ORBITAL MANEUVERS USING GRAVITATIONAL CAPTURE

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The objective of this paper is to study the problem of gravitational capture in the regularized restricted three-body problem. A gravitational capture occurs when a massless particle changes its two-body energy around one celestial body from positive to negative without the use of non-gravitational forces. We studied the importance of several of the parameters involved for a capture in the Earth-Moon system, including the time required for the capture and the effects of the periapse distance. Then, we generalize those results for other binary systems, like the Sun-Earth, Sun-Mars and Sun-Jupiter systems. Next, we cover the whole interval of the mass parameter μ (the mass ratio of the two primaries) and we study the gravitational capture in the interval $0.0 < \mu < 0.5$. The elliptical restricted problem is also considered as an option for the model.

1 - Introduction

The phenomenon called gravitational capture is a very interesting characteristic of some dynamical system, like the three- or four- body system in celestial mechanics. It is under investigation for some time now, specially by Belbruno (1987, 1990, 1992a, 1992b), Belbruno and Miller (1990a, 1990b), Miller and Belbruno (1991), Yamakawa et. al. (1992, 1993a, 1993b), Krish (1991), Krish et. al. (1992), Vieira-Neto and Prado (1998). The basic idea is that a slightly hyperbolic orbit (with a residual positive energy) around a celestial body can be transformed in a slightly elliptic orbit (with a residual negative energy) without the use of any propulsive system. The only forces responsible for this capture are gravitational perturbations from one or more other bodies. One of the most important applications of this property is the construction of trajectories to the Moon. In this maneuver, a spacecraft leaves a parking orbit around the Earth on its way to the Moon, makes a Swing-By with the Moon to go to a distant region and then, using the perturbations of the Sun and the Earth, it comes back to the Moon for a gravitational capture. This capture is only temporary, but an impulse can be applied during this temporary capture to make it permanent. The advantage is that this impulse has a magnitude smaller than the one required for a standard maneuver without the gravitational capture, and it means that there is a saving in fuel involved in this special type of maneuver.

In this paper, we show a general survey of this topic, including the major steps given in the recent years. We also show in more detail the gravitational capture in the planar circular and elliptical restricted three-body problem. In this model it is assumed the existence of three bodies: two primaries orbiting their center of mass in circular or elliptical orbits and a third particle with negligible mass traveling in the orbital plane of the two primaries, with its motion governed by them. We integrate backward in time trajectories that start close to the Moon (in the Earth-Moon system) that has a two-body (Moon-Spacecraft) energy slightly negative (so, in an elliptic orbit). Then we show our own results in this field, including the verification of in which cases an escape from the Moon occurs and the measurement of the time elapsed until the escape is complete (we define that an escape is complete when the spacecraft is at a distance of 100000 km from the Moon, to follow the same convention used by Yamakawa). Then, we make contour-plots to shown this elapsed time as a function of the angle between the initial velocity and the Earth-Moon line (horizontal axis) and the initial energy (vertical axis). We vary the angle from 0 to 360 degrees and the energy from 0 until the limit where we can find escapes. Those escape trajectories obtained by integration in backward time are equivalent of trajectories that result in gravitational capture in forward time. Those results allow us to make a table that shows the balance between time and savings obtained for each possible transfer. It also allow us to find trajectories that give us the maximum savings in the impulse that make the final capture for a given time limit for the capture. They can be used to construct Belbruno-Miller transfers to the Moon. We also extend our study to verify the influence of the initial distance from the Moon in those results and we also consider others systems, like the Sun-Earth and the Sun-Jupiter.

2 - Mathematical Model and Some Properties

The model used in most of this paper is the well-known planar circular restricted three-body problem. This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3), with negligible mass, is orbiting these two primaries. The motion of M_3 is supposed to stay in the plane of the motion of M_1 and M_2 and it is affected by both primaries, but it does not affect their motion (Szebehely, 1967). The standard canonical system of units associated with this model is used (the unit of distance is the distance M_1-M_2 and the unit of time is chosen such that the period of the motion of M_2 around M_1 is 2π). Under this model, the equations of motion are:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial U}{\partial x} = \frac{\partial \Omega}{\partial x} \qquad \ddot{y} + 2\dot{x} = y - \frac{\partial U}{\partial y} = \frac{\partial \Omega}{\partial y} \tag{1}$$

where Ω is the pseudo-potential function given by:

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{2}$$

and x and y are two perpendicular axes with the origin in the center of mass of the system, with x pointing from M_1 (that has coordinates $x = -\mu$, $y = 0$) to M_2 (that has coordinates $x = 1-\mu$, $y = 0$).

One of the most important reasons why the rotating frame is more suitable to describe the motion of M_3 in the three-body problem is the existence of an invariant, that is called Jacobi integral (or energy integral). There are many ways to define the Jacobi integral and the reference system used to describe this problem (see Szebehely, 1967, pg. 449). In this paper the definitions used by Broucke (Broucke, 1979) are followed. Under this version, the Jacobi integral is given by:

$$J = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \Omega(x, y) = \text{Const} \tag{3}$$

Another important property needed in this paper is the mirror image theorem (Miele, 60). It is an important and useful property of the planar circular restricted three-body problem. It says that: "In the rotating coordinate system, for each trajectory defined by $x(t), y(t), \dot{x}(t), \dot{y}(t)$ that is found, there is a symmetric (in relation to the "x" axis) trajectory defined by $x(-t), -y(-t), -\dot{x}(-t), -\dot{y}(-t)$".

3 - Lamaître Regularization

The equations of motion given by (1) are right, but they are not suitable for numerical integration in trajectories passing near one of the primaries. The reason is that the positions of both primaries are singularities in the potential U (since r_1 or r_2 goes to zero, or near zero) and the precision of the numerical integration is affected every time this situation occurs.

The solution for this problem is to use regularization, that consists in a substitution of the variables for position (x-y) and time (t) by another set of variables (ω_1, ω_2, τ), such that the singularities are eliminated in these new variables. Several transformations with this goal are available in the literature (see Szebehely, 1967), chapter 3), like Thiele-Burrau, Lamaître and Birkhoff. They are called "global regularization", to emphasize that both singularities are eliminated at the same time.

The case where only one singularity is eliminated at a time is called "local regularization". For the present research the Lamaître's regularization is used. To perform the required transformation, it is necessary first to define a new complex variable $q = q_1 + i * q_2$ (i is the imaginary unit), with q_1 and q_2 given by:

$$q_1 = x + 1/2 - \mu \quad q_2 = y \tag{4-5}$$

Now, in terms of q, the transformation involved in Lamaître regularization is given by:

$$q = f(\omega) = \frac{1}{4}\left(\omega^2 + \frac{1}{\omega^2}\right) \tag{6}$$

for the old variables for position (x-y) and:

$$\frac{\partial t}{\partial \tau} = \left|f'(\omega)\right|^2 = \frac{\left|\omega^4 - 1\right|^2}{4\left|\omega\right|^6} \tag{7}$$

where $f'(\omega)$ denotes $\frac{\partial f}{\partial \omega}$, for the time.

In these new variables the equation of motion of the system is:

$$\omega'' + 2i\left|f'(\omega)\right|^2 \omega' = \mathrm{grad}_{\omega}\Omega^* \tag{8}$$

where $\omega = \omega_1 + i{*}\omega_2$ is the new complex variable for position, ω' and ω'' denotes first and second derivatives of ω with respect to the regularized time τ, $\mathrm{grad}_{\omega}\Omega^*$ represents $\frac{\partial\Omega^*}{\partial\omega_1} + i\frac{\partial\Omega^*}{\partial\omega_2}$ and Ω^* is the transformed pseudo-potential given by:

$$\Omega' = \left(\Omega - \frac{C}{2}\right)\left|f'(\omega)\right|^2 \tag{9}$$

where $C = \mu(1-\mu) - 2J$.

Equation (8) in complex variable can be separated in two second order equations in the real variables ω_1 and ω_2 and organized in the standard first order form, that is more suitable for numerical integration. The final form, after defining the regularized velocity components ω_3 and ω_4 as $\omega_1' = \omega_3$ and $\omega_2' = \omega_4$, is:

$$\omega_1' = \omega_3 \qquad \omega_2' = \omega_4 \tag{10a-b}$$

$$\omega_3' = 2\omega_4\left|f'(\omega)\right|^2 + \frac{\partial\Omega^*}{\partial\omega_1} \quad \omega_4' = -2\omega_3\left|f'(\omega)\right|^2 + \frac{\partial\Omega^*}{\partial\omega_2} \tag{10c-d}$$

Another set of equations necessary for this research is the one that relates velocity components from one set of variables to another. They are:

$$\dot{q}_1 = \frac{f'(\omega)}{|f'(\omega)|^2}\omega_3 \qquad \dot{q}_2 = \frac{f'(\omega)}{|f'(\omega)|^2}\omega_4 \tag{11}$$

4 - The Gravitational Capture

To define the gravitational capture it is necessary to use a few basic concepts from the two-body celestial mechanics. Those concepts are:

a) Closed orbit: a spacecraft in a orbit around a central body is in a closed orbit if its velocity is not large enough to escape from the central body. It remains always inside a sphere centered in the central body;

b) Open orbit: a spacecraft in a orbit around a central body is in a open orbit if its velocity is large enough to escape from the central body. In this case the spacecraft can go to infinity, no matter what is its initial position.

To identify the type of orbit of the spacecraft it is possible to use the definition of the two-body energy (E) of a massless particle orbiting a central body. The equation is $E = \frac{V^2}{2} - \frac{\mu}{r}$, where V is the velocity of the spacecraft relative to the central body, μ is the gravitational parameter of the central body and r is the distance between the spacecraft and the central body.

With this definition it is possible to say that the spacecraft is in a open orbit if its energy is positive and that it is in a closed orbit if its energy is negative. In the two-body problem this energy remains constant and it is necessary to apply an external force to change it. This energy is no longer constant in the restricted three-body problem. Then, for some initial conditions, a spacecraft can alternate the sign of its energy from positive to negative or from negative to positive. When the variation is from positive to negative the maneuver is called a "gravitational capture", to emphasize that the spacecraft was captured by gravitational forces only, with no use of an external force, like the thrust of an engine. The opposite situation, when the energy changes from negative to positive is called a "gravitational escape". In the restricted three-body problem there is no permanent gravitational capture. If the energy changes from positive to negative, it will change back to positive in the future. The mechanism of this capture is very well explained in Yamakawa (1992).

5 - The Belbruno-Miller Trajectories

One of the most important applications of the gravitational capture can be found in the Belbruno-Miller trajectories Belbruno (1987, 1990, 1992a, 1992b), Belbruno and Miller

(1990a, 1990b), Miller and Belbruno (1991), Yamakawa et. al. (1992, 1993a, 1993b), Krish (1991), Krish et. al. (1992), Yamakawa (1992). The concept of gravitational capture is used together with the basic ideas of the gravity-assisted maneuver and the bi-elliptic transfer orbit to generate a trajectory that requires a fuel consumption smaller than the one required by the Hohmann (1925) transfer. This maneuver consists of the following steps: i) the spacecraft is launched from an initial circular orbit with radius r_0 to an elliptic orbit that crosses the Moon's path; ii) a Swing-By with the Moon is used to increase the apoapsis of the elliptic orbit. This step completes the first part of the bi-elliptic transfer, with some savings in ΔV due to the energy gained from the Swing-By; iii) With the spacecraft at the apoapsis, a second very small impulse is applied to rise the periapsis to the Earth-Moon distance. Solar effects can reduce even more the magnitude of this impulse; iv) The transfer is completed with the gravitational capture of the spacecraft by the Moon.

6 - The Elliptic Restricted Three-Body Problem

After studying the circular problem, we give attention to the elliptic problem. For this case, the equations of motion for the spacecraft are assumed to be the ones valid for the well-known planar restricted elliptic three-body problem. We also use the standard canonical system of units, which implies that:

1. The unit of distance is the semi-major axis of the orbit M_1 and M_2;
2. The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one;
3. The mass of the smaller primary (M_2) is given by $\mu = \dfrac{m_2}{m_1 + m_2}$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is (1-μ), to make the total mass of the system unitary;
4. The unit of time is defined such that the period of the motion of the two primaries is 2π;
5. The gravitational constant is one.

There are several systems that can be used to describe the elliptic restricted problem (Szebehely, 1967). In this section the fixed (inertial) and the rotating-pulsating systems are described.

In the fixed system the origin is located in the barycenter of the two heavy masses M_1 and M_2. The horizontal axis $\overline{x}$ is the line connecting M_1 and M_2 and the vertical axis $\overline{y}$ is perpendicular to $\overline{x}$. In this system the position of M_1 and M_2 is:

$$\overline{x}_1 = -\mu r \cos \nu \tag{12}$$

$$\overline{y}_1 = -\mu r \sin \nu \tag{13}$$

$$\overline{x}_2 = (1-\mu) r \cos \nu \tag{14}$$

$$\bar{y}_2 = (1-\mu) r \sin\nu \tag{15}$$

where r is the distance between the two primaries, given by $r = \dfrac{1-e^2}{1+e\cos\nu}$, and ν is the true anomaly of M_2.

Then, in this system, the equations of motion of the massless particle are:

$$\bar{x}'' = \frac{-(1-\mu)(\bar{x}-\bar{x}_1)}{r_1^3} - \frac{\mu(\bar{x}-\bar{x}_2)}{r_2^3} \tag{16}$$

$$\bar{y}'' = \frac{-(1-\mu)(\bar{y}-\bar{y}_1)}{r_1^3} - \frac{\mu(\bar{y}-\bar{y}_2)}{r_2^3} \tag{17}$$

where $(\ )''$ means the second derivative with respect to time, r_1 and r_2 are the distances from M_1 and M_2, given by:

$$r_1^2 = (\bar{x}-\bar{x}_1)^2 - (\bar{y}-\bar{y}_1)^2 \tag{18}$$

$$r_2^2 = (\bar{x}-\bar{x}_2)^2 - (\bar{y}-\bar{y}_2)^2 \tag{19}$$

Now, we will introduce the rotating-pulsating system of reference. In this system, the origin is again the center of mass of the two massive primaries. The horizontal axis (x) is the line that connect the two primaries. It rotates with a variable angular velocity in a such way that the two massive primaries are always in this axis. The vertical axis (y) is perpendicular to the x axis. Besides the rotation, the system also pulsates in a such way to keep the massive primaries in fixed positions. To achieve this situation we have to multiply the unit of distances for the instantaneous value of the distance between the two primaries (r). In a system like this one, the positions of the primaries are:

$$x_1 = -\mu,\ x_2 = 1-\mu,\ y_1 = y_2 = 0 \tag{20}$$

In this system, the equations of motion por the massless particle are:

$$\ddot{x} - 2\dot{y} = \frac{r}{p}\left(x - (1-\mu)\frac{x-x_1}{r_1^3} - \mu\frac{x-x_2}{r_2^3} \right) \tag{21}$$

$$\ddot{y} + 2\dot{x} = \frac{r}{p}\left(y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \right) \tag{22}$$

and we also have an equation to relate time and the true anomaly of the primaries:

$$\dot{t} = \frac{r^2}{p^{1/2}} \tag{23}$$

where the overdot means derivative with respect to the true anomaly of the primaries and p is the semi-lactus rectum of the ellipse.

The equations that relates one system to another are:

$$\bar{x} = rx\cos\nu - ry\sin\nu \tag{24}$$

$$\bar{y} = rx\sin\nu + ry\cos\nu \tag{25}$$

$$x = (\bar{x}\cos\nu + \bar{y}\sin\nu)/r \tag{26}$$

$$y = (\bar{y}\cos\nu - \bar{x}\sin\nu)/r \tag{27}$$

for the positions and:

$$\bar{x}' = x'r\cos\nu - y'r\sin\nu - \frac{x(1-e^2)\sin\nu}{(1+e\cos\nu)^2} - \frac{y(1-e^2)(e+\cos\nu)}{(1+e\cos\nu)^2} \tag{28}$$

$$\bar{y}' = x'r\sin\nu + y'r\cos\nu - \frac{y(1-e^2)\sin\nu}{(1+e\cos\nu)^2} + \frac{x(1-e^2)(e+\cos\nu)}{(1+e\cos\nu)^2} \tag{29}$$

$$x' = \frac{\bar{x}'\cos\nu}{r} + \frac{\bar{y}'\sin\nu}{r} - \frac{\bar{x}(\sin\nu + e\sin 2\nu)}{1-e^2} + \frac{\bar{y}(\cos\nu + 2e\cos^2\nu - e)}{1-e^2} \tag{30}$$

$$y' = \frac{\bar{y}'\cos\nu}{r} - \frac{\bar{x}'\sin\nu}{r} - \frac{\bar{y}(\sin\nu + e\sin 2\nu)}{1-e^2} - \frac{\bar{x}(\cos\nu + 2e\cos^2\nu - e)}{1-e^2} \tag{31}$$

for the velocities, where $p = 1 - e^2$ and $r = \dfrac{1-e^2}{1+e\cos\nu}$.

7 - Results

To quantify the "gravitational captures" we studied this problem under several different initial conditions. The assumptions made for the numerical examples presented in the first part of this section are (some of them will be changed later, to generalize our results):

i) The system of primaries used is the Earth-Moon system;

ii) The motion is planar everywhere;
iii) The starting point of each trajectory is 100 km from the surface of the Moon (r_p from the center of the Moon). Then, to specify the initial position completely it is necessary to give the value of one more variable. The variable used is the angle α, an angle measured from the Earth-Moon line, in the counter-clock-wise direction and starting in the side opposite to the Earth (see Fig. 1);
iv) The magnitude of the initial velocity is calculated from a given value of $C_3 = 2E = V^2 - \frac{2\mu}{r}$, where E is the two-body energy of the spacecraft with respect to the Moon, V is the velocity of the spacecraft, μ is the gravitational parameter of the Moon and r is the distance between the spacecraft and the center of the Moon. The direction of the velocity is assumed to be perpendicular to the line spacecraft-center of the Moon and pointing to the counter-clock-wise direction for a direct orbit and to the clock-wise direction for a retrograde orbit (see Fig. 1);
v) To consider that an escape occurred, we request two conditions (following the conditions used in Yamakawa (1992)): i) that the spacecraft reaches a distance of 100000 km (0.26 canonical units) from the center of the Moon in a time shorter than 50 days; ii) the energy is positive at this point. Fig. 1 shows the point P where the escape occurs. The angle that specifies this point is called the "entry position angle" and it is designated with the letter β.

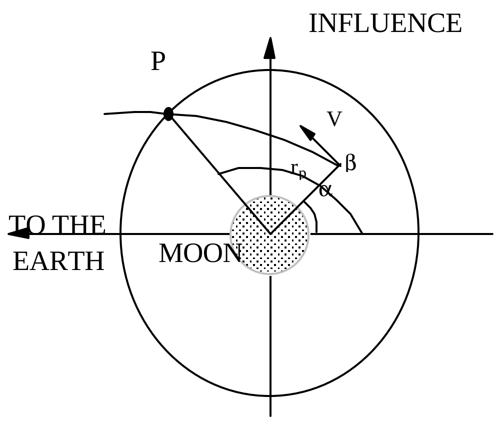


Fig. 1 - Variables to specify the initial conditions of the spacecraft.

Then, for each initial position the trajectories were numerically integrated backward in time. Every escape in backward time corresponds to a "gravitational capture" in forward time. The time of flight until an escape occurs is obtained. Then, the results are organized and plotted in the next figures. Fig. 2 shows C_3 plotted in a system of axis that has the time of flight for escape in canonical units in the horizontal axis and the angle α in degrees in the vertical axis. To avoid an excessive number of lines that would make the plots unclear, we split this figure in three parts. In the first part we plotted the cases where $C_3 = 0, -0.01, -0.02$ (the inner curve represents $C_3 = 0$ and the outer curve represents $C_3 = -0.02$). From this part of the figure it is possible to see the existence of two regions of minimum time of flight for a given value of C_3. Those minimums are close to 150° and 325°. They correspond to "windows" for the capture of the spacecraft. This phenomenon occurs for all the values of C_3. In the second part of this figure we plotted the cases where $C_3 = 0, -0.04, -0.08$, that confirm this behavior. In the third part of this figure we plotted the cases where $C_3 = 0, -0.1, -0.2$ to give an idea of whole interval studied. We can see that not all the lines are continuos. It means that, for some values of C_3, not all the values of α allow an escape to occur. For $C_3 \cong -0.2$ (close to the minimum value of C_3 that allows an escape) only the regions close to the ranges 0-20°, 160-220° and 320-360° has escapes. Those plots give important information to mission designers, because they have estimates for the time required for the gravitational capture.

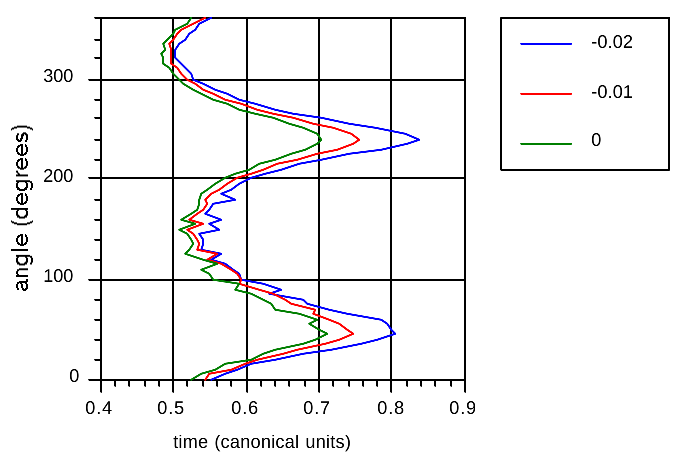


Fig. 2 - C_3 for "Gravitational Captures".

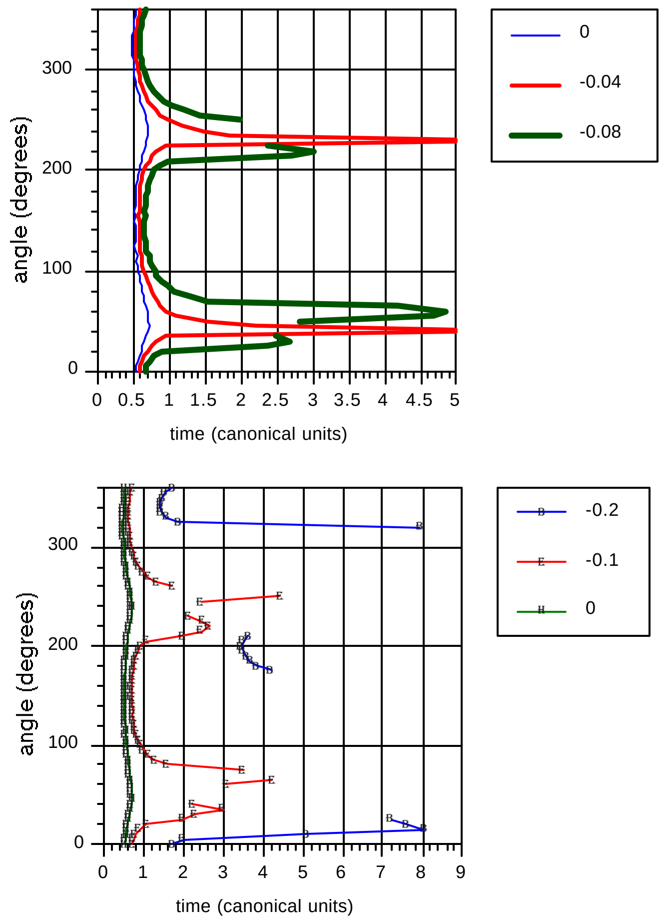


Fig. 2 (Cont.) - C_3 for "Gravitational Captures".

Then, we calculated the value of the minimum energy that allows an escape for every initial angle α. The results are shown in Fig. 3. The radial variable is the absolute value of the minimum value of C_3 and the angular variable is the angle α. Those results are very similar to the results obtained previously by Yamakawa (1992). We can see the existence of angles that provides maximum savings, like the interval 150-200°. We also can see that a direct orbit usually provides more savings than the retrograde orbits.

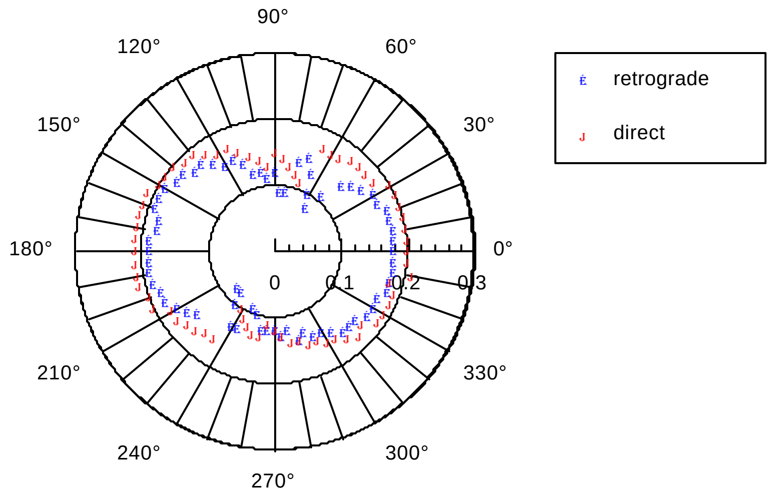


Fig. 3 - Absolute values of the minimum C_3 for each α (direct and retrograde orbits).

Fig. 4 shows the entry position angle β as a function of C_3 for several values of α (0, 60, 120, 180, 240, 300 degrees). The range for the angle β was changed from 0 to 360 degrees to -90 to 270 degrees to avoid discontinuities in some families. Yamakawa and his colleagues (Yamakawa 1992) show similar results for the case $\alpha = 180°$.

Then, we start to generalize our results a bit. Our next step is to study the effects of the variation of the initial distance (r_p) from the Moon. We calculate and plot the minimum C_3 as a function of the entry position angle for several values of the initial distance. Fig. 5 shows those results. The figure is separated in two parts: the first one shows the results when r_p is close to the surface of the Moon (1838 km to 20838 km) and the second one shows the regions more distant from the surface of the Moon (20838 km to 50838 km). The results show that the plots seem to rotate in the clock-wise direction when r_p increases. We can also see that, for some values of the angle α, the increase of r_p makes the absolute values of C_3 to increase (so, the real value decreases), like in the range 0-30°, and in some others it has the opposite effect, like in the 120-150° interval. So, there is no general rule to govern those effects.

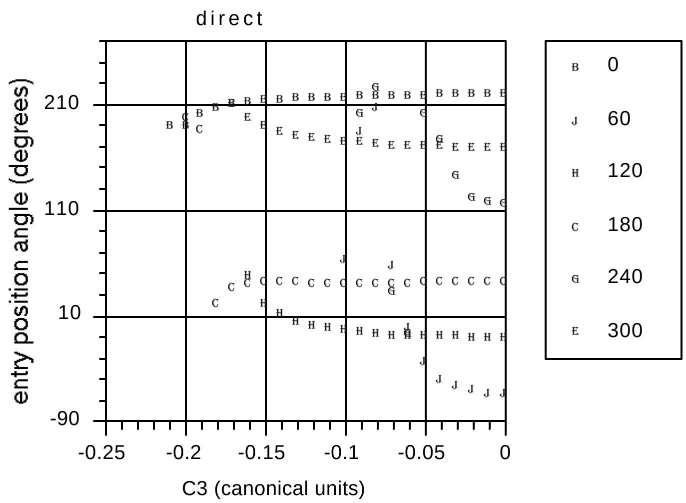


Fig. 4 - Entry position angle β as a function of C_3.

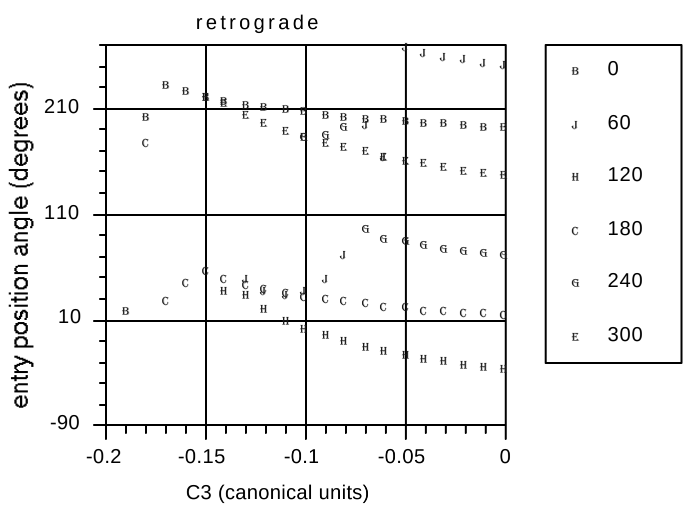


Fig. 4 (cont.) - Entry position angle β as a function of C_3.

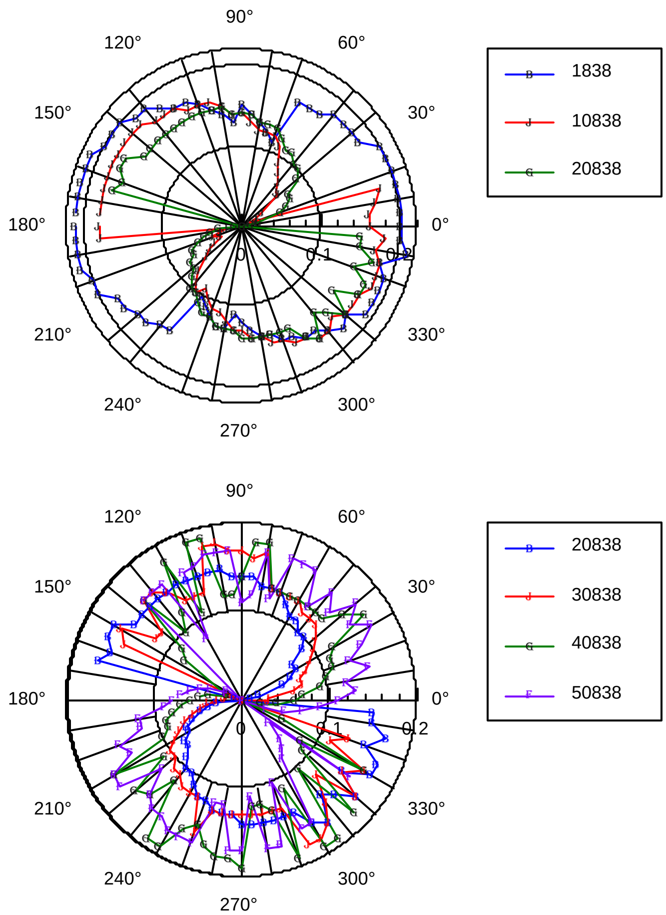


Fig. 5 - Absolute values of the minimum C_3 for each α (several values for the initial distance).

Our next generalization is to extended those results for other systems than the Earth-Moon. Fig. 6 shows the results for the Sun-Jupiter, Sun-Mars and Sun-Earth.

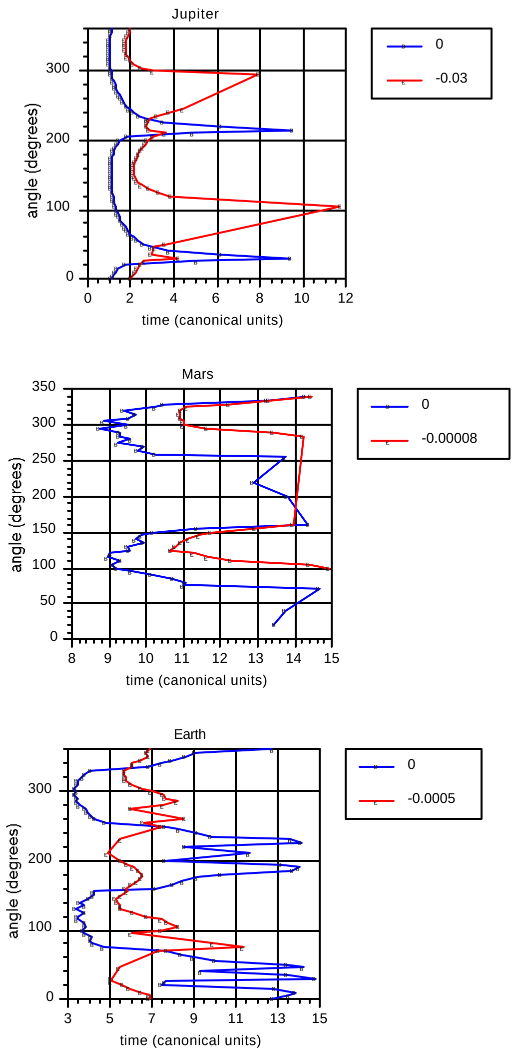


Fig. 6 - Results for the Sun-Jupiter, Sun-Mars and Sun-Earth systems.

We clearly see that the savings provided by the gravitational capture in those cases are a lot smaller than in the Earth-Moon case, but they still exist. The savings decrease when the mass of the planet decreases. The plots also show that the "windows" of shorter times are preserved.

As an example of the calculations that we made for th eelliptical case, we show the results for the cases where the eccentricity of the primaries is kept constant and the true anomaly assumes the values 0°, 90°, 180°, 270°. Figure 7 shows the numerical results in plots where the radial variable is the magnitude of C_3 and the angular variable is the angle α. We can see that the savings are greater where the secondary body is at periapse ($\gamma = 0°$), what is expected, since the smaller distance between the two primaries increase the effect of the third body (the main cause of the savings). We can also see the regions of maximum and minimum savings. In figure 8 we can see the direct effect of the eccentricity. In this figure the true anomaly is kept constant at $\gamma = 0°$ and the eccentricity assumes the values 0.0, 0.1, 0.5 and 0.8. We can see the regions of maximum and minimum savings and we can conclude that when the eccentricity increases, the magnitude of the savings also increases.

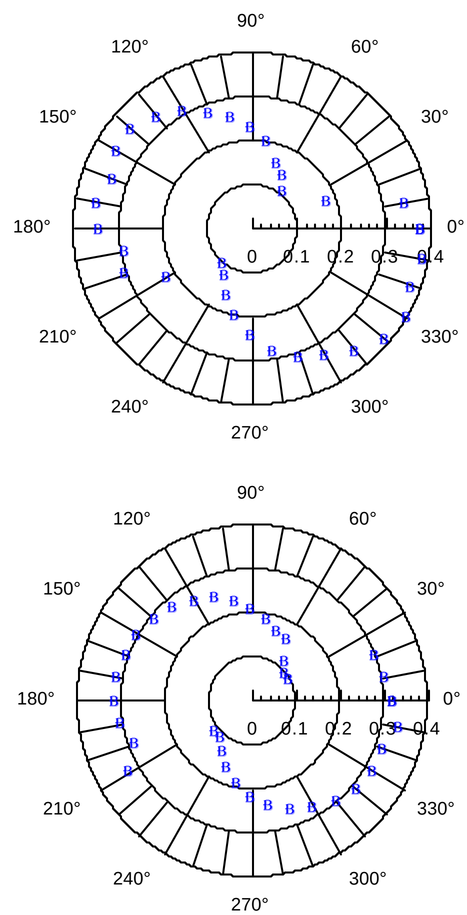
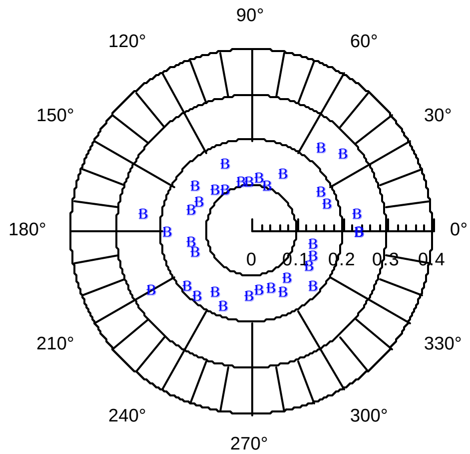


Figure 7 - Minimum C_3 for e = 0.5 and γ = 0°, 90°, 180°, 270°.



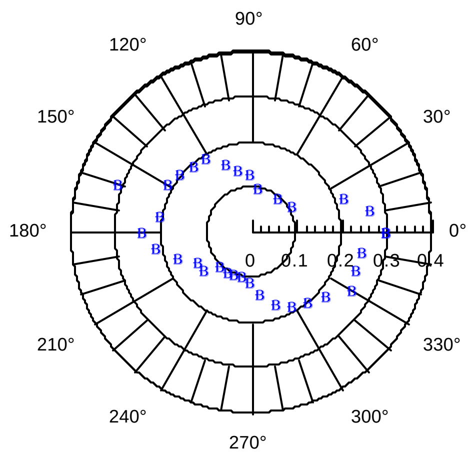


Figure 7 (Cont.) - Minimum C_3 for e = 0.5 and $\gamma = 0^\circ, 90^\circ, 180^\circ, 270^\circ$.

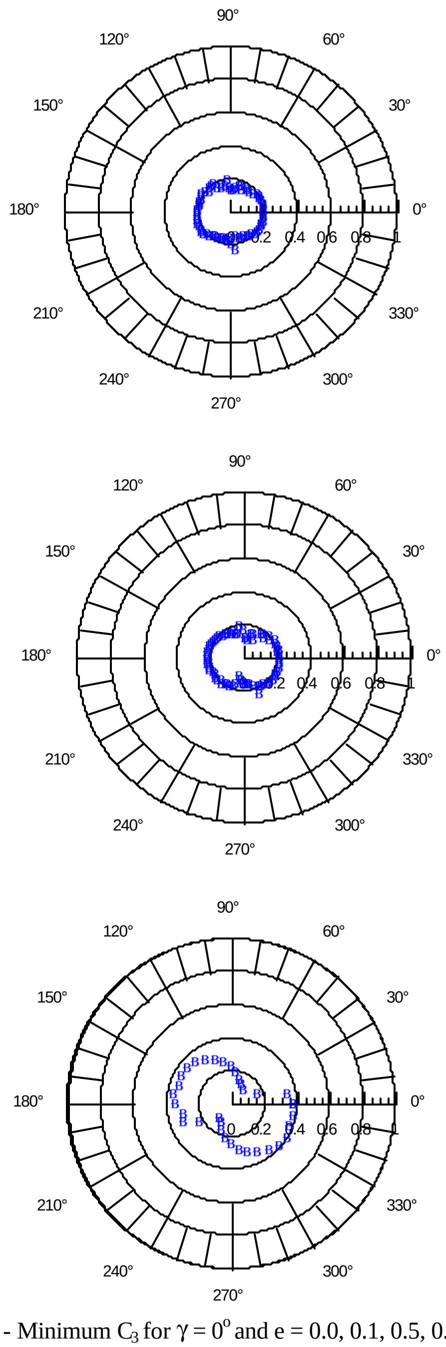


Figure 8 - Minimum C_3 for $\gamma = 0^o$ and e = 0.0, 0.1, 0.5, 0.8

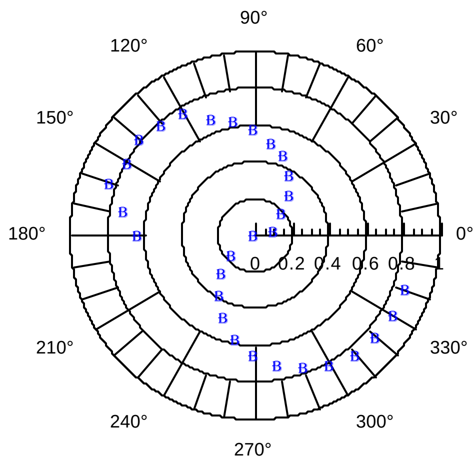


Figure 8 (Cont.) - Minimum C_3 for $\gamma = 0^o$ and e = 0.0, 0.1, 0.5, 0.8

8 - Conclusion

This paper studied the "gravitational capture" in the regularized restricted three-body problem. Our results confirmed the results found previously by Yamakawa and his colleagues, that showed the characteristics and importance of this problem for Earth-Moon transfers. We also showed the regions of minimum energy for the capture with the correspondent time of flight. We found the existence of "windows" with short time for capture. Then we extended those results by varying the initial distance from the Moon and quantifying its effect. Next, new results were shown for other systems in the Solar System, like the Sun-Earth, Sun-Mars and Sun-Jupiter systems. Those results showed the existence of savings in those cases, but with smaller magnitudes. We also generalized this research for the interval $0.0 < \mu < 0.5$ and we concluded that the magnitude of the C_3 minimum increases very much with μ. Those results are useful to design trajectories similar to the Belbruno-Miller transfers to the Moon. We also developed a numerical algorithm to study the problem of gravitational capture in the elliptical restricted three-body problem. The effect of the true anomaly for a fixed eccentricity and the effect of the eccentricity for a fixed true anomaly were studied. We showed the numerical results and we concluded that the savings increase with the eccentricity and when the true anomaly goes close to zero.

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