Comments on Local Electron Concentration Determination from Doppler Dispersion Measurements of Satellite Radio Beacons

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During the past several years the use of satellite radio beacons in the study of the ionosphere has become well established. In particular, the measurement of dispersion in the optical path length between a satellite source and a receiver on the ground (ionospheric Doppler shift) has been used extensively and successfully to determine the integrated electron content of the ionosphere over the propagation path, and to follow its variation as the satellite moves in the field of view of the receiver.

From time to time, and particularly recently, attempts have also been made to use these dispersion measurements for the determination of electron concentration in the vicinity of the satellite transmitter, including *Berning* [1956, 1959], *Graves* [1960], *Alpert* [1964], *Alpert and Sinelnikov* [1965], *Misyura et al.* [1966], among others. The inherent difficulties in these determinations have not always been fully recognized by the authors concerned, and as a consequence some of their published results are of doubtful validity.

Such measurements have been the subject of an active controversy [*Gringauz et al.*, 1965, 1966; *Alpert*, 1965], which apparently has not yet been resolved. It is the purpose of this note to call attention to the errors inherent in experiments of this kind and to urge extreme caution both in their use and in the acceptance of results derived from them. We do not intend to enter into a point-by-point discussion of the matters raised in these earlier papers, but rather we shall present a brief summary of our conclusions concerning this class of experiment.

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At high frequencies the reduction in the optical path length below its free space value can be approximated very closely by integrating the refractivity along the straight line path from transmitter to receiver. Furthermore, the refractivity is very nearly proportional to the electron concentration N, so that the optical path dispersion between two frequencies f_1, f_2 can be written as

$$\Delta P = K \int N \, ds \qquad (1)$$

where

$$K = \frac{e^2}{8\pi^2 \epsilon_0 m}\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)$$

$$= 40.3\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) \quad \text{in MKS units}$$

and the integral is taken over the straight line path. We shall make this high-frequency approximation in common with most of the workers listed above. A more exact formulation would result in greatly increased complexity but would not alter the essence of our discussion and conclusions.

The experimental observation that can be made is of the change of ΔP with time, which is the combined result of temporal changes in the ionosphere together with changes of length and direction of the propagation path in the nonuniform ionosphere. A description of these effects that shows that changes in ΔP can be resolved into five terms has been made by *Alpert* [1964]. After some convenient changes in notation and the correction of some minor errors, Alpert's result can be written

$$\frac{1}{K}\frac{d\Delta P}{dt} = N_s \sec \phi_s \frac{dR_s}{dt}$$

$$+ \tan \phi_s \frac{d\phi_0}{dt} \int N\left(\frac{R_0 R_s \cos \phi_0 \cos \phi_s}{R^2 \cos^2 \phi}\right) ds$$

$$+ \rho_s \sec \phi_s \frac{d\phi_0}{dt} \int \frac{\partial N}{\partial x} \frac{\rho \sec \phi}{\rho_s \sec \phi_s} ds$$

$$+ \frac{dy_s}{dt} \cdot \int \frac{\partial N}{\partial y} \frac{\rho}{\rho_s} ds + \int \frac{\partial N}{\partial t} ds \qquad (2)$$

where

R is the geocentric distance of a point on the ray.
ϕ is the zenith angle.
ρ is the distance from the receiver.
x, y denote horizontal directions, respectively, in and normal to the plane of incidence.
subscripts 0 and s refer, respectively, to values at receiver and satellite.

The first term on the right is proportional to the vertical component of velocity and to the local electron concentration at the satellite. It is from the contribution of this term that the local number density N_s must be found. The second term is proportional to the slant column electron content modified by a weighting function that is greater than unity at lower heights where the ray is more oblique and less than unity for the upper part of the ray path; the skewness of this weighting function increases with zenith angle. This term is usually much larger than the rest and permits the estimation of electron content, particularly when used near the zenith. The third and fourth terms are proportional to the horizontal gradients in the ionosphere, in and normal to the plane of incidence, respectively, weighted with distance from the receiver. The fifth term in equation 2 is the integrated time rate of change of electron concentration along the ray path.

It is instructive to examine further the way in which the direction of satellite motion affects the local density and electron content terms in the Doppler shift equation. For simplicity, assume that the ionosphere is spherically stratified and time invariant so that the last three terms in equation 2 are all zero. Then equation 2 may be written

$$\frac{1}{K} \frac{d(\Delta P)}{dt} = (V_r) N_s + \frac{V_c \sin \phi_s}{\rho_s} \cdot \int N \left(\frac{R_0 R_s \cos \phi_0 \cos \phi_s}{R^2 \cos^2 \phi} \right) ds \qquad (3)$$

where $V_r = \sec \phi_s (dR_s/dt)$ and $V_c = \rho_s \sec \phi_s (d\phi_0/dt)$.

The physical significance of these two velocities can be seen in Fig. 1. The total satellite velocity **V** has been split into two *nonorthogonal* components, V_r parallel to the ray and V_h in the horizontal direction; V_c is the inplane component of V_h. V_r should not be confused with the projection of **V** on the ray, nor should V_h be regarded as the total horizontal component of velocity in general. They assume these values only in the respective special cases of satellite motion entirely along the ray direction and of motion that is purely horizontal. Thus, to find the components of **V** that multiply the local density and integrated electron content, it is incorrect to split the vector into vertical and horizontal components or into components parallel and perpendicular to the ray as is sometimes done. Instead the appropriate nonorthogonal components V_r and V_h must be used.

Returning to the general form of equation 2, it is evident that the success of any attempt to deduce the local electron concentration N_s from measured values of $d\Delta P/dt$ must be in the extent to which the combined uncertainties in the last four terms of equation 2 can be considered small compared to the value of the first term. The assignment of uncertainties to the various

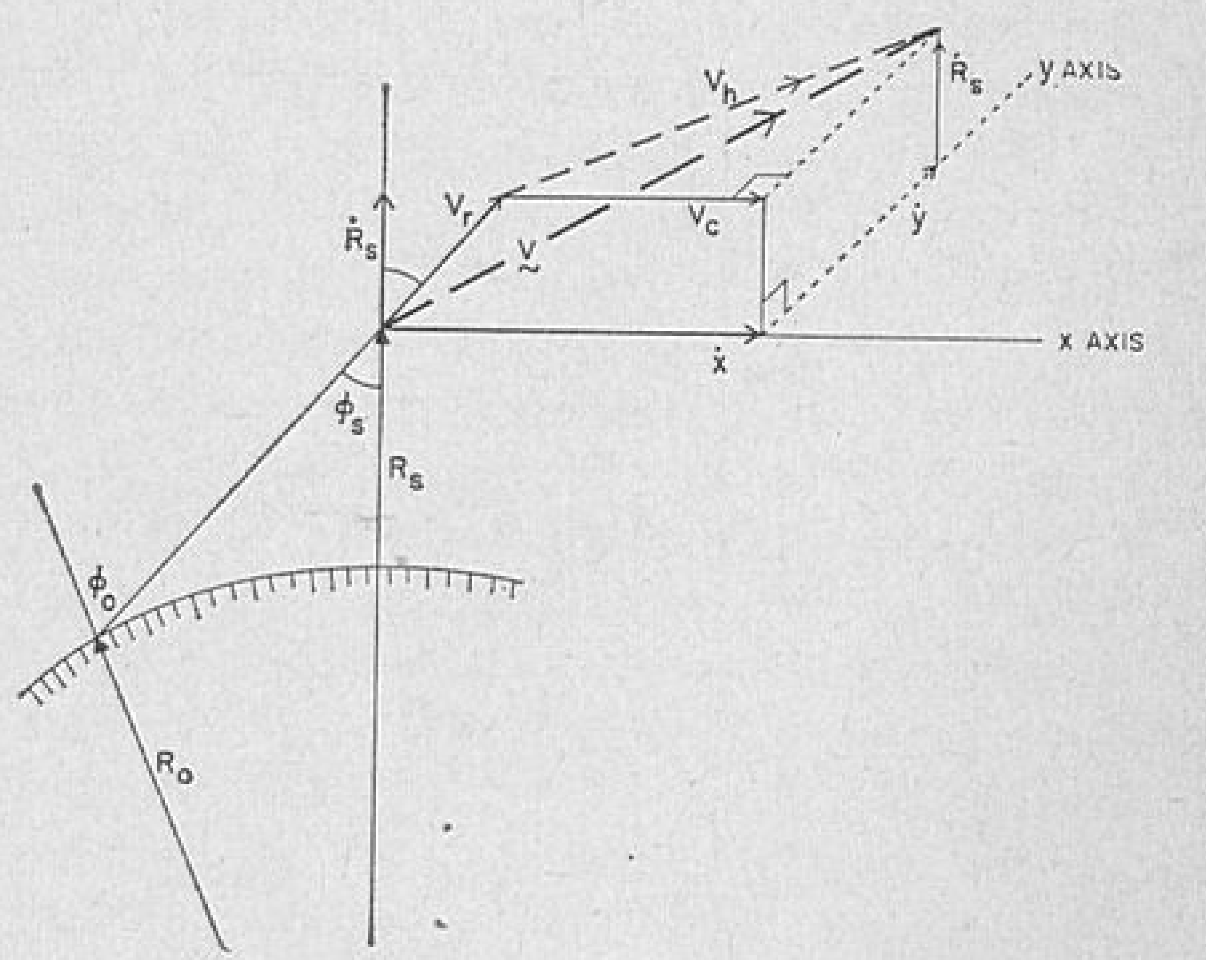

Fig. 1. Schematic diagram of satellite motion and its resolution into the velocity components V_r and V_c, which are coefficients of the local electron concentration and integrated electron content terms, respectively, in equation 3. All solid lines are in the vertical plane containing the observer, satellite, and ray path, while dotted lines are perpendicular to this plane and broken lines are oblique.

terms is a highly subjective matter since good statistical measures of many of the relevant parameters are not available. Even where the satellite beacon experiment is accompanied by other supporting experiments, for example from a chain of nearby ionospheric sounding stations, their information only approximates that which is needed, and there still remains some uncertainty. We shall discuss equation 2 with regard to a set of nominal values for the uncertainties in the various terms, which are supposed to persist after all corrections are made for known patterns of ionospheric dependence on position and time. We believe this set to be not unrealistic, although of course it may be possible to demonstrate in a particular instance that certain improvements have been achieved.

The ionosphere is supposed to have a vertical column electron content to the height of the satellite of 2×10^{17} m^{-2}, concentrated mainly about a mean height of 300 km. The content is considered to be uncertain to 5%, while the tolerance in its height distribution is represented by a uniform displacement of 30 km. The residual uncertainties in the horizontal gradients are taken to be 1% of the total electron content in 100 km, and the temporal change to be uncertain to a rate of 1% of the total content in 200 seconds. These are fairly conservative estimates; they may often be much larger.

We shall consider the satellite to be at 1000-km altitude moving with a horizontal velocity of 7 km/sec and with a vertical velocity component of 1 km/sec. We then calculate the uncertainties in the various terms of equation 2 and express the results in terms of the corresponding uncertainty in the value of N_s, which would be calculated from this equation. (Several of the cited papers have attempted to compute local electron concentrations when the vertical velocity component was less than half our nominal value, so that the uncertainty in their results is proportionately greater.)

The contributions of the various terms in equation 2 to the uncertainty in N_s depend on the zenith angle of the measurement and also on the direction of satellite motion relative to the plane of incidence. These conditions are illustrated in Figures 2 and 3 for two hypothetical satellite passes with the nominal orbital and ionospheric parameters listed above. Figure 2 approximates the case of an overhead pass where the satellite velocity is in the plane of incidence at all times, while Figure 3 is drawn for the case of a more distant pass where the zenith angle of the ray at the receiver reaches

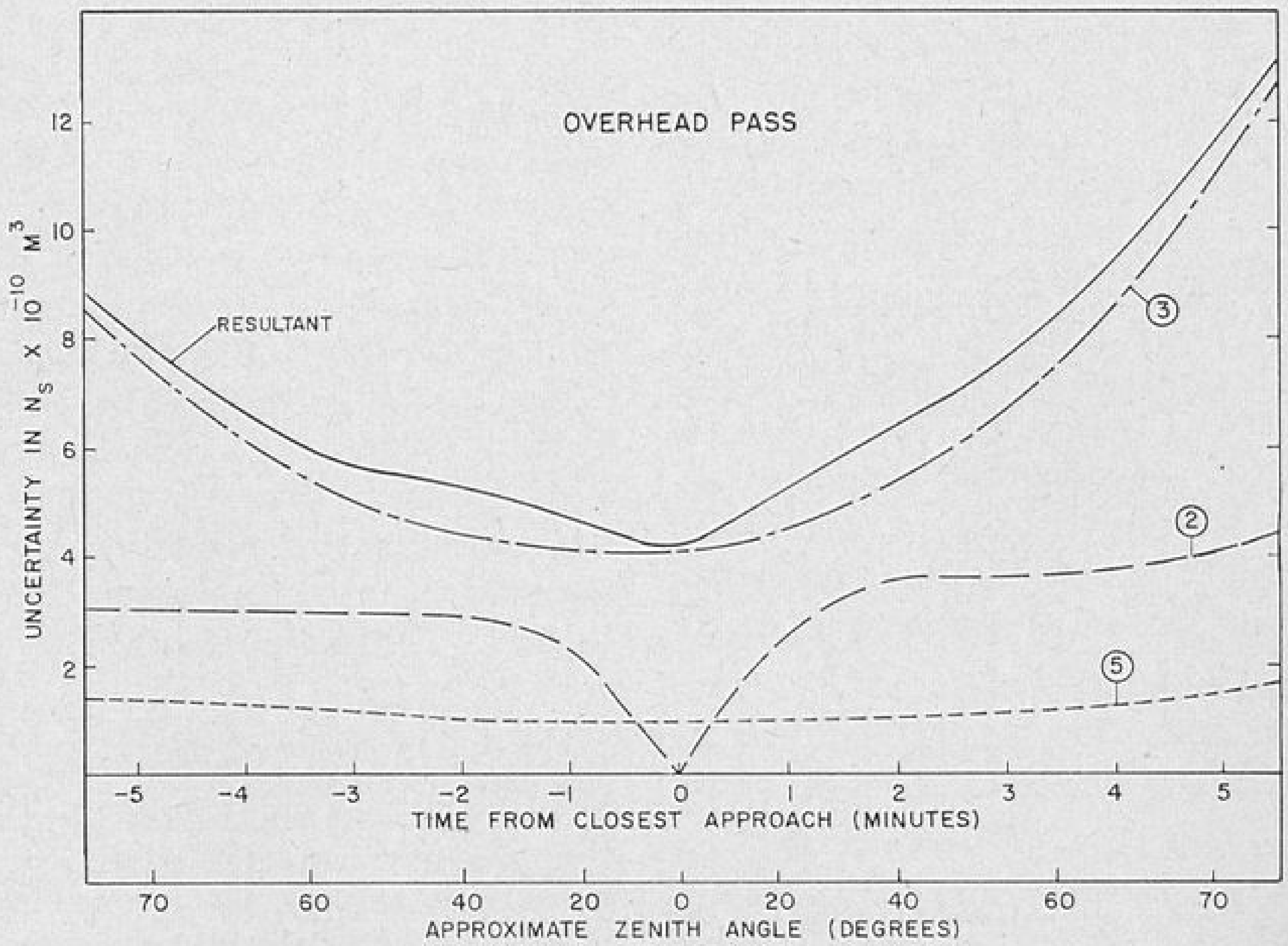

Fig. 2. Profiles of the nominal uncertainties in local electron concentration arising from the later terms of equation 2 for an overhead satellite pass. The numbered curves refer to the corresponding terms in equation 2.

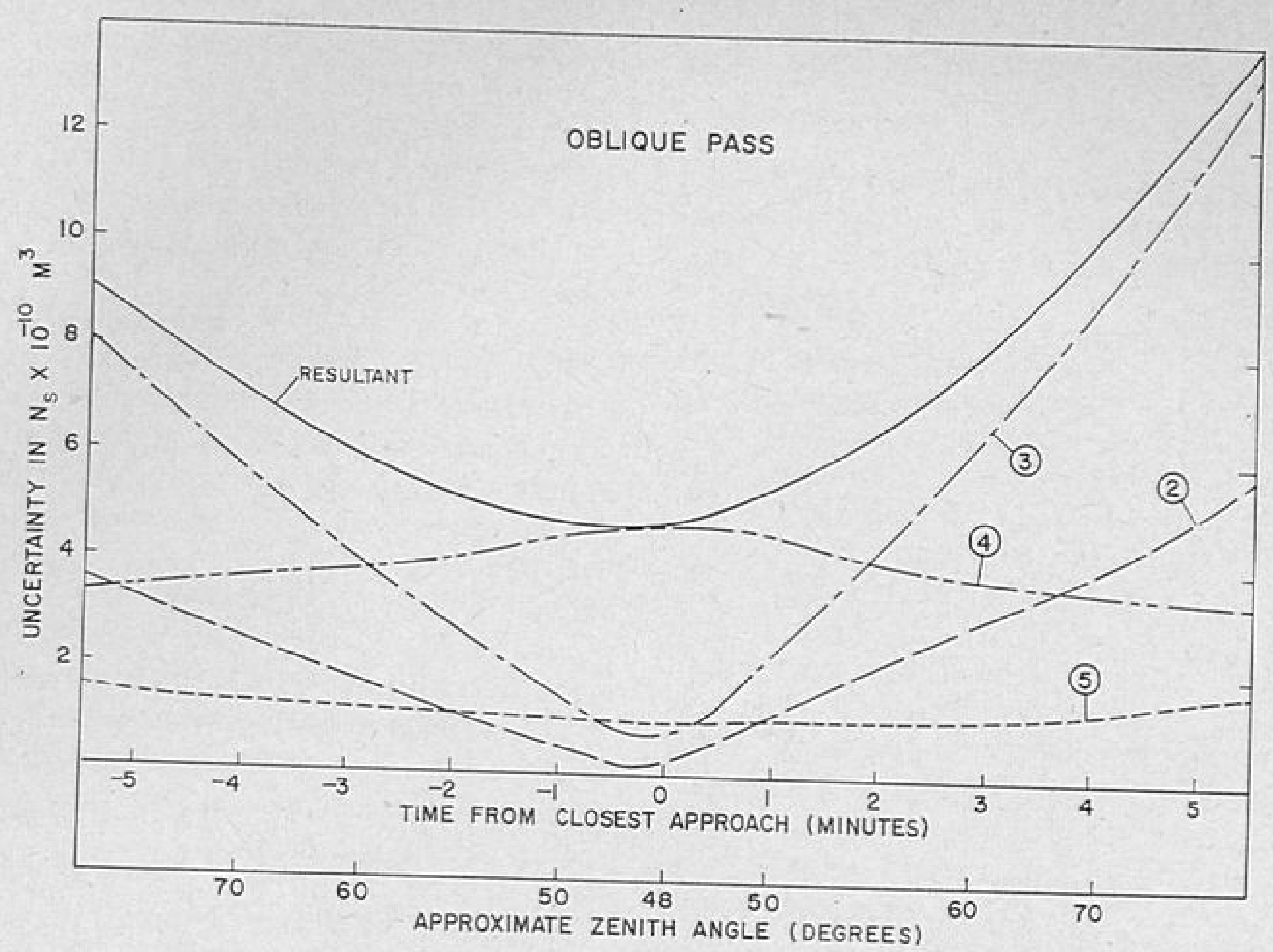

Fig. 3. Nominal uncertainties in local electron concentration for the case of an oblique satellite pass.

a minimum value of 48°. The contributions of the various terms in equation 2 are identified by number, and their quadratic sum is shown as a resultant curve in each case.

It can be seen that the uncertainty is dominated by one or other of the horizontal gradient terms at all times, with significant contributions from the other terms as well. For the oblique pass the in-plane gradient term vanishes at the point of minimum zenith angle, a fact which led *Alpert* [1964] and others to use this point for measurement of N_s, but it is evident that the out-of-plane gradient term is still large and cannot be ignored as these workers have done.

The most favorable time for measurement of N_s is near the point of closest approach, but even here our nominal parameters give an uncertainty of about 5×10^{10} m^{-3}, which is greater than the usual range of N_s at about 1000-km altitude of 1–3 $\times 10^{10}$ m^{-3} [e.g., *Brace et al.*, 1967]. The uncertainty of the measurement increases steadily toward the limits of the pass owing principally to the increasingly rapid horizontal movement of the 'ionosphere point' through the possible in-plane horizontal gradient.

This situation improves only slightly for lower satellite heights. Although the local density N_s may be much larger and the underlying electron content smaller, these advantages are largely offset by a decrease in the vertical velocity of the satellite and an increase in the angular velocity of the ray. The effects of horizontal gradients are still quite important and cannot be ignored.

For satellites in very eccentric orbits, there may be intervals during which the vehicle appears to be moving directly away from an observer on the earth. At these times the temporal uncertainty may be the largest contributor to equation 2, although even with radial velocities of 3 or 4 km/sec (typical of OGO-A), the uncertainty in N_s would be reduced to only 3×10^9 m^{-3}. A substantial improvement can be obtained by measuring the polarization rotation of the waves coincident with the Doppler measurements to further reduce the temporal uncertainty [*da Rosa*, 1965]. *Obayashi* [1965] has obtained one estimate in this manner, but too little information is available to determine the over-all uncertainty.

We conclude therefore, that the ionospheric Doppler shift method using satellite radio beacons cannot yield useful values of local electron concentration, in general. Any attempt to derive this quantity from the measurements must include a clear demonstration that the sources of error discussed above have been taken into account and their effects shown to be reasonably small.

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