Wave dissipation by electron Landau damping in low aspect ratio tokamaks with elliptic magnetic surfaces

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Longitudinal dielectric permittivity elements are derived for radio-frequency waves in an axisymmetric tokamak with elliptic magnetic surfaces, for arbitrary elongation and inverse aspect ratio. A collisionless plasma model is considered. Drift-kinetic equation is solved separately for untrapped (passing or circulating) and three groups of the trapped particles as a boundary-value problem. Bounce resonances are taken into account. A coordinate system with the "straight" magnetic field lines is used. Permittivity elements, evaluated in the paper, are suitable to estimate the wave dissipation by electron Landau damping (e.g., during the plasma heating and current drive generation) in the frequency range of Alfvén, fast magnetosonic, and lower hybrid waves, for both the large and low aspect ratio tokamaks. The dissipated wave power is expressed by the summation of terms including the imaginary parts of both the diagonal and nondiagonal elements of the longitudinal permittivity. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1499953]

Low aspect ratio tokamaks (or spherical tokamaks) represent a promising alternative route to magnetic thermonuclear fusion.[1–5] In order to achieve fusion conditions in these devices additional plasma heating must be employed. Effective schemes of heating and current drive in tokamak plasmas can be realized using rf waves. As is well known, the kinetic wave theory of any toroidal plasma should be based on the solution of Vlasov–Maxwell's equations. However, this problem is not simple even in the scope of linear theory since to solve the wave (or Maxwell's) equations it is necessary to use the correct dielectric (or wave conductivity) tensor valid in the given frequency range for the concrete two- or three-dimensional (2D or 3D) plasma model. In this paper, the longitudinal permittivity elements are derived for rf waves in a 2D axisymmetric tokamak with elliptic magnetic surfaces under the arbitrary elongation and arbitrary aspect ratio. The drift-kinetic equation is solved separately for untrapped particles, usual t-trapped particles, and two additional groups of the so-called d-trapped particles as a boundary-value problem, using an approach developed for low aspect ratio tokamak with circular magnetic surfaces[6] and for large aspect ratio tokamaks with elliptic magnetic surfaces.[7,8]

To describe a low aspect ratio tokamak with elliptic magnetic surfaces we use the new variables (r,θ',ϕ) instead of quasitoroidal coordinates (ρ,θ,ϕ) as

$$r=\rho\sqrt{(a^2/b^2)\sin^2\theta+\cos^2\theta},$$
$$\theta'=2\arctan\left[\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\tan\left(\frac{1}{2}\arctan\left(\frac{a}{b}\tan\theta\right)\right)\right],\quad \phi=\phi, \tag{1}$$

transforming the initial elliptic cross sections of the magnetic surfaces to circles with radius a of the external magnetic surface, where the Cartesian coordinates are $x=(R+\rho\cos\theta)\cos\phi$, $y=(R+\rho\cos\theta)\sin\phi$, $z=-\rho\sin\theta$. In the (r,θ') coordinates, the magnetic field lines become "straight," and the module of an equilibrium magnetic field, $H=|\mathbf{H}|$, is

$$H(r,\theta')=\sqrt{H_{\phi0}^2+H_{\theta0}^2}\,g(r,\theta'),$$
$$g(r,\theta')=\sqrt{(1-\varepsilon\cos\theta')^2+\lambda(\varepsilon-\cos\theta')^2/(1-\varepsilon^2)}, \tag{2}$$

where

$$\varepsilon=r/R,\quad \lambda=h_\theta^2\left(b^2/a^2-1\right),$$
$$h_\phi=H_{\phi0}/\sqrt{H_{\phi0}^2+H_{\theta0}^2},\quad h_\theta=H_{\theta0}/\sqrt{H_{\phi0}^2+H_{\theta0}^2}. \tag{3}$$

$H_{\varphi0}(r)$ and $H_{\theta0}(r)$ are the toroidal and poloidal projections of $\mathbf{H}$ for a given magnetic surface at the points $\theta=\pm\pi/2$; R is the major tokamak radius; and b and a are the major and minor semiaxes of the elliptic cross section of the external magnetic surface. In this model, all magnetic surfaces are similar to each other with the same elongation equal to b/a.

To solve the drift-kinetic equation for plasma particles we use the standard method[9–14] of switching to new variables associated with conservation integrals of energy and magnetic moment. Introducing the variables ν (particle energy) and μ (nondimensional magnetic moment) in velocity space instead of $\nu_\parallel$ and $\nu_\perp$:

$$\nu^2=\nu_\parallel^2+\nu_\perp^2,\quad \mu=\frac{\nu_\perp^2}{\nu_\parallel^2+\nu_\perp^2}\frac{\sqrt{H_{\phi0}^2+H_{\theta0}^2}}{H(r,\theta')}, \tag{4}$$

the perturbed distribution function of plasma particles (any kind of ions and electrons) can be found as

$$f(t,r,\theta',\phi,v_\parallel,v_\perp)=\sum_{s=\pm1} f_s(r,\theta',v,\mu)\exp(-i\omega t+in\phi), \quad (5)$$

where we have determined that the problem is uniform in both time t and toroidal angle ϕ. In the zeroth order over magnetization parameters (i.e., neglecting the finite Larmor radius effects and assuming that the wave frequency is much larger than both the drift frequency and the precession frequency), the linearized drift-kinetic equation for harmonics f_s can be written as

$$\frac{(1-\varepsilon\cos\theta')^2}{(1-\varepsilon^2)^{1.5}}\frac{\sqrt{1-\mu g(r,\theta')}}{g(r,\theta')}\left(\frac{\partial f_s}{\partial\theta'}+inqf_s\right)-i\frac{sr\omega}{h_\theta v}f_s$$
$$=2\,(erE_\parallel F/Mh_\theta v_T^2)\sqrt{1-\mu\cdot g(r,\theta')}, \quad (6)$$

where

$$F=\frac{N}{\pi^{1.5}v_T^3}\exp\left(-\frac{v^2}{v_T^2}\right),\quad v_T^2=\frac{2T}{M},\quad q=\frac{\varepsilon h_\phi}{h_\theta\sqrt{1-\varepsilon^2}}. \quad (7)$$

$E_\parallel=\mathbf{E}\cdot\mathbf{h}$ is the parallel (to $\mathbf{h}=\mathbf{H}/H$) electric field component; th steady-state distribution function F is given as Maxwellian with the particle density N, temperature T, charge e, and mass M. The index of particles species (ions and electrons) is omitted in Eq. (6). By the indexes $s=\pm1$ for f_s, we distinguish the perturbed distribution functions with positive and negative values of the parallel velocity $v_\parallel=sv\sqrt{1-\mu g(r,\theta')}$ relative to $\mathbf{H}$. Thus, the initial drift-kinetic equation is reduced to the first-order differential equation with respect to the poloidal angle θ', where the variables r, v, μ (as well as R, a, b, q, N, T) appear as the parameters. Note, using the smallness of $\varepsilon\ll1$ and $\lambda\ll1$, that Eq. (6) can be readily reduced to the initial drift-kinetic equation in Ref. 7 for plasma particles in the large aspect ratio tokamaks with small elongation.

After solving Eq. (6), the longitudinal component of the current density $j_\parallel=\mathbf{j}\cdot\mathbf{h}$ can be expressed as

$$j_\parallel(r,\theta')=\pi eg(r,\theta')\sum_s^{\pm1} s$$
$$\times\int_0^\infty v^3\int_0^{1/g(r,\theta')}f_s(r,\theta',v,\mu)\,d\mu\,dv. \quad (8)$$

Depending on μ and θ', the phase volume of plasma particles should be split in the phase volumes of untrapped, t-, and d-trapped particles by the following inequalities:

$$0\le\mu\le\mu_u,\quad -\pi\le\theta'\le\pi \quad \text{for untrapped particles}, \quad (9)$$

$$\mu_u\le\mu\le\mu_t,\quad -\theta_t\le\theta'\le\theta_t \quad \text{for } t\text{-particles}, \quad (10)$$

$$\mu_t\le\mu\le\mu_d,\quad -\theta_t\le\theta'\le-\theta_d \quad \text{for } d\text{-particles}, \quad (11)$$

$$\mu_t\le\mu\le\mu_d,\quad \theta_d\le\theta'\le\theta_t \quad \text{for } d\text{-particles}, \quad (12)$$

where [analyzing the condition $v_\parallel(\mu,\theta')=0$]

$$\mu_u=\frac{1-\varepsilon}{\sqrt{1+\lambda}},\quad \mu_t=\frac{1+\varepsilon}{\sqrt{1+\lambda}},\quad \mu_d=\sqrt{1+\frac{\varepsilon^2}{\lambda}}, \quad (13)$$

and the reflection points $\pm\theta_t$ and $\pm\theta_d$ for t- and d-trapped particles, respectively, are

$$\pm\theta_t=\pm\arccos\left\{\varepsilon(1+\lambda)/(\lambda+\varepsilon^2)-\sqrt{\frac{\varepsilon^2(1+\lambda)^2}{(\lambda+\varepsilon^2)^2}-\frac{1}{\lambda+\varepsilon^2}\left[1+\varepsilon^2\lambda-\left(\frac{1-\varepsilon^2}{\mu}\right)^2\right]}\right\}, \quad (14)$$

$$\pm\theta_d=\pm\arccos\left\{\varepsilon(1+\lambda)/(\lambda+\varepsilon^2)+\sqrt{\frac{\varepsilon^2(1+\lambda)^2}{(\lambda+\varepsilon^2)^2}-\frac{1}{\lambda+\varepsilon^2}\left[1+\varepsilon^2\lambda-\left(\frac{1-\varepsilon^2}{\mu}\right)^2\right]}\right\} \quad (15)$$

for the given magnetic field structure by $g(r,\theta')$ in Eq. (2).

Further, we solve Eq. (6), in the general case, for untrapped, t-trapped, and two groups of d-trapped particles, i.e., under the condition when the tokamak magnetic field configuration can be considered as a system with two local minimums of $\mathbf{H}(r,\theta')$. In this case, the existence criterion of the d-trapped particles is $\varepsilon<\lambda$ or

$$b/a>\sqrt{1+\varepsilon+q^2(1-\varepsilon^2)/\varepsilon}. \quad (16)$$

Otherwise, if $\varepsilon>\lambda$, the equilibrium magnetic field has only one minimum, and the d-trapped particles are absent at the given magnetic surface (as it is in tokamaks with circular magnetic surfaces where $b=a$, $\lambda=0$ and accordingly $\lambda<\varepsilon$). Of course, the d-trapped particles are characteristic only of elongated tokamaks. Moreover, in the D-shaped tokamaks, in the general case, the additional groups of trapped particles can appear. However, the description of the new groups of the trapped particles there will be more complicated.

The solution of Eq. (6) should be found by the specific boundary conditions of the trapped and untrapped particles. For untrapped particles, we use the periodicity of f_s over θ', whereas the boundary condition for the t- and d-trapped particles is the continuity of f_s at the corresponding stop points, Eqs. (14) and (15). As a result, we seek the perturbed distribution functions of untrapped, f_s^u, t-trapped, f_s^t, and d-trapped, f_s^d, particles as

$$f_s^u=\sum_p^{\pm\infty} f_{s,p}^u\exp\left[i2\pi(p+nq)\frac{\tau(\theta')}{T_u}-inq\theta'\right], \quad (17)$$

$$f_s^t=\sum_p^{\mp\infty} f_{s,p}^t\exp\left[i2\pi p\frac{\tau(\theta')}{T_t}-inq\theta'\right], \quad (18)$$

$$f_s^d=\sum_p^{\mp\infty} f_{s,p}^d\exp\left[i2\pi p\frac{\tau(\theta')-\tau(\theta_d)}{T_d}-inq\theta'\right], \quad (19)$$

where p is the (integer) number of the bounce resonances for trapped particles, and the transit resonances for untrapped particles,

$$\tau(\theta')=\int_0^{\theta'}\frac{(1-\varepsilon^2)g(r,\eta)\,d\eta}{(1-\varepsilon\cos\eta)^2\sqrt{1-\mu g(r,\eta)}}, \quad (20)$$

is the new time-like variable (instead of θ' to describe the bounce-periodic motion of untrapped, t-, and d-trapped particles along the magnetic field line with the corresponding periods $T_u=2\tau(\pi)$, $T_t=4\tau(\theta_t)$, and $T_d=2[\tau(\theta_t)-\tau(\theta_d)]$. The Fourier harmonics $f^u_{s,p}$, $f^t_{s,p}$, and $f^d_{s,p}$ for untrapped, t-, and d-trapped particles can be readily derived after the corresponding bounce-averaging.

To evaluate the dielectric tensor elements we use the Fourier expansions of the current density and electric field over the poloidal angle θ':

$$\frac{j_\parallel(\theta')}{(1-\varepsilon^2)g(r,\theta')}=\sum_m^{\pm\infty} j^m_\parallel \exp(im\theta'), \tag{21}$$

$$E_\parallel(\theta')\frac{(1-\varepsilon^2)g(r,\theta')}{(1-\varepsilon\cos\theta')^2}=\sum_{m'}^{\pm\infty} E^{m'}_\parallel \exp(im'\theta'). \tag{22}$$

As a result, the whole spectrum of the electric field, $E^{m'}_\parallel$, is present in the given mth harmonic $j^m_\parallel$ of the current density:

$$\frac{4\pi i}{\omega} j^m_\parallel=\sum_{m'}^{\pm\infty} \varepsilon^{m,m'}_\parallel E^{m'}_\parallel=\sum_{m'}^{\pm\infty}(\varepsilon^{m,m'}_{\parallel,u}+\varepsilon^{m,m'}_{\parallel,t}+\varepsilon^{m,m'}_{\parallel,d})E^{m'}_\parallel, \tag{23}$$

where $\varepsilon^{m,m'}_{\parallel,u}$, $\varepsilon^{m,m'}_{\parallel,t}$ and $\varepsilon^{m,m'}_{\parallel,d}$ are, respectively, the separate contributions of any kind of the untrapped, t-, and d-trapped particles, to the longitudinal (parallel) permittivity elements:

$$\varepsilon^{m,m'}_{\parallel.u}=\frac{\omega_p^2 r^2}{2h_\theta^2 v_T^2\pi^3}\sum_{p=-\infty}^{\infty}\int_0^{\mu_u}\frac{T_u A^m_p A^{m'}_p}{(p+nq)^2}[1+2u_p^2+2i\sqrt{\pi}u_p^3 W(u_p)]d\mu, \tag{24}$$

$$\varepsilon^{m,m'}_{\parallel.t}=\frac{\omega_p^2 r^2}{2h_\theta^2 v_T^2\pi^3}\sum_{p=1}^{\infty}\int_{\mu_u}^{\mu_t}\frac{T_t}{p^2}B^m_p B^{m'}_p[1+2\nu_p^2+2i\sqrt{\pi}\nu_p^3 W(\nu_p)]d\mu, \tag{25}$$

$$\varepsilon^{m,m'}_{\parallel.d}=\frac{\omega_p^2 r^2}{h_\theta^2 v_T^2\pi^3}\sum_{p=1}^{\infty}\int_{\mu_t}^{\mu_d}\frac{T_d}{p^2}(C^m_p C^{m'}_p+D^m_p D^{m'}_p)\times[1+2z_p^2+2i\sqrt{\pi}z_p^3 W(z_p)]d\mu. \tag{26}$$

Here we have used the following definitions:

$$W(z)=\exp(-z^2)\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z \exp(t^2)dt\right), \tag{27}$$

$$u_p=\frac{r\omega\sqrt{1-\varepsilon^2}T_u}{2\pi h_\theta|p+nq|v_T},\quad \nu_p=\frac{r\omega\sqrt{1-\varepsilon^2}T_t}{2\pi h_\theta p v_T},$$
$$z_p=r\omega\sqrt{1-\varepsilon^2}T_d/2\pi h_\theta p v_T,\quad \omega_p^2=4\pi Ne^2/M, \tag{28}$$

$$A^m_p=\int_0^\pi \cos\left[(m+nq)\eta-(p+nq)\pi\frac{\tau(\eta)}{\tau(\pi)}\right]d\eta, \tag{29}$$

$$B^m_p=\int_0^{\theta_t}\cos\left[(m+nq)\eta-p\frac{\pi\tau(\eta)}{2\tau(\theta_t)}\right]d\eta+(-1)^{p-1}\int_0^{\theta_t}\cos\left[(m+nq)\eta+p\frac{\pi\tau(\eta)}{2\tau(\theta_t)}\right]d\eta, \tag{30}$$

$$C^m_p=\int_{\theta_d}^{\theta_t}\sin[(m+nq)\eta]\cos\left[p\pi\frac{\tau(\eta)-\tau(\theta_d)}{\tau(\theta_t)-\tau(\theta_d)}\right]d\eta, \tag{31}$$

$$D^m_p=\int_{\theta_d}^{\theta_t}\sin[(m+nq)\eta]\sin\left[p\pi\frac{\tau(\eta)-\tau(\theta_d)}{\tau(\theta_t)-\tau(\theta_d)}\right]d\eta. \tag{32}$$

As was mentioned previously, Eqs. (24)–(26) describe the contribution of any kind of untrapped, t-, and d-trapped particles to the dielectric elements. The corresponding expressions for plasma electrons and ions can be obtained from (24)–(26) replacing T, N, M, e by the electron T_e, N_e, m_e, e_e and ion T_i, N_i, M_i, e_i parameters, respectively. To obtain the total expressions of the permittivity elements, as usual, it is necessary to carry out the summation over all species of plasma particles. It must be pointed out that the dielectric characteristics, Eqs. (24)–(26), are derived neglecting drift effects and the finite particle-orbit widths. These effects (as well as the finite pressure and Larmor radius corrections) can be accounted for in the next order(s) of perturbations over the magnetization parameter.

Note that Eqs. (24)–(32) have been written in the quite general form where the ellipticity is accounted implicitly by the functions $\tau(r,\theta)$, $g(r,\theta)$, and $\lambda(r)$ defined, respectively, in Eqs. (20), (2), and (3). In particular, for tokamaks with circular magnetic surfaces, where $\lambda=0$ and $\varepsilon^{m,m'}_{\parallel,d}\equiv 0$, the expressions $\varepsilon^{m,m'}_{\parallel,u}$ and $\varepsilon^{m,m'}_{\parallel,t}$ (and the corresponding phase coefficients A^m_p, B^m_p) can be simplified substantially[6] because the $\tau(r,\theta)$ functions for trapped and untrapped particles can be reduced to (i) the third kind elliptic integrals in low ($\varepsilon<1$) aspect ratio tokamaks, or (ii) the first kind elliptic integrals in large ($\varepsilon\ll 1$) aspect ratio tokamaks. An important feature of the dielectric characteristics of Eqs. (24)–(26) is the fact that, since the phase coefficients A^m_p, B^m_p, C^m_p and D^m_p are independent of the wave frequency ω and the particle energy ν, the analytical Landau integration of the perturbed distribution functions of both the trapped and untrapped particles in velocity space is possible. As a result, the longitudinal permittivity elements are written by summation of bounce-resonant terms including the well-known plasma dispersion function $W(z)$, i.e., by the probability integral of the complex argument, Eq. (27). After this, the numerical estimations of both the real and imaginary parts of the longitudinal permittivity elements become simpler, and their dependence of the wave frequency is defined only by the arguments u_p, ν_p, and z_p of the plasma dispersion functions $W(u_p)$, $W(v_p)$, and $W(z_p)$.

One of the main mechanisms of the radio frequency plasma heating is the electron Landau damping of waves due to the Cherenkov resonance interaction of $E_\parallel$ with untrapped, t-, and d-trapped electrons. Here, it should be taken into account that the Cherenkov resonance conditions are different for trapped and untrapped particles in the considered plasma model, see the arguments of the plasma dispersion functions (28), and have nothing in common with the wave–particle resonance condition in the cylindrical magnetized plasmas. Another important feature of tokamak plasmas is the contribution of all $E^{m'}_\parallel$ harmonics to the given $j^m_\parallel$ harmonic of the current density, Eq. (23). As a result, after averaging in time and poloidal angle, the wave power absorbed, $P=\mathrm{Re}(E_\parallel j_\parallel^*)$, due to the untrapped, t-trapped, and d-trapped electrons can be estimated by

$$P=\frac{\omega}{8\pi}\sum_m^{\pm\infty}\sum_{m'}^{\pm\infty}(\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}+\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}+\mathrm{Im}\,\varepsilon_{\parallel,d}^{m,m'})\times(\mathrm{Re}\,E_\parallel^m\mathrm{Re}\,E_\parallel^{m'}+\mathrm{Im}\,E_\parallel^m\mathrm{Im}\,E_\parallel^{m'}),\tag{33}$$

where $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}$, $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}$, and $\mathrm{Im}\,\varepsilon_{\parallel,d}^{m,m'}$ are the separate contributions of untrapped, t-, and d-trapped electrons to the imaginary part of the longitudinal permittivity elements: $\mathrm{Im}\,\varepsilon_\parallel^{m,m'}=\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}+\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}+\mathrm{Im}\,\varepsilon_{\parallel,d}^{m,m'}$,

$$\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}=\frac{\omega_p^2r^5\omega^3}{8h_\theta^5v_T^5\pi^{5.5}}(1-\varepsilon^2)^{1.5}\sum_{p=-\infty}^{\infty}\int_0^{\mu_u}\frac{T_u^4}{|p+nq|^5}\times A_p^mA_p^{m'}\exp(-u_p^2)d\mu,\tag{34}$$

$$\mathrm{Im}\,\varepsilon_{\parallel.t}^{m,m'}=\frac{\omega_p^2r^5\omega^3}{8h_\theta^5v_T^5\pi^{5.5}}(1-\varepsilon^2)^{1.5}\sum_{p=1}^{\infty}\int_{\mu_u}^{\mu_t}\frac{T_t^4}{p^5}B_p^mB_p^{m'}\times\exp(-\nu_p^2)d\mu,\tag{35}$$

$$\mathrm{Im}\,\varepsilon_{\parallel.d}^{m,m'}=\frac{\omega_p^2r^5\omega^3}{4h_\theta^5v_T^5\pi^{5.5}}(1-\varepsilon^2)^{1.5}\sum_{p=1}^{\infty}\int_{\mu_t}^{\mu_d}\frac{T_d^4}{p^5}(C_p^mC_p^{m'}+D_p^mD_p^{m'})\exp(-z_p^2)d\mu.\tag{36}$$

In the simplest case of toroidicity-induced Alfvén eigenmodes[15] (TAEs), describing the coupling of only two harmonics with m_0 and m_0-1, the terms with $m_0,\ m_0-1$ also should be accounted for in Eq. (23) to estimate the TAEs' absorption by the trapped and untrapped electrons. As a result, the dissipated power of TAEs by the electron Landau damping is expressed as

$$P=\frac{\omega}{8\pi}\sum_{m=m_0-1}^{m_0}\mathrm{Im}\,\varepsilon_\parallel^{m,m'}|E_\parallel^m|^2+\frac{\omega}{4\pi}\mathrm{Im}\,\varepsilon_\parallel^{m_0,m_0-1}\times(\mathrm{Re}\,E_\parallel^{m_0}\,\mathrm{Re}\,E_\parallel^{m_0-1}+\mathrm{Im}\,E_\parallel^{m_0}\,\mathrm{Im}\,E_\parallel^{m_0-1}),\tag{37}$$

where $|E_\parallel^m|^2=(\mathrm{Re}\,E_\parallel^m)^2+(\mathrm{Im}\,E_\parallel^m)^2$ is the squared module of the mth electric field harmonic. Of course, our dielectric characteristics, Eqs. (24)–(26) and Eqs. (34)–(36), can be applied as well to study, e.g., the excitation/dissipation of the kinetic Alfvén waves in toroidal plasmas as was made in Ref. 9 for a large aspect ratio tokamak with circular magnetic surfaces, and the ellipticity-induced Alfvén eigenmodes[16] in elongated tokamaks when the $(m_0\pm2)$ harmonics of the electric field and the permittivity elements $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m_0,m_0\pm2}$, $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m_0,m_0\pm2}$, $\mathrm{Im}\,\varepsilon_{\parallel,d}^{m_0,m_0\pm2}$ should be involved.

Note that the nondiagonal elements $\varepsilon_\parallel^{m,m'}|_{m\neq m'}$ are characteristic only for toroidal plasmas. For the one-mode (cylindrical) approximations, when $m=m'=m_0$, the nondiagonal elements vanish, i.e., $\mathrm{Im}\,\varepsilon_\parallel^{m,m'}|_{m\neq m'}=0$, and Eqs. (33) and (37) can be reduced to the well-known expression

$$P=(\omega/8\pi)\,\mathrm{Im}\,\varepsilon_\parallel^{m,m}|E_\parallel^m|^2.\tag{38}$$

The longitudinal permittivity elements evaluated in the paper are suitable for both the large and low aspect ratio tokamaks with elliptic magnetic surfaces and valid in the wide range of wave frequencies, mode numbers, and plasma parameters. Expressions (24)–(26), (34)–(36) have a natural limit to the corresponding results[6] for tokamak plasmas with circular magnetic surfaces, if $b=a$ and $\lambda\to0$. Since the drift-kinetic equation is solved as a boundary-value problem, the longitudinal permittivity elements (24)–(26) can be applied to study the wave processes with a regular frequency such as the wave propagation and wave dissipation during the plasma heating and current drive generation, when the wave frequency has being given, e.g., by the antenna-generator system. Of course, the best application of our dielectric characteristics is to develop a numerical code to solve the two-dimensional Maxwell's equations in elongated tokamaks for electromagnetic fields in the frequency range of Alfvén, fast magnetosonic, and lower hybrid waves. On the other hand, they can be analyzed independently of the solution of Maxwell's equation, by analogy with the computations in Refs. 6–8.

Note that in analyzing the collisionless wave dissipation by the plasma electrons one should remember other kinetic mechanisms of the wave–particle interactions, such as the transit time magnetic pumping and/or the cyclotron resonance damping, which can be described by the transverse and cross-off dielectric permittivity elements. Recently, a comprehensive theoretical analysis on the Cherenkov absorption of the magnetohydrodynamic waves (Alfvén and fast magnetosonic waves) by electrons has been done[17] for large aspect ratio tokamaks with circular magnetic surfaces. The corresponding approach can be used as well for elongated spherical tokamaks. However, this is a topic for additional investigation.

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