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The Presence of Bifurcation in a Closed-Loop Discrete Control of a Flexible Benchmark Plant analyzed from the Jury Stability Criterion

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ABSTRACT

Many interesting phenomena occur when the discrete control is applied over a flexible structure like the aliasing and hidden oscillations. In this work a particular phenomenon called our attention because was a bit strange: increasing the sampling period of a discrete proportional plus derivative closed-loop control (by Tustin or Bilinear method) over a harmonic oscillator (our flexible benchmark plant) we noted that happened regions of stability and instability. Initially we imagined that from some high value of sampling period this control system would instable and it stay instable. This work study analytically this phenomenon and show graphically this bifurcation.

INTRODUCTION

It was looking for a method that algorithmically could describe a stable behavior of a attitude digital control of an asymmetric satellite with 1.4 Ton in the rigid body and 49 kg of flexible appendage that happened the necessity of make some simple theoretical analysis before apply in the real case. Is very difficult to design a digital control of a flexible structure because is impossible ignore these fatal disturbs named aliasing, hidden oscillations, delays in inputs and outputs, quantization error, etc. In this work we consider initially the discrete time fading effects like aliasing and hidden oscillations. The aliasing and hidden oscillations ever will be present because the flexible plant has a infinite bandwidth, due its infinite vibration modes, in contrast with the limited bandwidth of the digital controller. We used the simplest flexible plant in this analysis: the harmonic oscillator. This choice was done to show in a most simple form these important results. From these information we could see the phenomenon showed in this work.

The idea of bifurcation is the study of the possible structural changes of the behavior of some dynamical system due some parameter change passing from the stability to the instability or in the reverse order. In a system with the presence of bifurcation a sudden change in behavior occurs as a parameter passes through a critical value called a bifurcation point. A system may contain more than one parameter each with its own bifurcation point so that it can display extremely complex behavior, and computer studies play an important part in providing a taxonomy for the behavior of such systems [1].

THE CLOSED LOOP DISCRETE CONTROL USED

The analog proportional plus derivative control (PD) is given by

$$D(s) = k_p + k_d.s \tag{1}$$

It have many methods to do the mapping from the analog world to the discrete world, for example: forward, backward, Tustin (trapezoidal or bilinear), Schneider, etc… The Tustin method between all the classical rules of mapping show better results in theoretical and experimental results. Because this fact we used the Tustin rule to describe the mapping from analog to discrete world [2] [8]. This integration method (see Figure 1) use the past output sample and the present sensor data input to predict or construct the present output of the controller mapping stable poles from the left region of s-plane to inside the unit circle in the discrete domain.

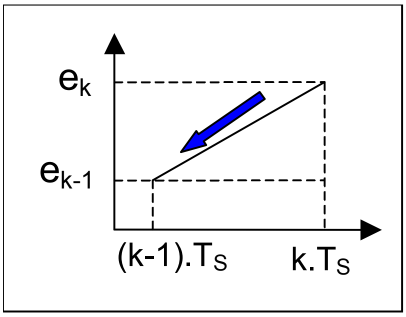

Figure 1: Tustin integrator in time domain.

The Tustin rule is a second order Adams-Moulton finite difference equation (2) and we can write as follow:

$$e_k = \frac{2}{T_s}.\nabla u_k - e_{k-1} = \frac{2}{T_s}.(u_k - u_{k-1}) - e_{k-1} \quad (2)$$

Applying the Z-Transform in (2) we obtain:

$$\frac{E(s)}{U(s)} = s \sim \frac{E(z)}{U(z)} = \frac{2}{T_s}.\frac{z-1}{z+1} \quad (3)$$

that is the Tustin rule of mapping of a derivative action of control.

JURY STABILITY CRITERION

The discrete time stability criterion equivalent of the Routh-Hurwitz analog method is the Schur-Cohn-Jury criterion. Mr. Jury has looked that when the Schur-Cohn-Jury criterion was applied over high order systems a great amount of calculations were necessary. Because this, Jury proposed a reduction method known Jury's Stability Criterion. Applying the Jury's criterion for some charateristic equation in closed-loop P(x) = 0 we can construct a table in witch elements are based in the P(z) coefficients. From this methodology we can found a analysis method to select the sampling period T_S to guarantee the asymptotic stability in discrete time control systems. If the characteristic polynomial assume the form:

$$P(z) = a_0 z^n + a_1 z^{n-1} + ... + a_{n-1} z + a_n \quad (4)$$

where $a_0 > 0$. The table of the Jury's Stability Criterion can be constructed in the Table 1.

TABLE 1: JURY GENERALIZED TABLE.

z^0	z^1	z^2	z^3	...	z^{n-2}	z^{n-1}	z^n
a_n	a_{n-1}	a_{n-2}	a_{n-3}	...	a_2	a_1	a_0
a_0	a_1	a_2	a_3	...	a_{n-2}	a_{n-1}	a_n
b_{n-1}	b_{n-2}	b_{n-3}	b_{n-4}	...	b_1	b_0	
b_0	b_1	b_2	b_3	...	b_{n-2}	b_{n-1}	
c_{n-2}	c_{n-3}	c_{n-4}	c_{n-5}	...	c_0		
c_0	c_1	c_2	c_3	...	c_{n-2}		
$\vdots$	$\vdots$	$\vdots$					
p_3	p_2	p_1	p_0				
p_0	p_1	p_2	p_3				
q_2	q_1	q_0					

Where,

$$b_k = \begin{vmatrix} a_0 & a_{k+1} \\ a_n & a_{n-1-k} \end{vmatrix} \quad ; k = 0,1,2,...,n-1 \quad (5)$$

$$c_k = \begin{vmatrix} b_{n-1} & b_{n-2-k} \\ b_0 & b_{k+1} \end{vmatrix} \quad ; k = 0,1,2,...,n-2 \quad (6)$$

$\vdots \; \vdots$

$$q_k = \begin{vmatrix} p_3 & p_{2-k} \\ p_0 & p_{k+1} \end{vmatrix} \quad ; k = 0,1,2 \quad (7)$$

Finally, the system will be asymptotically stable if the following constraints were all satisfact:

1. $|a_n| < |a_0|$ (8)

2. $P(z)|_{z=1} > 0$ (9)

3. $P(z)|_{z=-1} \begin{cases} > 0 & to \;\; n \;\; even \\ < 0 & to \;\; n \;\; odd \end{cases}$ (10)

4. $$\begin{matrix} |b_{n-1}| > |b_0| \\ |c_{n-2}| > |c_0| \\ \vdots \\ |q_2| > |q_0| \end{matrix} \quad (11)$$

THE BENCHMARK FLEXIBLE PLANT CONTROLLED FOR A DISCRETE PD BY TUSTIN RULE

The benchmark flexible plant used was the forced harmonic oscillator without damping as we can see in Figurer 2.

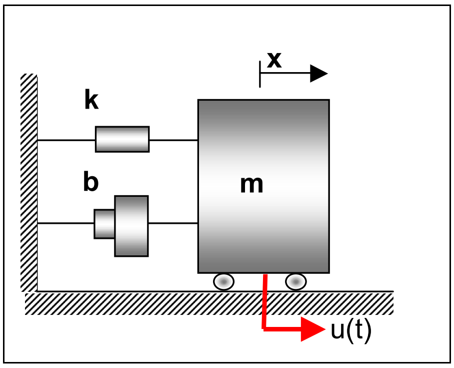

Figure 2: the harmonic oscillator.

The characteristic equation that can be extract [2] from the closed loop equation of the discrete time PD (proportional plus derivative) control using the Tustin rule as analog to discrete mapping and approximating by Zero Order Hold method the plant equation, is the following:

$$P(z) = z^2 + \left[\left(\frac{k_p.T_S + 2k_d}{T_S}\right).[1-\cos(\omega_n.T_S)] - 2.\cos(\omega_n.T_S)\right].z + \qquad (12)$$

$$+1 + \left(\frac{k_p.T_S - 2k_d}{T_S}\right).[1-\cos(\omega_n.T_S)]$$

ANALYTICAL AND EXPERIMENTAL RESULTS OBTAINED FOR THE SAMPLING PERIOD SELECTION

In this simple and important case the Jury's Stability criterion becomes:

$$1.\ |a_2| < |a_0|;\ \ 2.\ \ P(1) > 0;\ \ 3.\ \ P(-1) > 0; \qquad n = 2 \qquad (13)$$

From the first in (13) we can obtain:

$$i)\ \ -2 < \left(\frac{k_p.T_S - 2.k_d}{T_S}\right).(1-\cos(\omega_n.T_S)) < 0$$

(14)

From the second:

$$ii)\ \ k_p > -1$$

(15)

$$iii)\ \ \cos(\omega_n.T_s) < 1$$

(16)

that is an upper limit.
From the third:

$$iii)\ \ \cos(\omega_n.T_s) > \frac{2.k_d - T_s}{2.k_d + T_s}$$

(17)

that is a inferior limit.

As we can see all these situations (14 to 17) contrary the Jury's Criterion in the following form:

a) instability:

$$1)\ |a_2| < |a_0| \Rightarrow \left|\left(\frac{k_p T_s - 2k_d}{T_s}\right).(1-\cos(\omega_n.T_s)) + 1\right| < 1$$

(18)

Using numbers to be more close to the experimental satellite studied, we have for the PD control gains $k_p = 3.2$ and $k_d = 4.8$ (proportional and derivative, respectively); the non-damped mode frequency $\omega_n = 2\pi.0.1312 = 0{,}8244$ (rad/seg) (equivalent to the first vibration mode in the x axis of our asymmetric satellite [2] and a high sampling period of $T_S = 1{,}6$ seconds:

$1.1025 > 1$ (violation)

(19)

corresponding to a violation of the Jury's Stability Criterion, explaining the instability observed.

$2)\ P(1) > 0$

(20)

in numbers,

$6.3075 > 0$

(21)

$3)\ P(-1) > 0$

(22)

in numbers,

$-6{,}5125 > 0$ (violation)

that corresponds to more one violation of the Jury's Stability Criterion.

b) stability:

$$1)\ |a_2| < |a_0| \Rightarrow \left|\left(\frac{k_p T_s - 2k_d}{T_s}\right).(1-\cos(\omega_n.T_s)) + 1\right| < 1 \qquad (24)$$

In numbers, using a less sampling period of $T_S = 0.1$ sec we obtained the asymptotically stability:

$0{,}6848 < 1$

(25)

$2)\ P(1) > 0$

(26)

in numbers:

$0{,}0285 > 0$

(27)

3) $P(-1) > 0$

(28)

Em números,

$3{,}3411 > 0$

(29)

We can look now some approach of the stability regions of this attitude discrete control through the following functions $f_1(T_S)$, $f_2(T_S)$, $f_3(T_S)$ relative of the three stability constraints from the Jury's Criterion, using Tustin approximation to the PD controller and gains kp=3,2 e kd=4,8. From the first stability constraint we have:

$$f_1(T_S) = \left(\frac{k_p.T_S - 2.k_d}{T_S}\right).(1 - \cos(\omega_n.T_S)) + 1 \qquad (30)$$

and its stability condition is:

$-1 < f_1(T_S) < 1$

(31)

From the second stability condition:

$$f_2(T_S) = P(1) = 2.(1 + k_p).(1 - \cos(\omega_n.T_s))$$

(32)

and its stability condition is:

$f_2(T_S) > 0$

(33)

From the third stability condition:

$$f_3(T_S) = P(-1) = 2.\left(\left(1 - \frac{2.k_d}{T_S}\right) + \cos(\omega_n.T_s).\left(1 + \frac{2.k_d}{T_S}\right)\right) \qquad (34)$$

and its stability condition is:

$f_3(T_S) > 0$

(35)

The figures 3 to 5 show the results for the three functions (30), (33) and (35), plotting the 0.1 seconds mark and 1.6 seconds mark (instable).

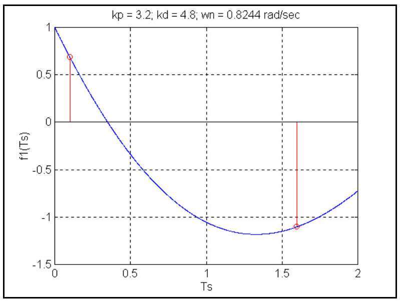

Figure 3 - $f_1(T_S)$.

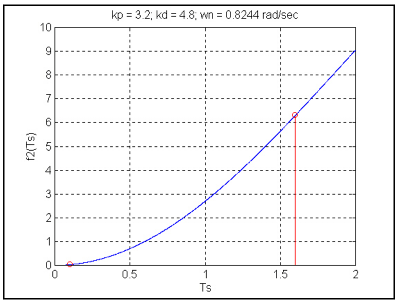

Figure 4 - $f_2(T_S)$.

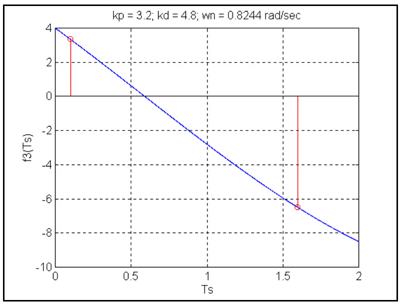

Figure 5 - $f_3(T_S)$.

These figures (3-5) show that the digital controller works with security if the sampling period does not violate the following constraint:

$$0 < T_S < 0{,}5 \quad \text{[secs]} \qquad (36)$$

PRESENCE OF BIFURCATION

In the figures 6, 7 and 8 we plotted the points considering high values of the sampling period. We may note the presence of instability and stability regions in the figures 6 (at the begin) and 8 characterizing the presence of a bifurcation being the sampling period T_S a parameter of our dynamical system.

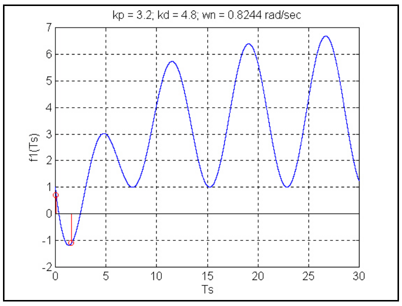

Figure 6 - $f_1(T_S)$.

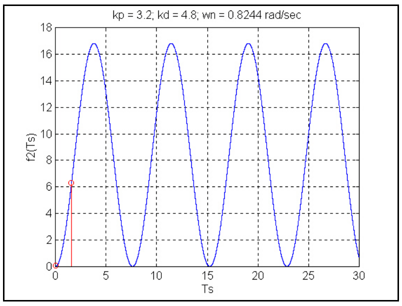

Figure 7 - $f_2(T_S)$.

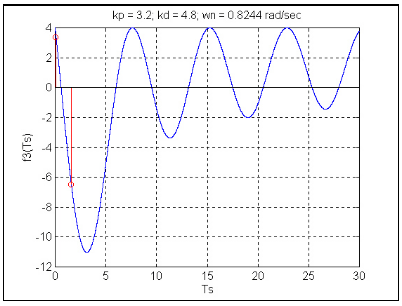

Figure 8 - $f_3(T_S)$.

CONCLUSION

We noted from the last section that the bifurcation effect can happens in some simple flexible plants discretely controlled, satisfacting the aiming of this work. We can predict the behavior of the functions $f_1(T_S)$, $f_2(T_S)$ e $f_3(T_S)$ for very high sampling periods if we use the limit of T_S near the infinity. As result we obtain that: a) $f_1(T_S)$ will oscillate between 1 and $(1+2.k_p)$; b) $f_2(T_S)$ will oscillate between 0 and $4(1+k_p)$; c) $f_3(T_S)$ will oscillate between 0 and 4.

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