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14. Abstract/Notes

This work presents the study of the capture of the equilibrium position for the first Brazilian satellite. After obtaining the kinetic, gravitational potential and elastic potential energies for the Lagrangean equations of motion. This system of equations is hybrid, nonautonomous, coupled and nonlinear, thus making an analytical approach not feasible. Performing simplifications on the modelling of the appendages and using the assumed modes method for the elastic displacements, the distributed parameters and the hybrid character of the set of equations are eliminated. Then, by numerical integration the attitude motion during and after the transient phase of the appendages deployment are established. The initial conditions in which the capture is possible are studied, by considering a large variation of parameters like deployment velocity, position of center of mass, moments and products of inertia, bending rigidity, etc.

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ANALYSIS OF THE CAPTURE OF THE NOMINAL EQUILIBRIUM POSITION FOR THE FIRST BRAZILIAN SATELLITE

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ABSTRACT

This work presents the study of the capture of the equilibrium position for the first Brazilian satellite. After obtaining the kinetic, gravitational potential and elastic potential energies for the Lagrangean functional, the Hamilton's principle is used to get the governing equations of motion. This system of equations is hybrid, nonautonomous, coupled and nonlinear, thus making an analytical approach not feasible.

Performing simplifications on the modelling of the appendages and using the assumed modes method for the elastic displacements, the distributed parameters and the hybrid character of the set of equations are eliminated. Then, by numerical integration the attitude motion during and after the transient phase of the appendages deployment are established. The initial conditions in which the capture is possible are studied, by considering a large variation of parameters like deployment velocity, position of center of mass, moments and products of inertia, bending rigidity, etc.

INTRODUCTION AND SYSTEM DEFINITION

In this work the attitude motion of the first Brazilian satellite is studied and the initial conditions for the capture of the nominal equilibrium position are determined. The capture consists of extension and retraction maneuvers of the stabilizer mast so that, in the presence of the gravity gradient torque, the satellite oscillates around the local vertical. The references [1], [2], [3] and [4] are the main works that served as basis to this study.

The first Brazilian satellite has a central rigid body, which is a prism of octogonal base, with its faces covered by solar cells. It weights 100 kg and will describe an eccentric orbit of approximately 800 km of height, with an inclination of 25^o. In the capture phase a 10 meters elastic mast will be deployed carrying a 3 kg tip mass. This procedure induces an increase in two of the principal moments of inertia, which assures the gravity gradient stabilization (Figure 1).

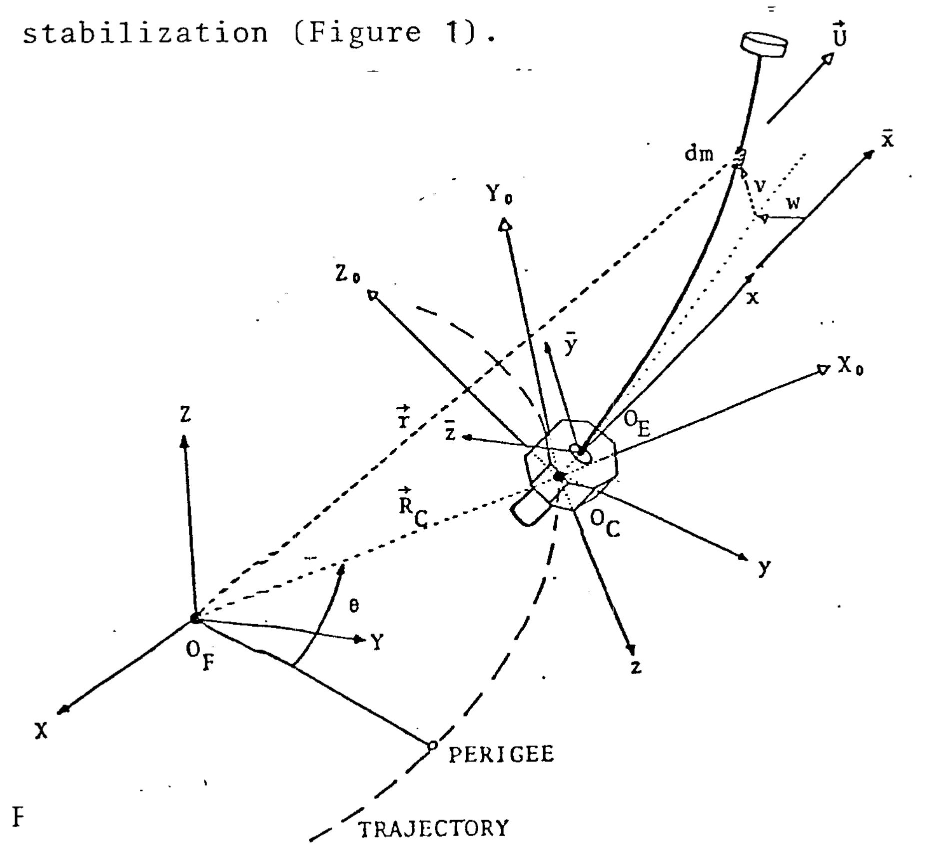

Fig. 1 - Satellite in the deformed state in orbit.

The instantaneous center of mass O_C describes an eccentric orbit with an eccentricity of approximately .01, around the center of force O_F. The elastic appendage is clamped in the central body and the system is free for the tridimensional movements of vibra rion and librational in the gravitational field.

The position vector $\vec{R}_C$ and the true anomaly θ define the position of O_C with respect to the inertial system $O_F(X,Y,Z)$. The axes X_0, Y_0 and Z_0 form an orthogonal system, with origin in O_C, where X_0 and Z_0 are respectively the vertical local vector and the normal vector to the orbit, with Y_0 forming a right handed triad.

The librational response of the vehicle is defined by the Euler rotations ψ, Λ and ϕ (angles of roll, yaw and pitch, re spectively) of the system $O_C(x,y,z)$, fixed to the body with respect to the orbital system $O_C(X_0, Y_0, Z_0)$.

The elastic deformations of the element of mass dm of the appendage are $v(\bar{x},t)$ and $w(\bar{x},t)$. This element is localized by $\vec{r}$, in the system $O_F(X,Y,Z)$, when the appendage is in the deformed state (Figure 1 and Figure 2).

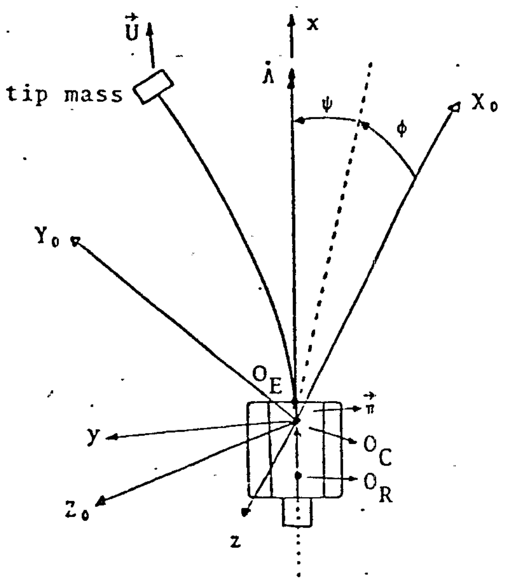

Fig. 2 - Satellite Euler angles.

The distance between the clamp point O_E and O_C is given by $|\vec{\pi}|$. The deployment velocity $\vec{U}$ is a vector aligned with the $\bar{x}$-axis, and its modulus can be variable in the time.

The mast has linear density ρ, modulus of elasticity E and moments of inertia of the transversal section J_T and J_Z.

ENERGY FORMULATION

The expression for the kinetic energy of the whole vehicle can be taken in the following form:

$$T = \frac{1}{2} M_{SAT} \left\{\frac{d\ R_c}{dt}\right\}^T \cdot \left\{\frac{d\ R_c}{dt}\right\} + \frac{1}{2} \{\omega\}^T[I]\{\omega\} + $$

$$+ \{\omega\}^T \int_m [\tilde{r}_d][\ell]^T\{V\}dm + \frac{1}{2}\int_m \{V\}^T\{V\}dm \ , \qquad (1)$$

where:

$\{\omega\}$ = absolute angular vector of the satellite;

$[I]$ = inertia tensor of the satellite;

$\{V\}$ = velocity of the appendage element of mass dm due to vibration ($\dot{v}$ and $\dot{w}$) and deployment (U);

$[\ell]$ = matrix of direction cosines of the $O_E(\bar{x},\bar{y},\bar{z})$ and $O_C(x,y,z)$;

$[\tilde{r}_d]$ = antisymmetric matrix to perform the cross product.

The gravitational potential has the form:

$$V_G = -\frac{\mu\ M_{SAT}}{R_C} - \frac{\mu}{2R_C^3}\,tr[I] + \frac{3}{2}\frac{\mu}{R_C^3}\{\ell_a\}[I]\{\ell_a\} \ , \qquad (2)$$

where:

$\{\ell_a\}$ = vector of the direction cosines of $\vec{R}_C$ with respect to the system $O_C(x,y,z)$;

tr[I] = trace of the inertia tensor [I].

The elastic potential energy may be expressed in the following manner:

$$V_{EB} = \frac{1}{2} \int_m \left[EJ_Y \left(\frac{\partial^2 v}{\partial \bar{x}^2} \right)^2 + EJ_Z \left(\frac{\partial^2 w}{\partial \bar{x}^2} \right)^2 \right] dm \; , \tag{3}$$

with the integral taken along the elastic domain and v and w are the elastic displacements in the directions $\bar{y}$ and $\bar{z}$ respectively.

The potential related to the axial load along the mast and the foreshortening due to elastic displacements is:

$$V_{EF} = \frac{1}{2} \int_m P \left[\left(\frac{\partial v}{\partial \bar{x}} \right)^2 + \left(\frac{\partial w}{\partial \bar{x}} \right)^2 \right] dm \; , \tag{4}$$

where:

P = axial force acting on the appendage.

With the expressions of the kinetic and potential energies, the Lagrangean functional can be obtained in the form:

$$L = T - V_G - V_{EB} - V_{EF} \; . \tag{5}$$

The hybrid character of the Lagrangean functional can be eliminated, transforming the partial derivatives into ordinary ones; the distributed parameter nature disappears, transforming the system into a set of ordinary differential equations, making the procedure easier for numerical integration.

At this stage, one makes use of the Rayleigh-Ritz method or the assumed modes method which is essentially a discretization process. The displacements $v(\bar{x},t)$ and $w(\bar{x},t)$ can be written in the following form:

$$v(\bar{x},t) = \sum_{j=1}^{m} f_j(\bar{x})\ \xi_j(t)$$

$$w(\bar{x},t) = \sum_{j=1}^{n} f_j(\bar{x})\ \eta_j(t) \quad , \tag{6}$$

where the functions $f_j(\bar{x})$ are known and depend only on the position of the element of mass. The functions $\xi_j(t)$ and $\eta_j(t)$ are the new generalized coordinates that will characterize the elastic movements.

Consequently, the vector of generalized coordinates for the ordinary system of equations is:

$$\vec{Q}^T = \{\phi,\ \Lambda,\ \psi,\ \xi_1,\ \eta_1\}, \tag{7}$$

where ϕ, Λ, ψ are the three attitude angles and ξ_1 and η_1 are the two coordinates related to the first mode of vibration.

The expressions in (6), for $j = 1$ is the substituted in the Lagrangean function, and the equations of motion are obtained in the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = F_k, \tag{8}$$

where $q_k \in \{\phi,\ \Lambda,\ \psi,\ \xi_1,\ \eta_1\}$, $k = 1,2,3,4,5$, and F_K are the generalized forces with respect to the q_k coordinate.

COMMENTS

The interactions between librational dynamics, flexibility and deployment are particularly emphasized. Based on the analysis, the following general conclusions can be made:

- The range of initial condition for the capture is reduced with the reduction of the deployment velocity.
- The capture with only one deployment maneuver can be assured for a large range of attitude angles.
- The capture in two phases of deployment assures librations in the steady state lesser than 5° of amplitude.
- The inversion maneuver can be established succesfully when the retraction and extension are adequately performed.
- There are critical combinations of flexibility, deployment rate and rotations for which the system tends to be unstable in vibration and libration.

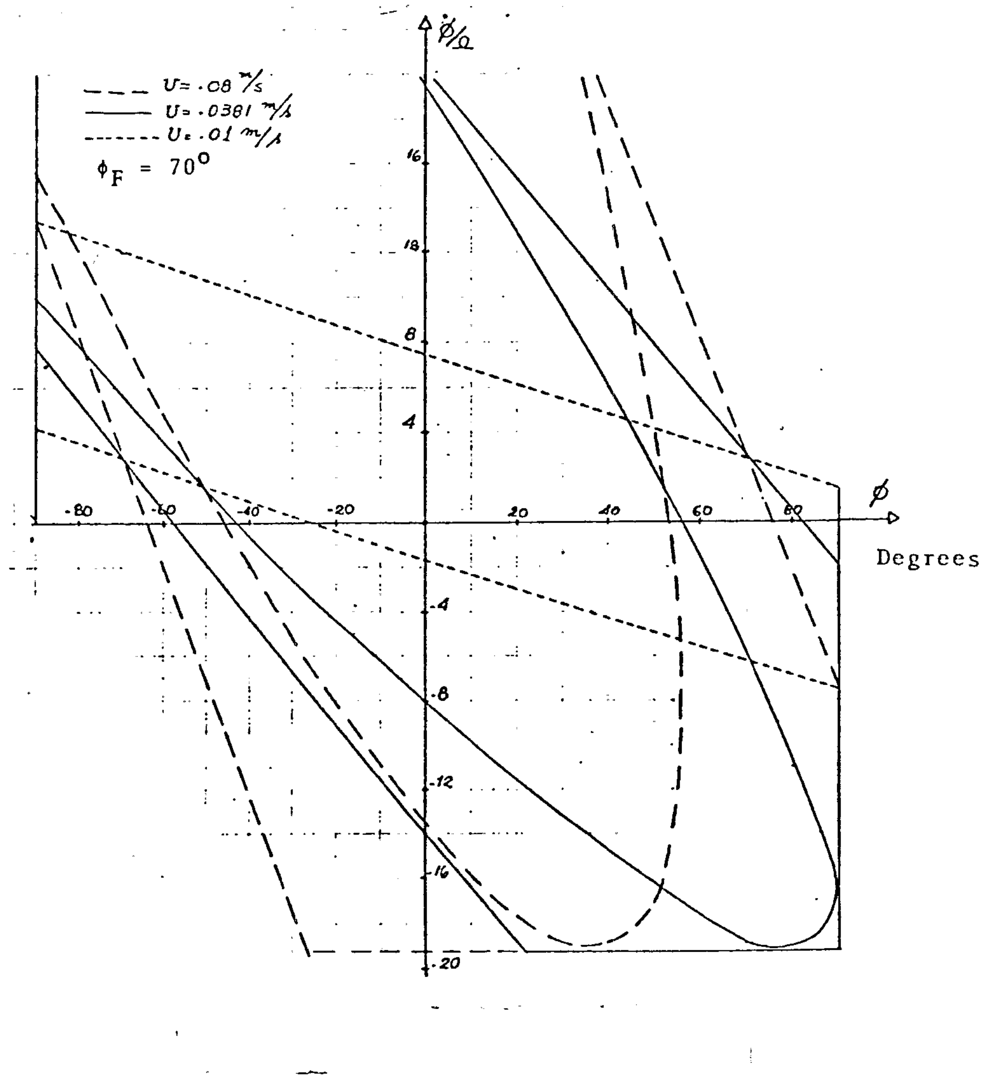

Fig. 3 - Envelops of maneuver for three deployment velocities.

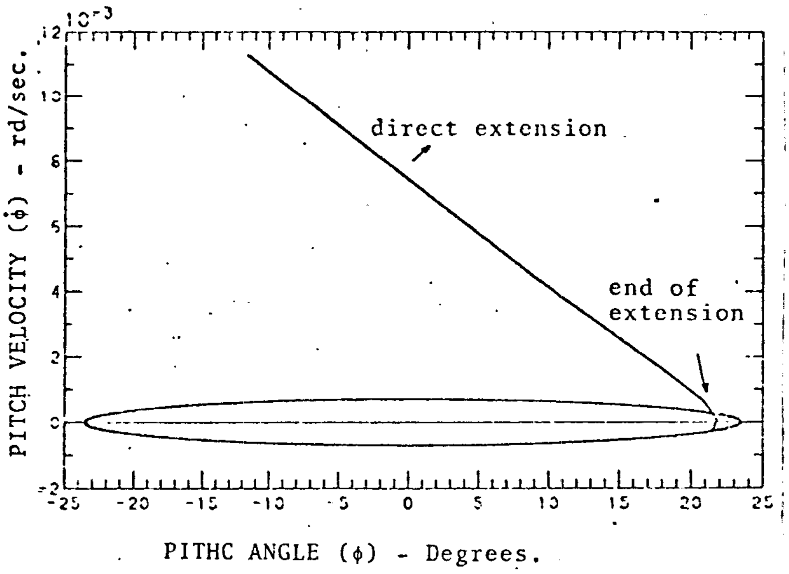

Fig. 4 - Direct capture of nominal equilibrium position.

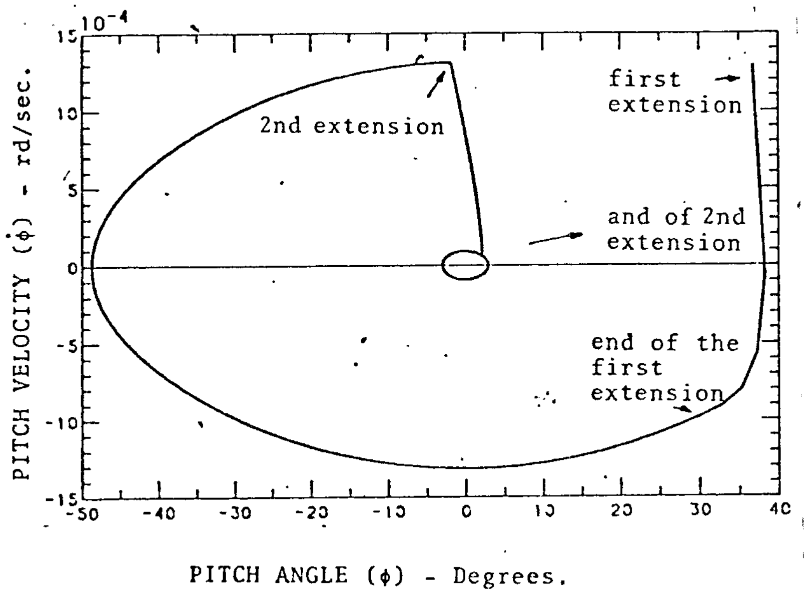

Fig. 5 - Capture in two phases of deployment.

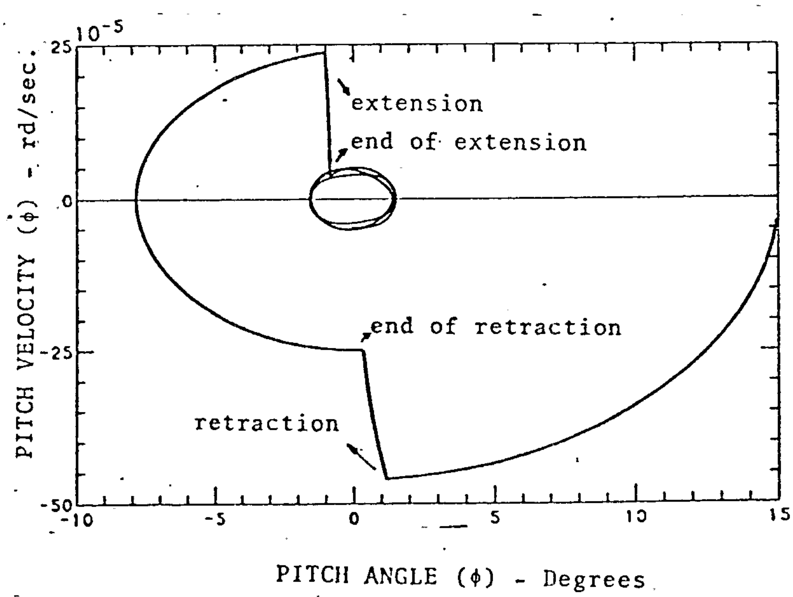

Fig. 6 - Dead beat maneuver

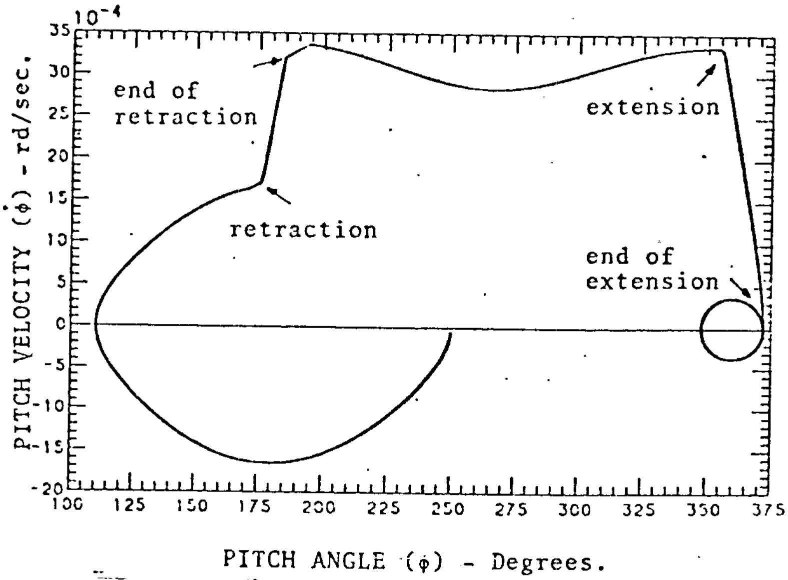

Fig. 7 - Inversion maneuver.

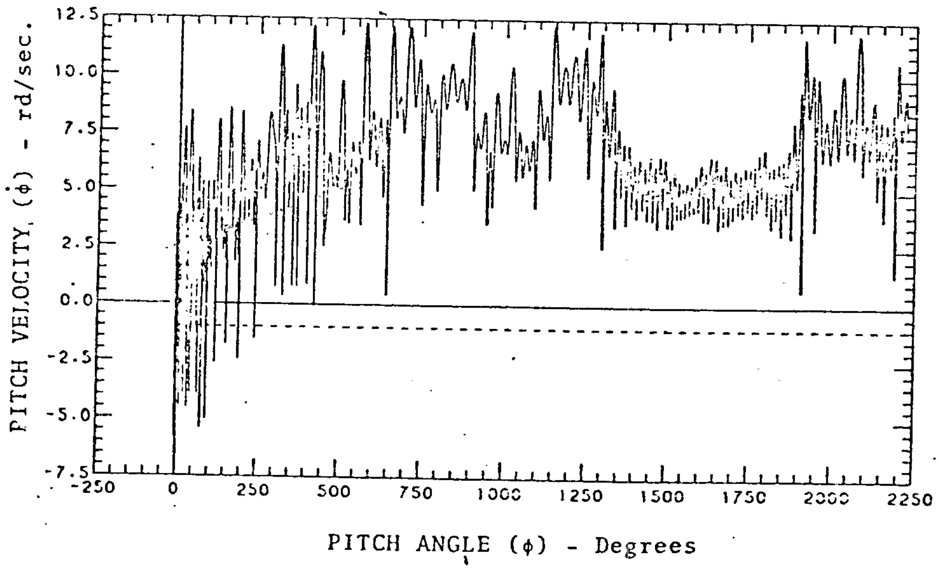

Fig. 8 - Attitude instability due to critical elastical displacement.

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