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# Harmonic analysis of the mean surface temperature of the northern hemisphere

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ABSTRACT. The mean surface temperatures of the northern hemisphere along the latitude circles for each 5° latitude from equator to 60°N have been subjected to harmonic analysis. The amplitudes and phases of the first 15 harmonics are given. The important seasonal changes from January to July are discussed.

## 1. Introduction

In order to investigate the possible relationship between the spectral properties of atmospheric fields and earth's orography, Peixoto *et al.* (1964) studied the harmonic analysis of mean topography of the earth along the parallels of the entire globe. Sankar Rao and Saltzman (1969) performed the spherical harmonic analysis of ground temperatures for the whole earth in their study of global monsoons. Their analysis was based on the data given in Haurwitz and Austin (1944) to represent land surface temperatures and in Sverdrup *et al.* (1942) for the ocean surface temperatures. With the availability of the latest mean surface temperatures it was decided to study one dimensional harmonic analysis of this data along the latitudes of the northern hemisphere from equator to 60°N. This spectral information can be of utility in studying the influence of heat sources and sinks in general circulation problems over the tropics and middle latitudes.

## 2. Mathematical Expressions

The temperature profile along a given latitude $\phi$ on the earth can be expressed in terms of Foureir expansion of the form—

$$T(\lambda, \phi) = < T(\phi) > + \sum_{n=1}^{\infty} H(n, \phi) \cos n \times \left[\lambda - \epsilon(n, \phi)\right]$$

where, $T(\lambda, \phi)$ = temperature of underlying surface, $\lambda$ = longitude, $H(n, \phi)$ = amplitude of the harmonic component of wave $n$, $\epsilon(n,\phi)$ = phase of the harmonic component of wave $n$, $n$ = wave number, and $< T(\phi) >$ = average of value $T(\lambda, \phi)$ around the latitude circle, and $\phi$ = constant.

$$< T(\phi) > = \tfrac{1}{2} H(o, \phi) = \frac{1}{2^{\pi}} \oint T(\lambda, \phi)\, d\lambda$$

For computational purpose,

$$< T(\phi) > = \frac{1}{M} \sum_{J=1}^{M} T(\lambda_J, \phi)$$

where, $M$ = number of equally spaced data points on a latitude circle. The amplitude and phase are computed as,

$$|H(n, \phi)| = [A_1^2(n, \phi) + A_2^2(n, \phi)]^{\frac{1}{2}} \text{ and}$$

$$\epsilon(n, \phi) = \frac{1}{n} \text{ arc tan } \frac{A_2(n, \phi)}{A_1(n, \phi)}$$

where,

$$A_1(n, \phi) = \frac{2}{M} \sum_{J=1}^{M} T(\lambda_J, \phi) \cos n \lambda_J$$

$$A_2(n, \phi) = \frac{2}{M} \sum_{J=1}^{M} T(\lambda_J, \phi) \sin n \lambda_J$$

## 3. Source of data

Mean surface temperatures are obtained from the analysed charts of atlases of land and oceans. Land temperatures are picked out from the atlas NAVAIR 50, 1C-52 (1966), while the ocean surface temperatures are picked out from the atlases published by Naval Oceanographic Office, Washington D.C., separately for the North Pacific Ocean (1966) , North Atlantic Ocean (1967) and the Indian Ocean (1967). The

**TABLE 1**

**Amplitude and phase spectrums of the mean surface temperature for the Northern Hemisphere**

Amplitude is in degrees Fahrenheit and Phase in degrees measured eastward from the Greenwich Meridian

| Lat. | Wave number 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (°N) | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. |
| | | | | | | | | JANUARY | | | | | | | | |
| 0 | 3·5 | 126 | 2·0 | 306 | 1·7 | 265 | 2·3 | 273 | 0·3 | 302 | 1·2 | 63 | 1·7 | 26 | 1·9 | 359 |
| 5 | 2·2 | 162 | 1·4 | 331 | 0·5 | 270 | 2·6 | 266 | 1·1 | 273 | 1·0 | 56 | 1·1 | 49 | 1·4 | 324 |
| 10 | 2·8 | 175 | 1·6 | 287 | 1·0 | 323 | 1·1 | 302 | 0·8 | 62 | 1·4 | 55 | 1·0 | 11 | 0·2 | 148 |
| 15 | 3·6 | 183 | 2·1 | 258 | 0·6 | 179 | 1·1 | 285 | 1·0 | 6 | 1·3 | 46 | 0·7 | 310 | 0·5 | 237 |
| 20 | 5·1 | 189 | 2·9 | 273 | 3·5 | 199 | 1·9 | 230 | 1·4 | 359 | 0·8 | 41 | 0·9 | 226 | 0·4 | 172 |
| 25 | 6·1 | 227 | 3·3 | 308 | 5·8 | 177 | 3·3 | 229 | 3·2 | 346 | 1·2 | 114 | 2·1 | 215 | 1·3 | 13 |
| 30 | 10·5 | 244 | 8·9 | 329 | 3·8 | 161 | 5·3 | 190 | 2·3 | 354 | 0·3 | 99 | 3·2 | 206 | 1·0 | 193 |
| 35 | 13·9 | 261 | 13·3 | 339 | 6·7 | 153 | 9·4 | 174 | 2·1 | 45 | 0·4 | 7 | 4·3 | 163 | 4·5 | 164 |
| 40 | 12·2 | 278 | 16·4 | 350 | 5·7 | 208 | 4·6 | 193 | 4·4 | 44 | 0·9 | 337 | 1·4 | 104 | 1·0 | 107 |
| 45 | 14·0 | 283 | 19·9 | 11 | 6·4 | 237 | 7·1 | 242 | 2·0 | 36 | 0·8 | 205 | 2·1 | 136 | 0·6 | 357 |
| 50 | 15·0 | 277 | 24·8 | 6 | 9·0 | 266 | 5·3 | 254 | 4·1 | 106 | 3·4 | 228 | 1·7 | 89 | 0·4 | 356 |
| 55 | 9·8 | 292 | 26·6 | 7 | 11·9 | 260 | 5·8 | 258 | 3·9 | 78 | 4·4 | 185 | 2·1 | 53 | 3·4 | 52 |
| 60 | 17·0 | 328 | 23·9 | 6 | 14·4 | 248 | 0·9 | 250 | 2·6 | 145 | 1·3 | 303 | 1·6 | 77 | 4·5 | 311 |
| | | | | | | | | JULY | | | | | | | | |
| 0 | 4·6 | 134 | 1·7 | 291 | 2·2 | 231 | 1·4 | 215 | 0·5 | 118 | 1·0 | 59 | 1·2 | 43 | 0·8 | 34 |
| 5 | 3·3 | 170 | 0·7 | 320 | 1·6 | 265 | 1·5 | 279 | 0·9 | 202 | 1·2 | 55 | 2·0 | 100 | 1·3 | 268 |
| 10 | 2·4 | 167 | 1·1 | 262 | 0·1 | 64 | 0·5 | 248 | 0·2 | 359 | 0·7 | 348 | 0·6 | 343 | 0·1 | 72 |
| 15 | 1·0 | 105 | 0·7 | 323 | 2·7 | 60 | 1·3 | 17 | 1·0 | 77 | 0·3 | 89 | 0·7 | 268 | 0·6 | 276 |
| 20 | 4·1 | 56 | 2·7 | 358 | 3·2 | 77 | 2·2 | 46 | 1·5 | 6 | 1·5 | 346 | 1·5 | 314 | 0·8 | 353 |
| 25 | 5·2 | 57 | 2·2 | 15 | 3·8 | 127 | 1·6 | 40 | 3·7 | 339 | 1·4 | 2 | 1·0 | 45 | 2·0 | 341 |
| 30 | 3·0 | 25 | 2·6 | 0 | 5·3 | 105 | 1·7 | 102 | 3·9 | 302 | 3·2 | 15 | 4·3 | 63 | 0·2 | 358 |
| 35 | 3·4 | 342 | 2·5 | 3 | 6·3 | 109 | 3·2 | 145 | 3·2 | 263 | 2·3 | 5 | 3·0 | 53 | 1·3 | 172 |
| 40 | 4·3 | 13 | 2·7 | 171 | 2·3 | 89 | 1·1 | 82 | 2·0 | 236 | 1·5 | 73 | 2·3 | 37 | 0·3 | 25 |
| 45 | 2·2 | 49 | 2·6 | 107 | 3·3 | 75 | 1·0 | 311 | 2·3 | 258 | 1·8 | 101 | 1·2 | 126 | 1·1 | 142 |
| 50 | 5·5 | 41 | 6·3 | 136 | 2·6 | 47 | 0·8 | 228 | 3·4 | 269 | 1·1 | 48 | 1·7 | 157 | 0·3 | 257 |
| 55 | 5·6 | 54 | 6·4 | 135 | 3·5 | 18 | 1·0 | 295 | 1·0 | 276 | 0·2 | 286 | 1·1 | 195 | 0·8 | 128 |
| 60 | 7·7 | 88 | 6·0 | 102 | 4·9 | 346 | 1·1 | 200 | 0·3 | 133 | 0·7 | 3 | 1·6 | 220 | 1·1 | 155 |

Amp. = Amplitude

**TABLE 1** (*contd*)

| Lat. | Wave number | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9 | | 10 | | 11 | | 12 | | 13 | | 14 | | 15 | |
| (°N) | Amp. | Pha. | Amp | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. | Amp. | Pha. |
| | | | | | | | JANUARY | | | | | | | |
| 0 | 0·2 | 303 | 1·1 | 151 | 0·9 | 119 | 0·6 | 30 | 0·5 | 248 | 0·9 | 223 | 0·2 | 189 |
| 5 | 1·3 | 315 | 0·7 | 124 | 1·2 | 133 | 0·3 | 352 | 0·9 | 333 | 0·4 | 215 | 0·6 | 173 |
| 10 | 1·0 | 166 | 0·6 | 253 | 0·9 | 308 | 0·4 | 3 | 0·3 | 82 | 0·3 | 319 | 0·6 | 299 |
| 15 | 0·8 | 109 | 0·6 | 324 | 1·3 | 290 | 0·4 | 126 | 1·1 | 111 | 0·2 | 244 | 1·1 | 247 |
| 20 | 1·0 | 27 | 1·0 | 281 | 0·9 | 272 | 1·8 | 58 | 1·1 | 71 | 1·4 | 225 | 0·8 | 231 |
| 25 | 1·3 | 75 | 2·1 | 229 | 1·1 | 343 | 2·1 | 43 | 0·1 | 151 | 0·4 | 185 | 0·6 | 110 |
| 30 | 0·6 | 125 | 1·0 | 302 | 0·8 | 100 | 2·2 | 173 | 1·6 | 259 | 1·3 | 21 | 1·3 | 69 |
| 35 | 1·7 | 215 | 0·5 | 60 | 1·8 | 96 | 1·9 | 141 | 3·0 | 176 | 1·0 | 106 | 2·2 | 72 |
| 40 | 2·9 | 199 | 1·7 | 155 | 1·0 | 334 | 1·0 | 283 | 0·8 | 112 | 0·1 | 95 | 0·7 | 209 |
| 45 | 2·3 | 296 | 0·7 | 116 | 0·2 | 331 | 0·7 | 277 | 0·5 | 138 | 0·4 | 234 | 1·2 | 223 |
| 50 | 3·1 | 300 | 0·5 | 40 | 1·0 | 297 | 0·8 | 176 | 1·0 | 151 | 1·1 | 12 | 1·1 | 152 |
| 55 | 2·1 | 226 | 2·8 | 339 | 2·3 | 197 | 2·3 | 229 | 2·4 | 74 | 2·0 | 234 | 1·3 | 46 |
| 60 | 3·5 | 185 | 0·9 | 322 | 1·7 | 178 | 2·1 | 55 | 2·6 | 331 | 1·4 | 239 | 1·6 | 294 |
| | | | | | | | JULY | | | | | | | |
| 0 | 0·1 | 181 | 0·3 | 152 | 0·8 | 124 | 0·2 | 32 | 0·4 | 321 | 0·4 | 216 | 0·3 | 180 |
| 5 | 0·6 | 326 | 1·3 | 191 | 0·8 | 304 | 0·6 | 28 | 0·9 | 307 | 1·0 | 80 | 1·2 | 161 |
| 10 | 0·4 | 136 | 0·3 | 142 | 0·7 | 231 | 0·7 | 255 | 0·4 | 291 | 0·7 | 343 | 0·8 | 343 |
| 15 | 0·8 | 282 | 0·8 | 320 | 0·4 | 338 | 0·6 | 23 | 0·4 | 77 | 0·7 | 145 | 0·7 | 193 |
| 20 | 1·3 | 343 | 1·5 | 288 | 1·3 | 253 | 0·8 | 331 | 1·1 | 322 | 0·9 | 238 | 0·5 | 190 |
| 25 | 2·0 | 328 | 0·8 | 273 | 1·3 | 273 | 2·5 | 314 | 1·8 | 316 | 0·9 | 260 | 1·1 | 285 |
| 30 | 1·7 | 300 | 2·6 | 24 | 1·8 | 51 | 0·6 | 225 | 0·6 | 275 | 1·5 | 329 | 1·7 | 319 |
| 35 | 0·5 | 325 | 2·4 | 9 | 0·4 | 84 | 1·1 | 102 | 0·8 | 44 | 0·7 | 198 | 0·4 | 207 |
| 40 | 1·4 | 117 | 1·0 | 182 | 1·8 | 264 | 1·9 | 2 | 1·7 | 33 | 1·1 | 158 | 1·2 | 204 |
| 45 | 1·8 | 43 | 0·8 | 32 | 0·5 | 202 | 0·9 | 258 | 0·5 | 173 | 1·7 | 96 | 0·4 | 218 |
| 50 | 1·6 | 87 | 0·5 | 346 | 1·0 | 318 | 1·2 | 203 | 0·7 | 290 | 0·9 | 71 | 0·5 | 142 |
| 55 | 1·7 | 57 | 0·4 | | 0·9 | 312 | 0·7 | 163 | 0·8 | 215 | 1·2 | 66 | 0·7 | 82 |
| 60 | 2·1 | 82 | 1·2 | 298 | 0·9 | 258 | 1·7 | 177 | 1·4 | 104 | 1·6 | 18 | 1·1 | 293 |

Amp. = Amplitude Pha. = Phase

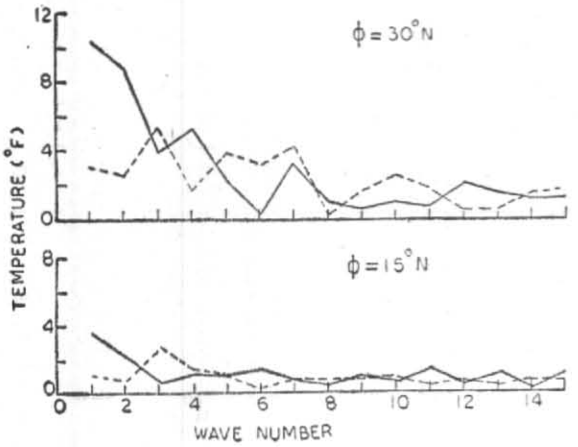


Fig. 1(a)

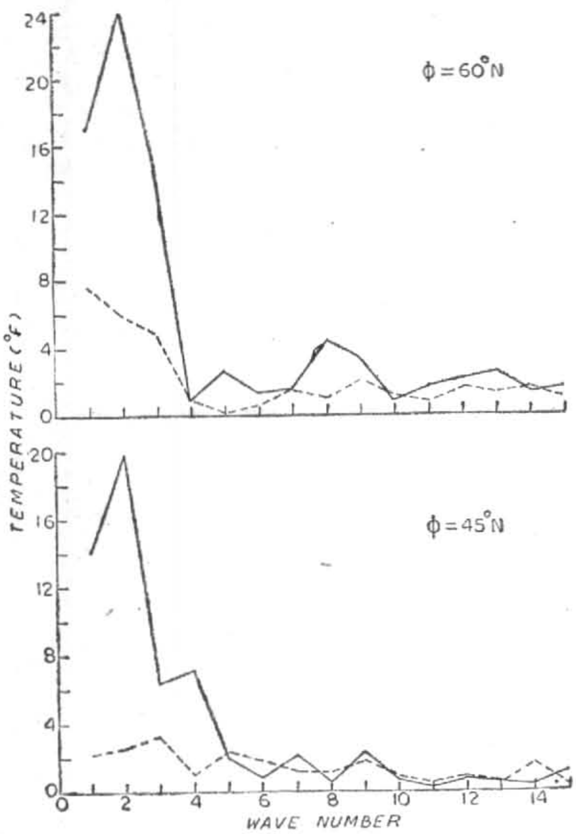


Fig. 1(b)

**Amplitude spectra of surface mean temperature denoted by |T|**

Solid line indicates January mean and dashed line July mean

**TABLE 2**

**Mean surface temperature (°F)**

| Latitude $\phi$ (degrees North) | January | July | Diff. |
|---|---|---|---|
| 0 | 79·8 | 79·0 | —0·8 |
| 5 | 79·8 | 79·6 | —0·2 |
| 10 | 78·2 | 81·6 | 3·4 |
| 15 | 76·6 | 82·1 | 5·5 |
| 20 | 71·7 | 82·2 | 10·5 |
| 25 | 66·5 | 81·6 | 15.1 |
| 30 | 57·7 | 79·0 | 23·3 |
| 35 | 49·5 | 74·1 | 24·6 |
| 40 | 40·5 | 70·7 | 30·2 |
| 45 | 29·3 | 66·9 | 37·6 |
| 50 | 19·2 | 61·3 | 42·1 |
| 55 | 16·4 | 57·4 | 41·0 |
| 60 | 6·8 | 54·2 | 47·4 |

temperatures are collected from equator to 60°N, at 5° latitude interval. Beyond 60° N, the picking up of the data accurately from the charts is not possible, as such the analysis is restricted upto this latitutde only. Harmonic analysis is performed based on interpolated values at 72 points around a latitude circle. January and July are chosen as the representative months of winter and summer seasons.

#### 4. Results

In Table 1, the amplitudes and phase angles for different harmonics are given at each latitude circle. The latitudinal variation of the mean surface temperature and the percentage variance accounted by the first 15 harmonics are given in Tables 2 and 3 respectively. The amplitude spectra for the four selected latitudes, *i.e.*, 50°, 30°, 45°, 60°N and the latitudinal variation of mean surface temperature in January and July are shown in Figs. 1 and 2.

It is seen from Table 1 that, for January, wave number 2 is prominent from 40° to 60°N, and 1 from equator to 35°N except at 5°N, where wave number 4 is dominant. In case of July, wave number 2 is prominent at 50° and 55°N only. At other latitudes it is 1 and 3. Sankar Rao and Saltzman (*loc. cit.*) also found the dominance of zonal wave number 2 in the winter northern hemisphere. The prominence of wave number 2 at higher latitudes in winter may be due to land-ocean contrasts. It can be seen from the four selected latitudes that the seasonal variation of amplitudes is highest at 60°N, decreasing southwards. In general the variations of amplitudes in middle and higher latitudes are more in January than in July.

TABLE 3

Percentage of variance accounted by first 15 harmonics

| Lat. (°N) | Wave number 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | Total variance* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JANUARY | | | | | | | | | | | | | | | | |
| 0 | 29 | 9 | 7 | 13 | 0 | 3 | 7 | 9 | 0 | 3 | 2 | 1 | 1 | 2 | 0 | 86 |
| 5 | 15 | 6 | 1 | 22 | 4 | 3 | 3 | 6 | 5 | 2 | 5 | 0 | 3 | 0 | 1 | 76 |
| 10 | 29 | 9 | 4 | 5 | 2 | 8 | 4 | 0 | 4 | 1 | 3 | 1 | 0 | 0 | 1 | 71 |
| 15 | 39 | 13 | 1 | 3 | 3 | 5 | 1 | 1 | 2 | 1 | 5 | 0 | 3 | 0 | 3 | 80 |
| 20 | 37 | 12 | 17 | 5 | 3 | 1 | 1 | 0 | 1 | 1 | 1 | 5 | 2 | 3 | 1 | 90 |
| 25 | 28 | 8 | 25 | 8 | 8 | 1 | 3 | 1 | 1 | 3 | 1 | 3 | 1 | 0 | 0 | 90 |
| 30 | 41 | 29 | 5 | 10 | 2 | 0 | 4 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 96 |
| 35 | 30 | 28 | 7 | 14 | 1 | 0 | 3 | 3 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 90 |
| 40 | 28 | 51 | 6 | 4 | 4 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 96 |
| 45 | 28 | 56 | 6 | 7 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 99 |
| 50 | 22 | 61 | 8 | 3 | 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 98 |
| 55 | 9 | 65 | 13 | 3 | 1 | 2 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 97 |
| 60 | 25 | 50 | 18 | 0 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 98 |
| JULY | | | | | | | | | | | | | | | | |
| 0 | 53 | 7 | 12 | 5 | 1 | 3 | 3 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 88 |
| 5 | 20 | 1 | 5 | 4 | 2 | 3 | 7 | 3 | 1 | 3 | 1 | 1 | 1 | 2 | 3 | 57 |
| 10 | 33 | 7 | 0 | 2 | 0 | 3 | 2 | 0 | 1 | 0 | 2 | 3 | 1 | 3 | 4 | 61 |
| 15 | 4 | 2 | 35 | 8 | 5 | 0 | 2 | 1 | 3 | 3 | 1 | 2 | 1 | 3 | 2 | 72 |
| 20 | 25 | 11 | 15 | 7 | 3 | 3 | 3 | 1 | 3 | 3 | 2 | 1 | 2 | 1 | 0 | 80 |
| 25 | 27 | 5 | 14 | 3 | 14 | 2 | 1 | 4 | 4 | 1 | 2 | 6 | 3 | 1 | 1 | 88 |
| 30 | 7 | 5 | 23 | 2 | 12 | 8 | 15 | 0 | 2 | 5 | 3 | 0 | 0 | 2 | 2 | 86 |
| 35 | 10 | 5 | 34 | 8 | 8 | 5 | 8 | 1 | 0 | 5 | 0 | 1 | 1 | 0 | 0 | 86 |
| 40 | 24 | 9 | 7 | 1 | 5 | 3 | 7 | 0 | 2 | 1 | 4 | 5 | 4 | 2 | 2 | 76 |
| 45 | 10 | 14 | 23 | 2 | 11 | 6 | 3 | 2 | 6 | 1 | 1 | 2 | 0 | 6 | 0 | 87 |
| 50 | 30 | 38 | 7 | 1 | 11 | 1 | 3 | 0 | 2 | 0 | 1 | 1 | 0 | 1 | 0 | 96 |
| 55 | 31 | 41 | 13 | 1 | 1 | 0 | 1 | 1 | 3 | 0 | 1 | 1 | 1 | 1 | 0 | 96 |
| 60 | 40 | 25 | 17 | 1 | 0 | 0 | 2 | 1 | 3 | 1 | 1 | 2 | 1 | 2 | 1 | 97 |

*Total variance accounted by first 15 wave numbers

Fig. 2 shows the mean surface temperature in January and July. The difference Jul-Jan increases with latitude except at 55°N. At the equator the temperature variation between summer and winter is negligible. In July, the mean temperature increases slightly from equator to 10°N, remains nearly at the same value upto Lat. 25°N, and decreases to 54°F at 60°N. In January, the temperature decreases slowly upto 15°N, and thereafter rapidly.

In Table 3, the percentage variance accounted by the harmonics was given. The first 15 harmonics are sufficient to account nearly

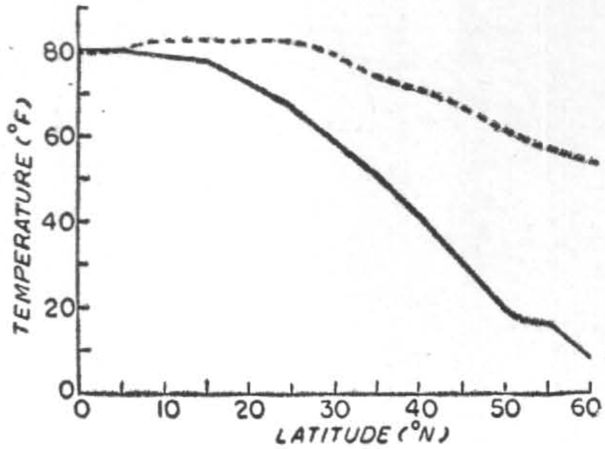


Fig. 2

Variation of surface mean temperature in °F with latitude. Solid line indicates January mean and dashed line July mean

95 per cent of variation at higher latitudes. At lower latitudes, they could account only upto 60 per cent in some cases.

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