes a nearly time invariant function of w, not distinguishable from any other repeatable error function of w. Therefore, equation (5) may be rewritten in a more compact form:

$$\Delta\varphi_{i,k}^{p} = \Delta\overline{\varphi}_{i,k}^{p} + 2\pi K_{i,k}^{p} + \delta\varphi_{i}(w)\,|_{w=w_k^p} + \varepsilon_{i,k}^{p} \quad . \tag{6}$$

where the under script k has been removed from the multipath delay term to indicate its time invariance. $\delta\varphi_i$ may be seen as generalized distortion functions of the GPS line of sight in the body frame that includes the antenna phase center variation, hardware and multipath delays altogether. Such a distortion is accountable for most of the inaccuracy on attitude determination by GPS and can be estimated by empirical curve fitting as described in the next section.

GROUND CALIBRATION

From this point one considers that the integer ambiguity has been solved. Also, deterministic errors that are repeatable function of a GPS line of sight are represented by the multi-path distortion function $\delta\varphi_i$, with no loss of generality. Under these assumptions, Eqs. 4 and 6 yields:

$$Y_{i,k}^{p} = \frac{2\pi}{\lambda} r_i^{\,T} w_k^{p} + \delta\varphi_i(w)\,|_{w=w_k^p} + \varepsilon_{i,k}^{p} \ , \tag{7}$$

where $Y_{i,k}^{p} \equiv \Delta\varphi_{i,k}^{p} - 2\pi K_{i,k}^{p}$.

The distortion function is usually modeled by a series of surface spherical harmonics[18] given by:

$$\begin{aligned}\delta\varphi_i \;=\; & J_0^i + \sum_{n=1}^{O} \Big\{ J_n^i P_{n,0}(\cos\theta) + \\ & \sum_{m=1}^{n} P_{n,m}(\cos\theta)\big[C_{n,m}^i \cos m\phi + S_{n,m}^i \sin m\phi\big] \Big\}\end{aligned} \tag{8}$$

where O is the order of the series; $P_{n,m}$ are the Legendre associate functions; $J_n^i, C_{n,m}^i, S_{n,m}^i$ are respectively the zonal and tesseral calibration coefficients to be determined (note that the antenna label i became here an upper script for the sake of the clarity); φ and θ are respectively the azimuth and co-elevation of the line of sight w:

$$w = \begin{Bmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ \sin\theta \end{Bmatrix} . \tag{9}$$

Assuming that on ground the attitude matrix is a priori known during the calibration phase, the calibration coefficients may be estimated by least squares:

$$\text{Min} \sum_{\forall k} \sum_{\forall p \in \wp_k} \left(\delta Y_{i,k}^{p}\right)^2$$

$$\delta Y_{i,k}^{p} \equiv Y_{i,k}^{p} - \left[\frac{2\pi}{\lambda} r_i^{\,T} A_k u_k^{p} + \delta\varphi_i(w)\,|_{w=w_k^p}\right] , \tag{10}$$

The observations $Y_{i,k}^{p}$ could be theoretically predicted from a physical diffraction model[13]. The approach would allow a more evenly distribution data basis over the antenna hemisphere. The other possibility uses real measurements on the ground, as reported by Cohen and Parkinson[12] (1991). Both possibilities suffer from non-modeled disturbances. In any case, if T^p is the sample size to the p-th GPS satellite the total number of observed phase single-differences and calibration coefficients are:

$$\text{Number of observations: } \sum_{\forall p \in \wp_*} T^p \ , \quad \wp_* \equiv \left\{ \bigcup_{\forall k} \wp_k \right\} \tag{11}$$

$$\text{Number of coefficients: } 1 + O\frac{3+O}{2} . \tag{12}$$

To have it in numbers, for instance, for a set of 8 GPS satellites continuously locked (not necessarily simultaneous) by 50 sampling times each and an 8th order calibration function, it gives 400 observations and 45 calibration coefficients to be separately estimated for every baseline. Here estimate equations are uncoupled.

Figure 2 shows typical antenna phase patterns generated by the spherical harmonics model. Their coefficients were based on an empirical curve fitting of measurements taken during a GPS experiment section at DLR, where two sheet metal plates were placed in a baseline with two antennas to intentionally induce a strong multi-path scenario[14].

HINDRANCES IN THE ON-BOARD CALIBRATION

In general, distortion may be due to thermo-mechanical or optical-electric-magnetic effects. Although the importance of ground calibration can not be neglected, the space environment is not easily reproduced on ground and is subject to temporal variations. Strong vibrations during the launch, micro-gravity effects and thermal cycles may also reduce the effectiveness of ground calibrations. Furthermore, sometimes a realistic and careful ground calibration can not be accomplished due to operational aspects among other possible reasons. Therefore, on board calibration may be necessary if the accuracy of a GPS based attitude determination has to be improved.

When the satellite has other attitude sensors besides the GPS one, they could be used to independently determine the a priori attitude to the calibration process. In this case the on board calibration process becomes identical to the ground one, but the achieved accuracy is of course limited by the accuracy of the a priori attitude. This situation may be useful when the other attitude sensors present observation outages, predicted or not, like temporary star sensor blockage, or sensor failures.

When an independent, a priori attitude information is either not available or not accurate enough to be taken into account, some important differences apply that would have to be carefully analyzed.

As a first immediate consequence, the problem becomes coupled and it would be necessary to estimate the attitude together with the whole set of calibration coefficients:

$$\text{Min} \sum_{\forall k} \sum_{\forall i} \sum_{\forall p \in \wp_k} \left(\delta Y_{i,k}^{p}\right)^2 \quad , \tag{13}$$

subject to the constraint given by Equation 1.

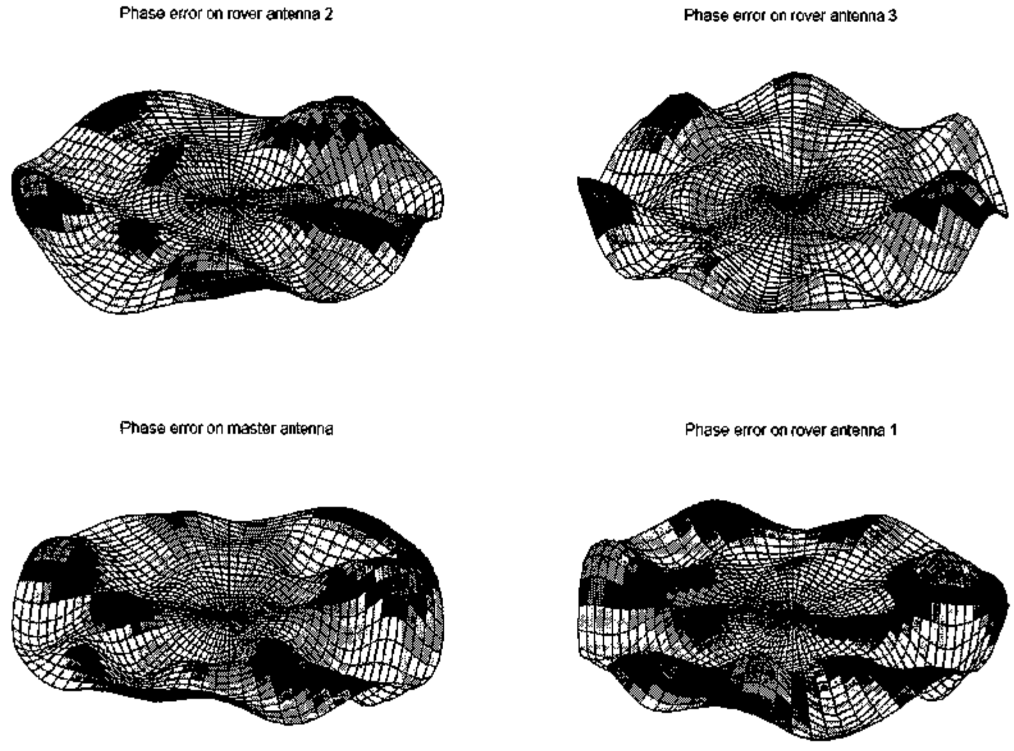

Fig. 2 – Examples of antenna phase patterns due to multipath delay.

Now the number of observations and coefficients are:

- Number of observations: $(N-1) \sum_{\forall p \in \wp_*} T^p$; (14)
- Number of independent coefficients:

$$(N-1)\left(1 + O\frac{3+O}{2}\right) + 3T^* \; ; \tag{15}$$

where T^* represents the total number of sampling times. Using the same values of the previus section example, for a given set of 3 baselines and an optimistic assumption that 8 GPS satellites (not necessarily the same ones) remain simultaneously locked during the whole sample period, it would amount 1200 observations and 285 coefficients to be jointly estimated. This means a considerable increase of computational effort.

The second important difference is subtle but may lead to divergent or meaningless results. The problem is related with the system observability. It happens that the solution of the minimization problem stated in equation 13 is not unique. Actually, for an order high enough, any attitude true motion could be represented as well by a series of spherical harmonics within an arbitrary accuracy level. Therefore, as it is known for instance in the star sensor calibration field, misalignments can not be distinguished from distortion effects in space[15].

At last, but not at least, if the number of baselines N-1 is greater than 2, the set of calibration coefficients that minimizes equation (13) is not unique, even under the false assumption that a unique attitude solution could be virtually fitted, and consequently the set of GPS line of sights consistently estimated.

Summarizing, the lack of a priori attitude knowledge in space has a coupling effect on the calibration equations, the system degrees of freedom becomes unrealistically high and the estimates of the calibration coefficients consequently inconsistent.

THE ON-BOARD CALIBRATION FUNCTION

From what has been above depicted, it is clear that calibration algorithms need to be adapted to autonomous on board applications. In this section one presents a new approach to on board consistent estimation of calibration coefficients. The multipath is represented by its indirect overall distortion effect on the GPS line of sight derived from the phase observable rather than by its direct effect on those measurements. Furthermore, such new "global" distortion function must be unable to represent an infinitesimal rotation about any body axis, following a recipe recommended on the similar problem of on-board calibration of star sensors[15].

Some advantages of using series of spherical harmonics to model the multi-path delay are:

- It is suitable to represent a continuous function on the sphere with no distortion;
- It is a linear combination of the calibration coefficients;
- There are recurrent formulas to the Legendre polynomials and associated functions.

Nevertheless, regarding the present applications, it has also a handicap: it presents a singularity in both coordinate system poles, where all azimuth coordinates collapse to a single point. Now, in most of the attitude determination proposed procedures the antenna baselines are coplanar, the antenna zenith being normal to the baselines. Therefore, the pole is towards the antenna zenith, a region of fundamental importance to the proposed algorithm. For this reason, the spherical harmonics model does not seem to be a convenient way to represent the proposed global calibration function and was so discarded.

A Taylor series on the stereographic azimuthal projection of the sphere (see Figure 3) was found to suit better to the problem for three main reasons:

- It is also able to represent a continuous function on the sphere, linear combination of the calibration coefficients;
- It has no singularities on the antenna boresight;
- It has no distortion close to the antenna zenith, where multi-path delay is remarkably small.

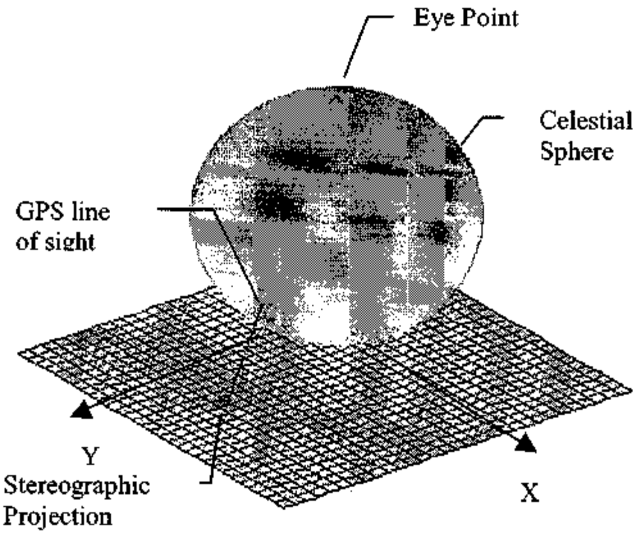

Fig. 3 – The Stereographic Projection

CONSISTENT ON-BOARD CALIBRATION

Based on such a global distortion model, in this section one proposes a recursive algorithm in four steps, as follows.

a) Initialization step:

- Set calibration coefficients $a_{n,m}$ and $b_{n,m}$ to zero;
- Estimate non-calibrated line of sight vectors w_k^p from the phase observable:

$$\underset{w}{\text{Min}} \sum_{\forall i} \left\{ Y_{i,k}^p - \frac{2\pi}{\lambda} r_i^T w_k^p \right\}^2 \quad , \ \forall k \ , \ \forall p \in \wp_k \quad , \qquad (16)$$

$$\text{Subject to: } \left| w_k^p \right| = 1 \ \text{ and } \ E_x w_k^p > \sin\mu \quad , \qquad (17)$$

where μ is the antenna mask angle and $E_z \equiv [0 \ \ 0 \ \ 1]$.

b) Calibration step:

- Evaluate planar coordinates of line of sight using the stereographic projection:

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \frac{2E_{xy}w_k^p}{1+E_z w_k^p}\ , \text{ where } E_{xy} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\ ; \qquad (18)$$

- Correct planar coordinates of line of sight using the global calibration function given by a two-dimensional Taylor series up to order O with calibration coefficients $a_{n,m}$ and $b_{n,m}$:

$$\begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix} = \begin{Bmatrix} x \\ y \end{Bmatrix} + \delta P\ , \text{ where } \delta P = \sum_{n=0}^{O}\sum_{m=0}^{O-n} \begin{Bmatrix} a_{n,m} \\ b_{n,m} \end{Bmatrix} x^n y^m\ ; \qquad (19)$$

- Evaluate calibrated cartesian coordinates of line of sight $\hat{w}_k^p$:

$$\hat{w}_k^p = \begin{Bmatrix} 4\hat{x} \\ 4\hat{y} \\ 4-\hat{x}^2-\hat{y}^2 \end{Bmatrix} \frac{1}{4+\hat{x}^2+\hat{y}^2}\ ; \qquad (20)$$

c) Attitude estimation step:

- Given $\hat{w}_k^p$ and u_k^p, solve the Wahba[19] problem using the well known algorithm QUEST[20], for instance.

d) Coefficient updating step:

- Estimate calibration coefficients $a_{n,m}$ and $b_{n,m}$ of the global distortion function under the Shuster's constraint[15]:

$$\underset{a,b}{\mathrm{Min}} \sum_{\forall k} \sum_{\forall p \in \wp_k} \left| \begin{Bmatrix} x-\bar{x} \\ y-\bar{y} \end{Bmatrix} + \delta P(a,b) \right|^2\ , \qquad (21)$$

$$\text{Subject to } \begin{cases} \delta P|_{0,0} = 0 \\ \nabla \times \delta P|_{0,0} = 0 \end{cases}\ , \qquad (22)$$

$$\text{or equivalently: } \begin{cases} a_{0,0} = 0 \\ b_{0,0} = 0 \\ a_{0,1} - b_{1,0} = 0 \end{cases} \qquad (23)$$

$$\text{where } \begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} = \frac{2E_{xy}A_k u_k^p}{1+E_z A_k u_k^p}\ , \qquad (24)$$

Steps b, c and d are repeated until a convergence criterion is achieved.

The last step is the critical one in terms of computational effort. The number of observations and independent parameters to be estimated are:

- Number of observations: $2\sum_{\forall p \in \wp_*} T^p$; (25)
- Number of independent parameters:

$$2\left(1+O\frac{3+O}{2}\right)+3T^*-3\ ; \qquad (24)$$

The numerical example in this case has 800 observations and 237 independent parameters to be estimated. There are less parameters here than on section 5, but also less observations. For a pessimistic scenario with 4 GPS satellites at sight during 100 sampling times the number of parameters to be determined would be 387 to the same number of observations.

NUMERICAL RESULTS

In this section numerical results from digital simulations are presented. Table 1 summarizes the simulation scenario. The performance of the proposed algorithm is compared with that of the ground calibration using the same algorithm but without the attitude determination step, since the attitude is already known in this case. In both cases, during the calibration section the algorithm estimates the calibration coefficients, while at performance evaluation section only the first three steps are executed.

Figure 4 shows what happens when no constraint is applied to the distortion model during on board calibration without any a priori attitude estimate. Estimates a_0, b_0 and $(a_{0,1}-b_{1,0})/2$ are not consistent due to the already mentioned lack of observability (see section 5).
Nevertheless, the mean value of coefficients $a_{0,1}$ and $b_{1,0}$ converges. When the Shuster's constrain is imposed, the only retained part ($a_{0,1} \equiv b_{1,0}$) still converges consistently.

Figure 4 shows the GPS tracks for both periods and a comparison of the distortion effect on the GPS line of sight on stereographic projection coordinates. The advantage of calibration is clear in both cases. One should note the existence of a region not observed during the 24 hours and still empty on the next 12 hours. The phenomenon is due to the GPS constellation orbit geometry. Therefore, even if one can not expect a good calibration of this region it has no practical consequence, as far as the attitude remains stable.

Figure 6 compares the performance of both algorithms in terms of error probability distribution observed during the simulation and shows the effect of an increasing number of GPS satellites on attitude error. Because the proposed distortion function was constrained to be free of misalignment-like terms, the performance is slightly worse in terms of attitude global error.

Remarkably, the residual on the GPS line of sight are practically the same. Indeed, both solutions achieved the same index of performance, what confirms that the minimization problem does not has a unique solution if the distortion function is not constrained, as previously explained in section 5.

Finally, Table 2 summarizes the algorithm performance during this numerical example.

CONCLUSIONS

A new algorithm for on board calibration of multi-path and any other type of phase delay dependent on the body frame coordinates only was presented. The global distortion effect on the GPS line of sight written in stereographic projection planar coordinates was modeled using two-dimensional Taylor's series. It was shown that observability problems arise when attitude has to be estimated together with the calibration coefficients. Specifically, coefficients that simulate a three degree of freedom misalignment do not converge. The proposed recipe is to purge them from the distortion model. On simulation results based on experimental measurements in an intentionally strong multi-path scenario, the algorithm yielded a considerable reduction of the attitude global error, only slightly worse than the a priori attitude knowledge would allow.

ACKNOWLEDGES

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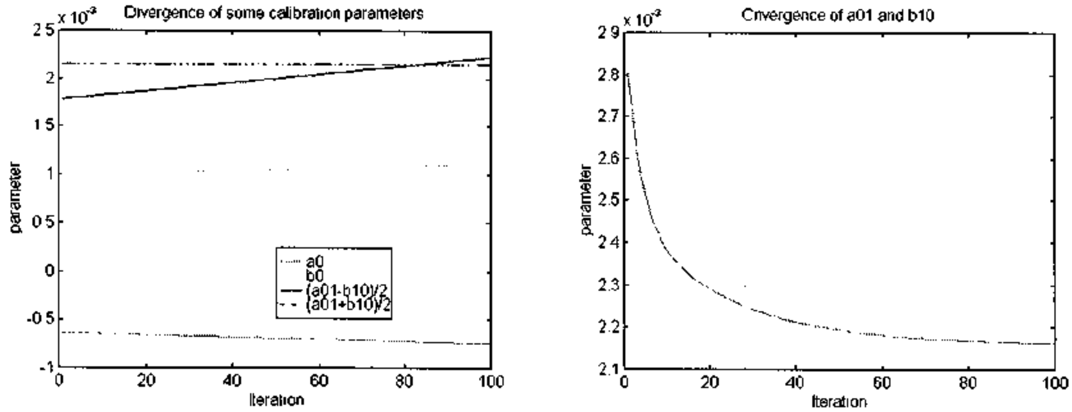

Fig 4 – Estimation of some calibration coefficients: a) unconstrained distortion model: conergence of $(a_{0,1}+b_{1,0})/2$ but divergence of a_0, b_0, and $(a_{0,1}-b_{1,0})/2$; b) constrained distortion model: convergence of $a_{0,1} \equiv b_{1,0}$.

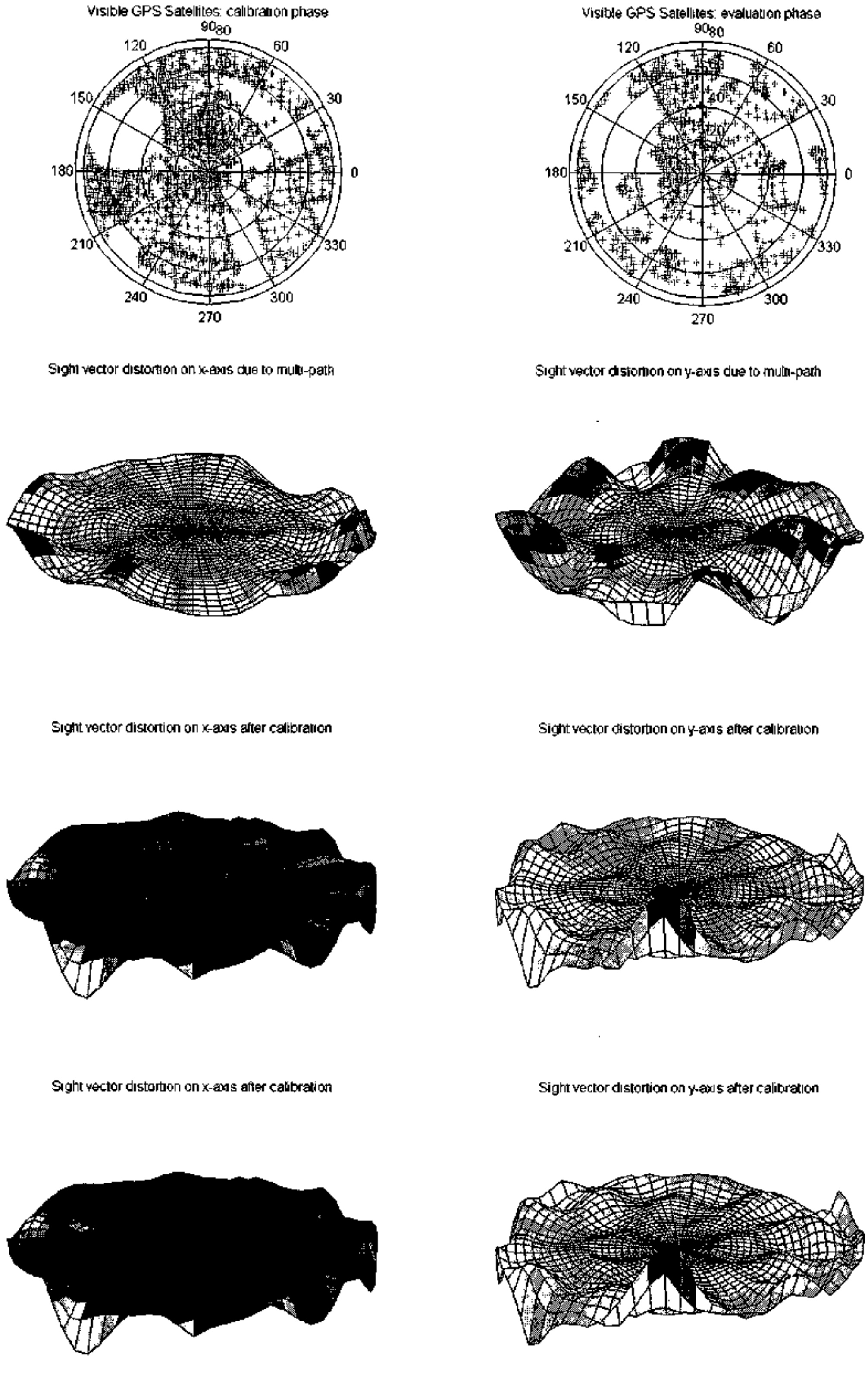

Fig. 5 – Antenna tracks of GPS satellites during: a) calibration period; b) performance evaluation period; **and distortion on stereographic projection coordinates:** c) before calibration, *x*-axis; d) before calibration, *y*-axis; e) after on board calibration, *x*-axis; f) after on board calibration, *y*-axis; g) after ground calibration, *x*-axis; h) after ground calibration, *y*-axis.

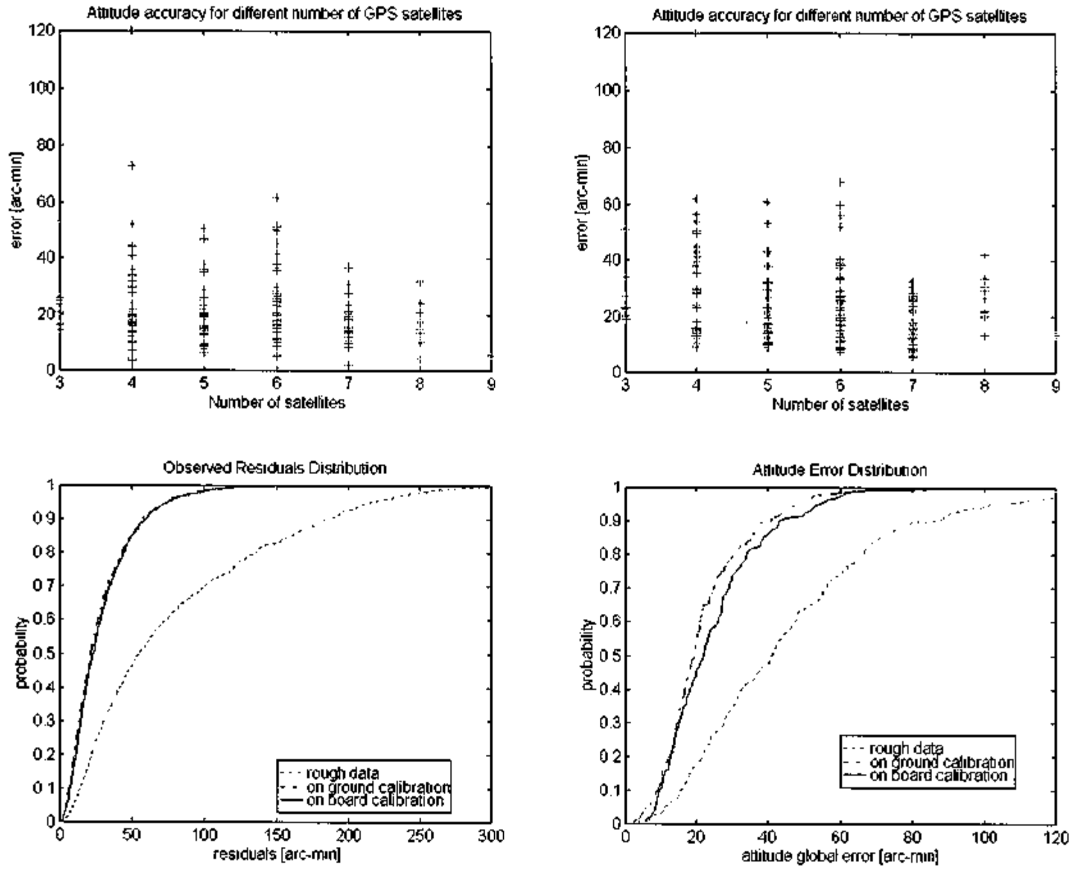

Fig. 6 – Influence of number of GPS locked satellites on attitude global error: a) ground calibration; b) on board calibration; **and error distribution on 12 hours of simulation:** c) residuals on GPS line of sight; d) attitude determination global error.

Table 1 – Simulation Scenario

Number of antennas:	4
Antennas configuration:	coplanar, at the corners of a square with side 80 cm. long.
Antenna mask angle:	15°
Carrier phase noise:	increases linearly from .33mm at zenith to 3.3mm at horizon (1σ)
Multi-path:	spherical harmonics model 15^{th} order, based on DLR's experiment with intentionally strong multi-path environment (see Fig. 2)
Calibration model:	two dimensional Taylor's series model, 10^{th} degree
Sample period:	24hr on calibration phase and 12hr on performance evaluation phase
Sample interval:	300s
Orbit & attitude:	circular; altitude: 625Km; inclination: 23°; attitude inertially stabilized

Table 2 – Performance of Multi-Path Calibration during 12 hours

Calibration type	Distortion on line of sight [arc-min]		Global attitude error [arc min]	
	rms	95%	rms	95%
rough data	104	215	53	104
on board	37	74	30	53
ground	36	74	27	47

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Consistent On-Board Multipath Calibration for GPS-Based Spacecraft Attitude Determination

Roberto V. F. Lopes and Paulo G. Milani
National Institute for Space Research – INPE, Brazil

BIOGRAPHY

Roberto Lopes was born in Rio de Janeiro, Brazil, in 1955. B.S. in Aeronautical Engineering in 1977 from the Aeronautical Institute of Technology and M.Sc. and Dr. degrees in Space Sciences in 1982 and 1898 from the National Institute for Space Research, INPE, Brazil. Post-doctoral fellow at The University of Maryland in 1994. Senior Staff at INPE where he works since 1978.

Paulo Milani was born in 1955, received the Dr. degree in Space Sciences since 1997 and the M. Sc. in the same area in 1988, both from INPE. He is the current manager of the Sub-orbital Platform (PSO) at INPE, a micro-gravity platform launched on sounding rockets. His interests are in real time simulation, guidance, control and GPS.

ABSTRACT

Aspects of the on-board calibration of spacecraft three-axis attitude determination systems employing the GPS phase-observable signal are examined. More specifically, a new consistent on-board estimation algorithm is proposed that uses a special set of calibration coefficients for multipath mitigation. A two-dimensional Taylor series in a convenient planar-projection coordinate system is used to model the global distortion effect of the GPS line of sight due to multipath. This selected projection avoids distortion and singularities close to the antenna zenith. Based on a previous solution to the similar problem of on-board star-sensor calibration, the model is purged of all misalignment-like components to cope with the inherent lack of observability. The algorithm was tested in the presence of a strong multipath effect using an empirical model based on a ground experiment data set. The test considered an inertially-stabilized satellite in a low-Earth orbit with low inclination. The data span covered 24 hours of continuous simulation with no *a priori* attitude information. The results were compared with those obtained when exact *a priori* attitude knowledge was included using a ground calibration algorithm. Both algorithms present the same significant gain in accuracy. The simulation results show that this methodology should be employed in order to assure precise autonomous on-board attitude determination using the GPS.

INTRODUCTION

The research on autonomous navigation, guidance and control of low Earth orbiting satellites has been greatly improved with the advent of the Global Positioning System (GPS). Several GPS-based schemes for autonomous navigation solution, attitude determination and other related subjects have been developed[1-11]. In particular, the important aspect of multipath interference on GPS observable has been analyzed and ground calibration algorithms proposed[12-14].

Multipath delay is the most relevant error source in attitude measurements from GPS interferometry. Baseline and antenna-phase-center uncertainties are also among the factors limiting attitude-determination accuracy. In addition, the space environment reduces the effectiveness of pre-launch calibrations. On the other hand, the problem of on-board calibration of spacecraft attitude sensors presents observability aspects that has not yet been carefully considered. Specifically, it is shown that in space those errors are not completely separable from the attitude itself or from three-axis misalignment of the antenna frame. Furthermore, they are not separable from one another. Therefore, in such scenarios ground calibration schemes may even lead to meaningless, inconsistent estimates.

The aim of this paper is to present a consistent on board calibration algorithm for multipath mitigation in order to allow a precise autonomous attitude determination for a spacecraft using the GPS. The paper states the problem similarity with the on board calibration problem for star sensors and proposes a constrained calibration function based on the ideas presented in a previous work[15].

In the following, basic concepts on GPS observables and their main error sources are overviewed as an introduction to the well-known technique of GPS interferometry for attitude determination. Afterwards the problem of multipath ground calibration is reviewed and differences with respect to the on board calibration problem are highlighted. A new algorithm to on board autonomous multipath calibration is then proposed in section 6 and its mathematical development presented in section 7. Finally, the performance of the new algorithm is compared with that obtained by a ground calibration algorithm that uses a priori attitude knowledge. The multipath simulation model to test the algorithms is based on experimental results[14].

THE GPS OBSERVABLES

According to the specialized literature (see Leick[16], 1995, for instance), all GPS satellites broadcast their messages in two frequencies, namely L1 and L2, which are respectively 154 and 120 times the fundamental frequency of 10.23 MHz. Both L1 and L2 carriers are phase modulated by the GPS navigation message at 50 bps. The navigation message contains ephemeris data of the GPS constellation and clock correction parameters. The carriers are also phase modulated by the pseudo-random noise codes. Those codes provide range information. Because each GPS satellite transmit a different segment of the pseudo-random noise (PRN) code at a time, GPS receivers can lock each GPS satellite separately. GPS satellites are so labeled by the respective week reference number of the PRN code they use. The encrypted P(Y) precision code is available for military navigation only and modulates both L1 and L2. The coarse acquisition code C/A is available for civilian users and modulates L1.

GPS receivers use the same pseudo-random noise codes to lock the satellites by means of cross-correlation techniques. Once a GPS satellite is locked its navigation message is demodulated and the code delay (the so called pseudo-range) is measured.

Code observable is suitable for orbit determination, but is affected by error sources like ionosphere and troposphere delays and the intentional degradation of the C/A code, namely the selected availability (SA). A position fix and the receiver clock bias can be evaluated from the C/A code observable if at least four GPS satellites are simultaneously locked. For high accuracy (sub-meter level) orbit determination, the carrier phase observable becomes necessary.

Also for attitude determination the code observable is not acceptable, especially in spacecraft applications where typical baselines are too short in face of the pseudo-range accuracy. In this case, the phase observable is rather appropriate, as described in the next section. This paper considers the carrier phase observable L1 with no loss of generality.

Although phase observable is more accurate than code observable it is still affected by clock instability and by ionosphere and troposphere delays, as well as by hardware and multi-path delays.

GPS INTERFEROMETRY

Let $\wp_k$ be the set of PRN of all GPS locked satellites at a given sample time t_k; T the sample size; $p \in \wp_k$ the upper script referring to the p-th GPS satellite; u the line of sight unit vector of a GPS satellite from the body position in the reference frame; w the same unit vector in the body frame and A_k the spacecraft attitude matrix at t_k. The attitude matrix is such that:

$$w_k^p = A_k u_k^p \;,\; \forall p \in \wp_k \;,\; k \in [1,T] \tag{1}$$

Attitude determination can be performed if the line of sight of at least two GPS locked satellites are known in both reference and body frames at every sampling time. Since the spacecraft position can be determined from the pseudo-range data set and the GPS satellite ephemeris are provided by the navigation message, their line of sight vectors in the reference frame u_k^p can be straightforwardly computed.

As for w_k^p, they are usually obtained from the carrier phase observable by means of the interferometry technique, as follows. A set of N antennas (usually 3 or 4) linked to a common GPS receiver are placed on a suitable satellite surface, one of them arbitrarily chosen as the master antenna (labeled by the under script 0) and the others taken as slave ones (labeled by the under script i). Thus, a set of N-1 non co-linear master-slave baselines are formed. The single difference of carrier phase for every antenna baseline allow the evaluation of w_k^p as better explained in the sequence.

Let $\varphi_{i,k}^p$ be the carrier phase L1 of the p-th GPS satellite received by the i-th antenna at the k-th sample time. Let $\Delta\varphi_{i,k}^p$ be the between-antenna single-difference of the carrier phase observable L1:

$$\Delta\varphi_{i,k}^p \equiv \varphi_{i,k}^p - \varphi_{0,k}^p \;, \forall i \in [1, N-1],\, p \in \wp_k,\; k \in [1,T], \tag{2}$$

Let r_i be the position vector of the i-th antenna in the body frame and Δr_i the relative position of the i-th slave antenna with respect to the master one:

$$\Delta r_i = r_i - r_0 \ , \ \forall i \in [1, N-1]. \qquad (3)$$

Ideally, the single difference of carrier phase is a linear observation of the respective line of sight unit vector (see Figure 1):

$$\Delta\overline{\varphi}_{i,k}^{p} = \frac{2\pi}{\lambda} \Delta r_i^{\,T} w_k^p \ , \forall i \in [1, N-1], p \in \wp_k \ , k \in [1, T]], \qquad (4)$$

where the over bar indicates ideal value; the upper symbol T indicates transpose; and λ is the L1 wavelength. In principle, given a set of single differences from at least two non co-linear baselines, it would be straightforward to solve equation (4) for w_k^p.

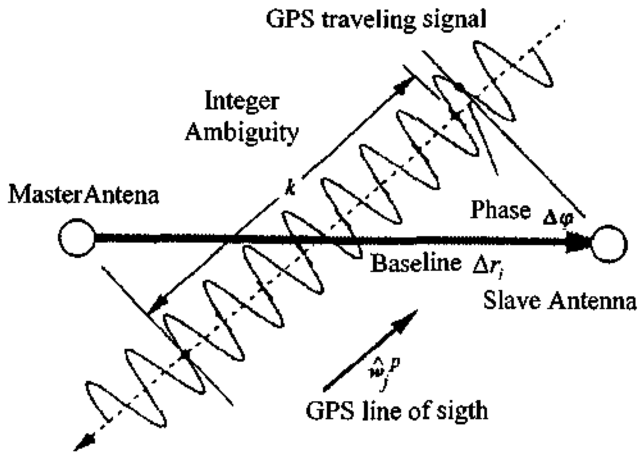

Fig. 1 – Geometry of the carrier phase as an attitude observation.

Once one has a set of w_k^p and u_k^p Eq. 1 can be solved for the attitude matrix A as usual in most attitude determination problems. When using GPS observations a dual problem is often considered. It comes from the fact that the scalar product is invariant to rotations. So the term $\Delta r_i^{\,T} w_k^p$ may be replaced by its counterpart $\Delta s_i^{\,T} u_k^p$, where Δs_i is the antenna baseline vector in the reference frame. If two or more GPS satellites are locked, Eq. 4 may be solved for Δs_i and the attitude matrix determined from the fact that $\Delta r_i = A_k \Delta s_{i,k}$. The first approach is followed in this paper.

In any case the carrier phase observable presents a number of drawbacks that ask for different treatments. First, there is an integer ambiguity due the fact that there may be a limited but unknown integer number $K_{i,k}^p \in \mathbf{Z}$ of whole wavelengths between the antennas. They are due to the receiver arbitrary initialization of an internal counter and are also subject to unpredicted cycle slips.

Second, the attitude observation technique provided by GPS interferometry is affected by the following internal error sources: receiver clock instability; antenna phase center displacements (mechanical or electromagnetic); hardware (cables and receiver electronics) delay; and thermal random noise.

Finally, the carrier phase observable is corrupted by external error sources like ionosphere, troposphere and multi-path delays.

Actually, some of the above mentioned error sources affect the carrier phase evenly over all the antennas and therefore are canceled out when the single difference is performed. They are the atmosphere related delays (which are actually not relevant to space applications) and the receiver clock instability (assuming that the antennas are linked to a common receiver since otherwise they would present independent clock instability). Therefore the phase single-difference may be more realistically represented by:

$$\begin{aligned}\Delta\varphi_{i,k}^{p} = \Delta\overline{\varphi}_{i,k}^{p} + 2\pi K_{i,k}^{p} + \frac{2\pi}{\lambda}\delta r_i^{\,T} w_k^p + d_i \\ + \delta\varphi_{i,k}(w)\big|_{w=w_k^p} + \varepsilon_{i,k}^{p}\end{aligned} \ . \qquad (5)$$

where δr_i represents the error on the i-th baseline due to phase center variations; d_i represents the hardware delay; $\delta\varphi_{i,k}$ the multi-path delay and $\varepsilon_{i,k}^p$ the random noise.

Integer ambiguity resolution is a subject out of the scope of this article, but a reliable algorithm for it is given in[17].

Antenna phase center variation is a constant bias plus a repeatable function of the line of sight unit vector, while baseline misalignment δr_i is constant. In principle, their effect on $\Delta\varphi_{i,k}^p$ could be ground calibrated. The hardware delay is a scalar, constant bias and could be ground calibrated as well. Indeed, some GPS receivers offer a self-positioning feature that allows to determine the baseline length and the hardware delay during the equipment setup section that may last for tens of minutes. Within the least squares approach the effect of random noise is not critical as well, for it decreases when the sample size increases.

Multipath is in general more difficult to deal with and has been considered the accuracy limiting factor in current algorithms for GPS based attitude determination. Carrier diffraction on the dynamically changing geometry of the surrounding environment causes multipath. So, $\delta\varphi_{i,k}$ is a deterministic, temporal function of the GPS line of sight in the body frame. Multipath mitigation techniques usually recommended include a careful choice of the antennas placement and the use of ground planes and chokers. Calibration algorithms have also been proposed and claimed to be the key point for a precise attitude determination using GPS.

In space applications the multipath becomes a nearly time invariant function of w, not distinguishable from any other repeatable error function of w. Therefore, equation (5) may be rewritten in a more compact form:

$$\Delta\varphi_{i,k}^{p} = \Delta\overline{\varphi}_{i,k}^{p} + 2\pi K_{i,k}^{p} + \delta\varphi_i(w)|_{w=w_k^p} + \varepsilon_{i,k}^{p} \quad . \tag{6}$$

where the under script k has been removed from the multipath delay term to indicate its time invariance. $\delta\varphi_i$ may be seen as generalized distortion functions of the GPS line of sight in the body frame that includes the antenna phase center variation, hardware and multipath delays altogether. Such a distortion is accountable for most of the inaccuracy on attitude determination by GPS and can be estimated by empirical curve fitting as described in the next section.

GROUND CALIBRATION

From this point one considers that the integer ambiguity has been solved. Also, deterministic errors that are repeatable function of a GPS line of sight are represented by the multi-path distortion function $\delta\varphi_i$, with no loss of generality. Under these assumptions, Eqs. 4 and 6 yields:

$$Y_{i,k}^{p} = \frac{2\pi}{\lambda} r_i^T w_k^p + \delta\varphi_i(w)|_{w=w_k^p} + \varepsilon_{i,k}^{p} \ , \tag{7}$$

where $Y_{i,k}^{p} \equiv \Delta\varphi_{i,k}^{p} - 2\pi K_{i,k}^{p}$.

The distortion function is usually modeled by a series of surface spherical harmonics[18] given by:

$$\begin{aligned}\delta\varphi_i \ = \ & J_0^i + \sum_{n=1}^{O}\Big\{J_n^i P_{n,0}(\cos\theta) + \\ & \sum_{m=1}^{n} P_{n,m}(\cos\theta)\big[C_{n,m}^i \cos m\phi + S_{n,m}^i \sin m\phi\big)\big]\Big\}\end{aligned} \tag{8}$$

where O is the order of the series; $P_{n,m}$ are the Legendre associate functions; $J_n^i, C_{n,m}^i, S_{n,m}^i$ are respectively the zonal and tesseral calibration coefficients to be determined (note that the antenna label i became here an upper script for the sake of the clarity); φ and θ are respectively the azimuth and co-elevation of the line of sight w:

$$w = \begin{Bmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ \sin\theta \end{Bmatrix} . \tag{9}$$

Assuming that on ground the attitude matrix is a priori known during the calibration phase, the calibration coefficients may be estimated by least squares:

$$\text{Min} \sum_{\forall k} \sum_{\forall p \in \wp_k} \left(\delta Y_{i,k}^{p}\right)^2$$

$$\delta Y_{i,k}^{p} \equiv Y_{i,k}^{p} - \left[\frac{2\pi}{\lambda} r_i^T A_k u_k^p + \delta\varphi_i(w)|_{w=w_k^p}\right] , \tag{10}$$

The observations $Y_{i,k}^{p}$ could be theoretically predicted from a physical diffraction model[13]. The approach would allow a more evenly distribution data basis over the antenna hemisphere. The other possibility uses real measurements on the ground, as reported by Cohen and Parkinson[12] (1991). Both possibilities suffer from non-modeled disturbances. In any case, if T^p is the sample size to the p-th GPS satellite the total number of observed phase single-differences and calibration coefficients are:

$$\text{Number of observations: } \sum_{\forall p \in \wp_*} T^p \ , \quad \wp_* \equiv \Big\{\bigcup_{\forall k} \wp_k\Big\} \tag{11}$$

$$\text{Number of coefficients: } 1 + O\frac{3+O}{2} . \tag{12}$$

To have it in numbers, for instance, for a set of 8 GPS satellites continuously locked (not necessarily simultaneous) by 50 sampling times each and an 8th order calibration function, it gives 400 observations and 45 calibration coefficients to be separately estimated for every baseline. Here estimate equations are uncoupled.

Figure 2 shows typical antenna phase patterns generated by the spherical harmonics model. Their coefficients were based on an empirical curve fitting of measurements taken during a GPS experiment section at DLR, where two sheet metal plates were placed in a baseline with two antennas to intentionally induce a strong multi-path scenario[14].

HINDRANCES IN THE ON-BOARD CALIBRATION

In general, distortion may be due to thermo-mechanical or optical-electric-magnetic effects. Although the importance of ground calibration can not be neglected, the space environment is not easily reproduced on ground and is subject to temporal variations. Strong vibrations during the launch, micro-gravity effects and thermal cycles may also reduce the effectiveness of ground calibrations. Furthermore, sometimes a realistic and careful ground calibration can not be accomplished due to operational aspects among other possible reasons. Therefore, on board calibration may be necessary if the accuracy of a GPS based attitude determination has to be improved.

When the satellite has other attitude sensors besides the GPS one, they could be used to independently determine the a priori attitude to the calibration process. In this case the on board calibration process becomes identical to the ground one, but the achieved accuracy is of course limited by the accuracy of the a priori attitude. This situation may be useful when the other attitude sensors present observation outages, predicted or not, like temporary star sensor blockage, or sensor failures.

When an independent, a priori attitude information is either not available or not accurate enough to be taken into account, some important differences apply that would have to be carefully analyzed.

As a first immediate consequence, the problem becomes coupled and it would be necessary to estimate the attitude together with the whole set of calibration coefficients:

$$\text{Min} \sum_{\forall k} \sum_{\forall i} \sum_{\forall p \in \wp_k} \left(\delta Y_{i,k}^{p}\right)^2 \quad , \tag{13}$$

subject to the constraint given by Equation 1.

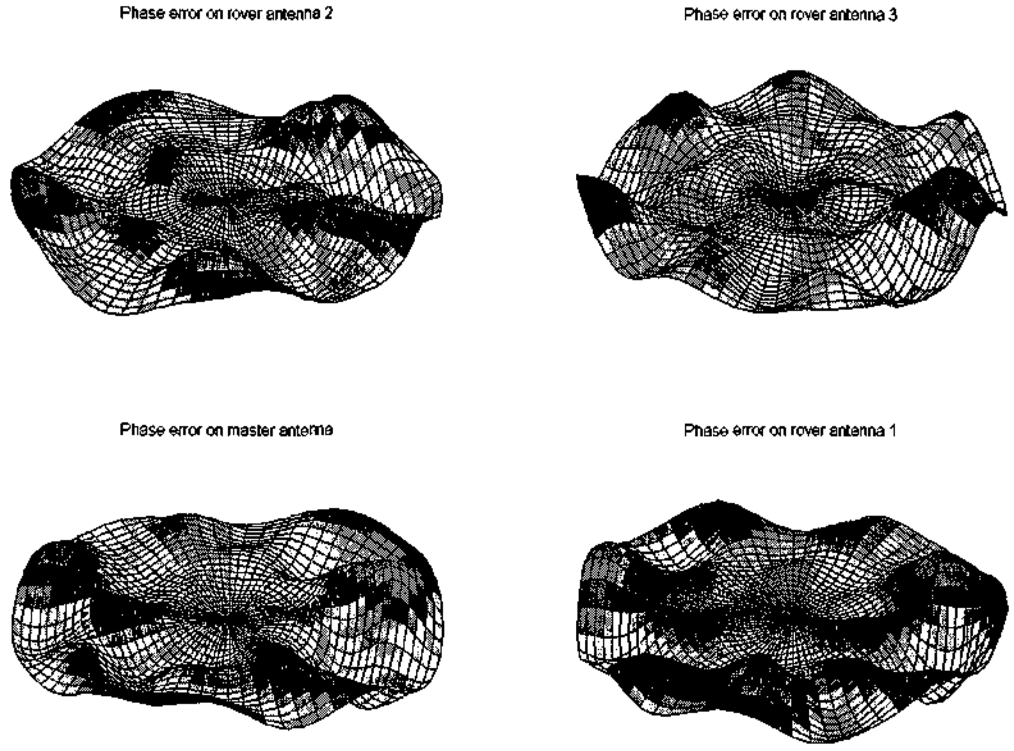

Fig. 2 – Examples of antenna phase patterns due to multipath delay.

Now the number of observations and coefficients are:

- Number of observations: $(N-1) \sum_{\forall p \in \wp_*} T^p$; (14)
- Number of independent coefficients:

$$(N-1)\left(1 + O\frac{3+O}{2}\right) + 3T^* \; ; \tag{15}$$

where T^* represents the total number of sampling times. Using the same values of the previus section example, for a given set of 3 baselines and an optimistic assumption that 8 GPS satellites (not necessarily the same ones) remain simultaneously locked during the whole sample period, it would amount 1200 observations and 285 coefficients to be jointly estimated. This means a considerable increase of computational effort.

The second important difference is subtle but may lead to divergent or meaningless results. The problem is related with the system observability. It happens that the solution of the minimization problem stated in equation 13 is not unique. Actually, for an order high enough, any attitude true motion could be represented as well by a series of spherical harmonics within an arbitrary accuracy level. Therefore, as it is known for instance in the star sensor calibration field, misalignments can not be distinguished from distortion effects in space[15].

At last, but not at least, if the number of baselines *N*-1 is greater than 2, the set of calibration coefficients that minimizes equation (13) is not unique, even under the false assumption that a unique attitude solution could be virtually fitted, and consequently the set of GPS line of sights consistently estimated.

Summarizing, the lack of a priori attitude knowledge in space has a coupling effect on the calibration equations, the system degrees of freedom becomes unrealistically high and the estimates of the calibration coefficients consequently inconsistent.

THE ON-BOARD CALIBRATION FUNCTION

From what has been above depicted, it is clear that calibration algorithms need to be adapted to autonomous on board applications. In this section one presents a new approach to on board consistent estimation of calibration coefficients. The multipath is represented by its indirect overall distortion effect on the GPS line of sight derived from the phase observable rather than by its direct effect on those measurements. Furthermore, such new "global" distortion function must be unable to represent an infinitesimal rotation about any body axis, following a recipe recommended on the similar problem of on-board calibration of star sensors[15].

Some advantages of using series of spherical harmonics to model the multi-path delay are:

- It is suitable to represent a continuous function on the sphere with no distortion;
- It is a linear combination of the calibration coefficients;
- There are recurrent formulas to the Legendre polynomials and associated functions.

Nevertheless, regarding the present applications, it has also a handicap: it presents a singularity in both coordinate system poles, where all azimuth coordinates collapse to a single point. Now, in most of the attitude determination proposed procedures the antenna baselines are coplanar, the antenna zenith being normal to the baselines. Therefore, the pole is towards the antenna zenith, a region of fundamental importance to the proposed algorithm. For this reason, the spherical harmonics model does not seem to be a convenient way to represent the proposed global calibration function and was so discarded.

A Taylor series on the stereographic azimuthal projection of the sphere (see Figure 3) was found to suit better to the problem for three main reasons:

- It is also able to represent a continuous function on the sphere, linear combination of the calibration coefficients;
- It has no singularities on the antenna boresight;
- It has no distortion close to the antenna zenith, where multi-path delay is remarkably small.

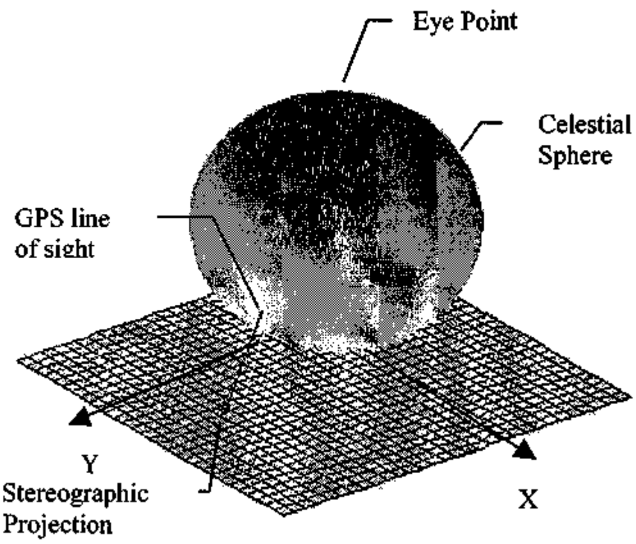

Fig. 3 – The Stereographic Projection

CONSISTENT ON-BOARD CALIBRATION

Based on such a global distortion model, in this section one proposes a recursive algorithm in four steps, as follows.

a) Initialization step:

- Set calibration coefficients $a_{n,m}$ and $b_{n,m}$ to zero;
- Estimate non-calibrated line of sight vectors w_k^p from the phase observable:

$$\underset{w}{\text{Min}} \sum_{\forall i} \left\{ Y_{i,k}^p - \frac{2\pi}{\lambda} r_i^T w_k^p \right\}^2 \ , \ \forall k \ , \ \forall p \in \wp_k \ , \qquad (16)$$

$$\text{Subject to: } \left| w_k^p \right| = 1 \ \text{ and } \ E_z w_k^p > \sin\mu \ , \qquad (17)$$

where μ is the antenna mask angle and $E_z \equiv [0 \ 0 \ 1]$.

b) Calibration step:

- Evaluate planar coordinates of line of sight using the stereographic projection:

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \frac{2E_{xy} w_k^p}{1 + E_z w_k^p} \text{ , where } E_{xy} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ ;} \quad (18)$$

- Correct planar coordinates of line of sight using the global calibration function given by a two-dimensional Taylor series up to order O with calibration coefficients $a_{n,m}$ and $b_{n,m}$:

$$\begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix} = \begin{Bmatrix} x \\ y \end{Bmatrix} + \delta P \text{ , where } \delta P = \sum_{n=0}^{O} \sum_{m=0}^{O-n} \begin{Bmatrix} a_{n,m} \\ b_{n,m} \end{Bmatrix} x^n y^m \text{ ;} \quad (19)$$

- Evaluate calibrated cartesian coordinates of line of sight $\hat{w}_k^p$:

$$\hat{w}_k^p = \begin{Bmatrix} 4\hat{x} \\ 4\hat{y} \\ 4 - \hat{x}^2 - \hat{y}^2 \end{Bmatrix} \frac{1}{4 + \hat{x}^2 + \hat{y}^2} \text{ ;} \quad (20)$$

c) Attitude estimation step:

- Given $\hat{w}_k^p$ and u_k^p, solve the Wahba[19] problem using the well known algorithm QUEST[20], for instance.

d) Coefficient updating step:

- Estimate calibration coefficients $a_{n,m}$ and $b_{n,m}$ of the global distortion function under the Shuster's constraint[15]:

$$\underset{a,b}{\text{Min}} \sum_{\forall k} \sum_{\forall p \in \wp_k} \left\| \begin{Bmatrix} x - \bar{x} \\ y - \bar{y} \end{Bmatrix} + \delta P(a, b) \right\|^2 \text{ ,} \quad (21)$$

$$\text{Subject to } \begin{cases} \delta P|_{0,0} = 0 \\ \nabla \times \delta P|_{0,0} = 0 \end{cases} \text{ ,} \quad (22)$$

$$\text{or equivalently: } \begin{cases} a_{0,0} = 0 \\ b_{0,0} = 0 \\ a_{0,1} - b_{1,0} = 0 \end{cases} \quad (23)$$

$$\text{where } \begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} = \frac{2E_{xy} A_k u_k^p}{1 + E_z A_k u_k^p} \text{ ,} \quad (24)$$

Steps b, c and d are repeated until a convergence criterion is achieved.

The last step is the critical one in terms of computational effort. The number of observations and independent parameters to be estimated are:

- Number of observations: $2 \sum_{\forall p \in \wp_*} T^p$; (25)
- Number of independent parameters:

$$2\left(1 + O\frac{3 + O}{2}\right) + 3T^* - 3 \text{ ;} \quad (24)$$

The numerical example in this case has 800 observations and 237 independent parameters to be estimated. There are less parameters here than on section 5, but also less observations. For a pessimistic scenario with 4 GPS satellites at sight during 100 sampling times the number of parameters to be determined would be 387 to the same number of observations.

NUMERICAL RESULTS

In this section numerical results from digital simulations are presented. Table 1 summarizes the simulation scenario. The performance of the proposed algorithm is compared with that of the ground calibration using the same algorithm but without the attitude determination step, since the attitude is already known in this case. In both cases, during the calibration section the algorithm estimates the calibration coefficients, while at performance evaluation section only the first three steps are executed.

Figure 4 shows what happens when no constraint is applied to the distortion model during on board calibration without any a priori attitude estimate. Estimates a_0, b_0 and $(a_{0,1} - b_{1,0})/2$ are not consistent due to the already mentioned lack of observability (see section 5).
Nevertheless, the mean value of coefficients $a_{0,1}$ and $b_{1,0}$ converges. When the Shuster's constrain is imposed, the only retained part ($a_{0,1} \equiv b_{1,0}$) still converges consistently.

Figure 4 shows the GPS tracks for both periods and a comparison of the distortion effect on the GPS line of sight on stereographic projection coordinates. The advantage of calibration is clear in both cases. One should note the existence of a region not observed during the 24 hours and still empty on the next 12 hours. The phenomenon is due to the GPS constellation orbit geometry. Therefore, even if one can not expect a good calibration of this region it has no practical consequence, as far as the attitude remains stable.

Figure 6 compares the performance of both algorithms in terms of error probability distribution observed during the simulation and shows the effect of an increasing number of GPS satellites on attitude error. Because the proposed distortion function was constrained to be free of misalignment-like terms, the performance is slightly worse in terms of attitude global error.

Remarkably, the residual on the GPS line of sight are practically the same. Indeed, both solutions achieved the same index of performance, what confirms that the minimization problem does not has a unique solution if the distortion function is not constrained, as previously explained in section 5.

Finally, Table 2 summarizes the algorithm performance during this numerical example.

CONCLUSIONS

A new algorithm for on board calibration of multi-path and any other type of phase delay dependent on the body frame coordinates only was presented. The global distortion effect on the GPS line of sight written in stereographic projection planar coordinates was modeled using two-dimensional Taylor's series. It was shown that observability problems arise when attitude has to be estimated together with the calibration coefficients. Specifically, coefficients that simulate a three degree of freedom misalignment do not converge. The proposed recipe is to purge them from the distortion model. On simulation results based on experimental measurements in an intentionally strong multi-path scenario, the algorithm yielded a considerable reduction of the attitude global error, only slightly worse than the a priori attitude knowledge would allow.

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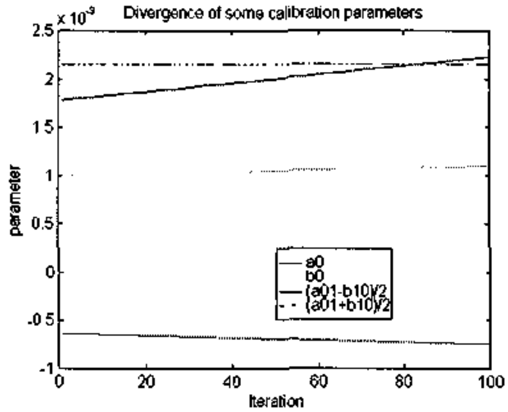

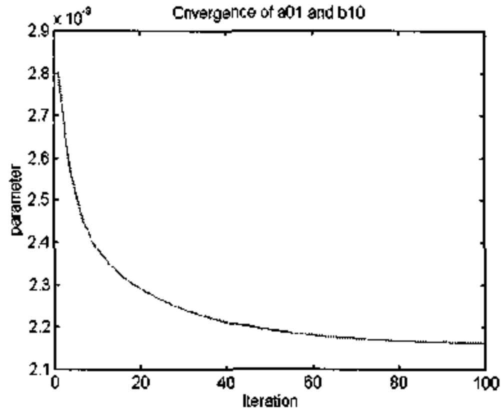

Fig 4 – Estimation of some calibration coefficients: a) unconstrained distortion model: conergence of $(a_{0,1}+b_{1,0})/2$ but divergence of a_0, b_0, and $(a_{0,1}-b_{1,0})/2$; b) constrained distortion model: convergence of $a_{0,1} \equiv b_{1,0}$.

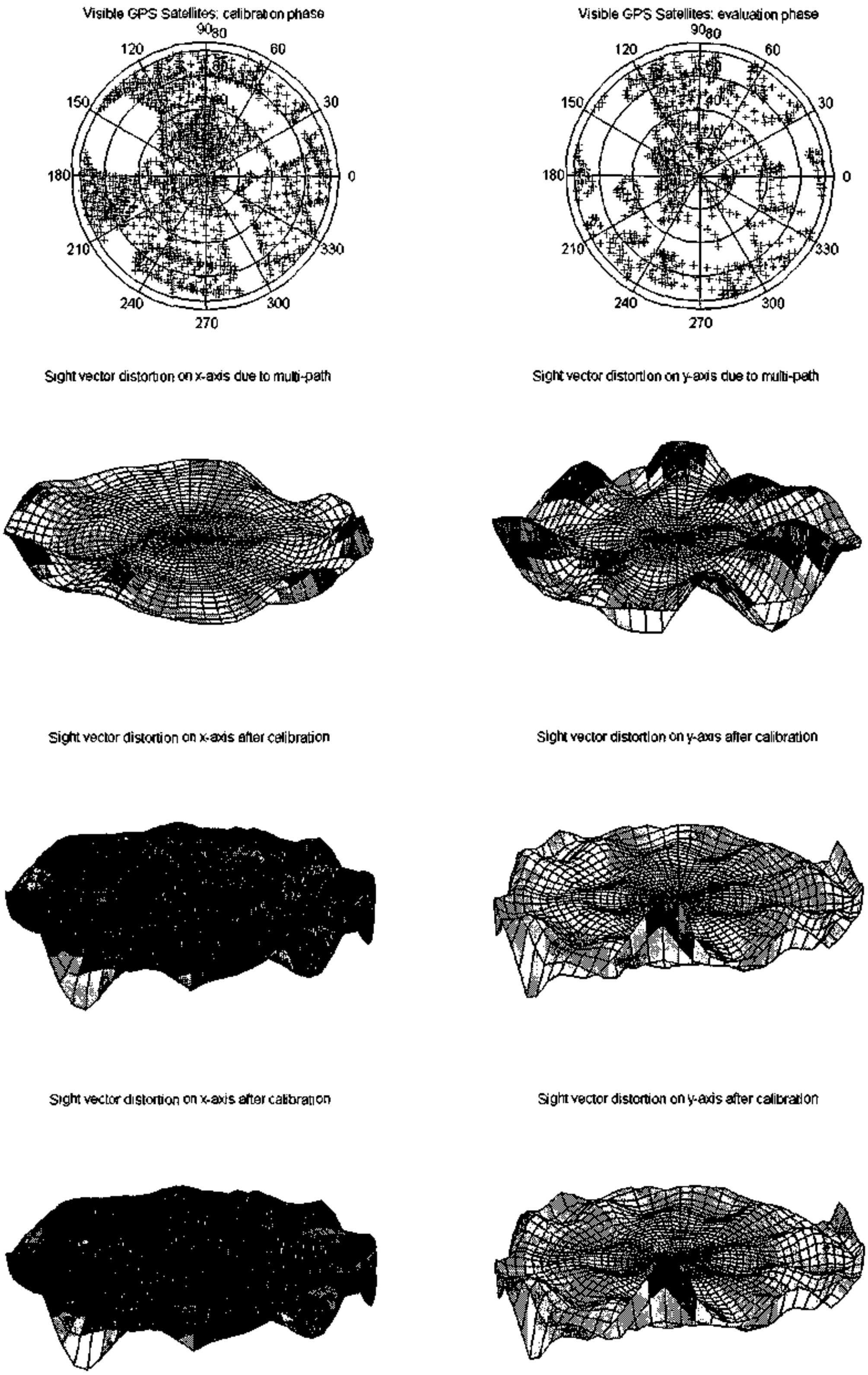

Fig. 5 – Antenna tracks of GPS satellites during: a) calibration period; b) performance evaluation period; **and distortion on stereographic projection coordinates:** c) before calibration, x-axis; d) before calibration, y-axis; e) after on board calibration, x-axis; f) after on board calibration, y-axis; g) after ground calibration, x-axis; h) after ground calibration, y-axis.

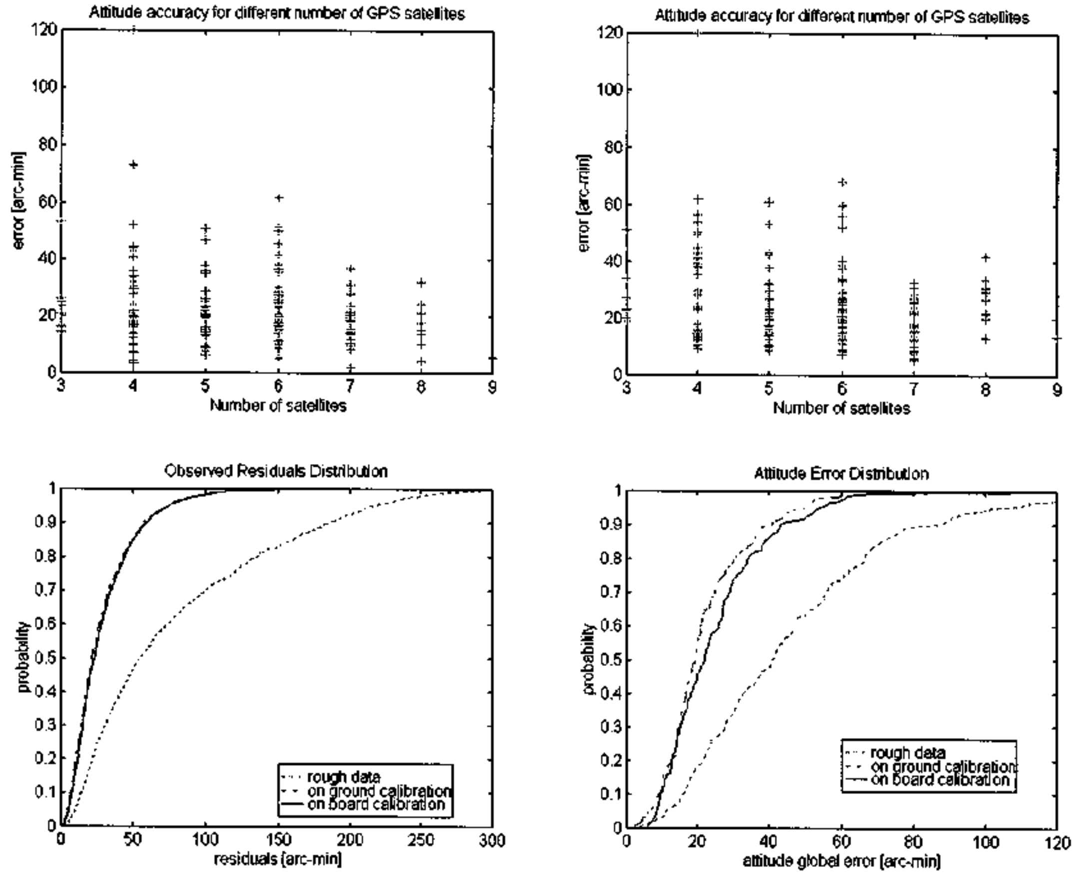

Fig. 6 – Influence of number of GPS locked satellites on attitude global error: a) ground calibration; b) on board calibration; **and error distribution on 12 hours of simulation:** c) residuals on GPS line of sight; d) attitude determination global error.

Table 1 – Simulation Scenario

Number of antennas:	4
Antennas configuration:	coplanar, at the corners of a square with side 80 cm. long.
Antenna mask angle:	15°
Carrier phase noise:	increases linearly from .33mm at zenith to 3.3mm at horizon (1σ)
Multi-path:	spherical harmonics model 15[th] order, based on DLR's experiment with intentionally strong multi-path environment (see Fig. 2)
Calibration model:	two dimensional Taylor's series model, 10[th] degree
Sample period:	24hr on calibration phase and 12hr on performance evaluation phase
Sample interval:	300s
Orbit & attitude:	circular; altitude: 625Km; inclination: 23°; attitude inertially stabilized

Table 2 – Performance of Multi-Path Calibration during 12 hours

Calibration type	Distortion on line of sight [arc-min]		Global attitude error [arc min]	
	rms	95%	rms	95%
rough data	104	215	53	104
on board	37	74	30	53
ground	36	74	27	47

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