Equation (17) can be reduced to:

$$U(\beta,\omega,\tau) = \frac{1}{2}\,\mathrm{col}(-\hat{\beta}^T\dot{\omega}, \beta_0\dot{\omega} + \hat{\beta}\times\dot{\omega}) -$$

$$- \frac{1}{4}\,\mathrm{col}(-\beta_0(\omega-\Omega)^T(\omega-\Omega) - 2\omega^T(\hat{\beta}\times\Omega), \qquad (18)$$

$$2[\omega\Omega^T + \Omega\omega^T]\hat{\beta} - [(\omega+\Omega)^T(\omega+\Omega)]\hat{\beta} + 2\beta_0(\omega\times\Omega))$$

4. REDUCED STATE EQUATIONS USING RELATIVE QUATERNIONS

The transformations above are not invertible, since X has eight dimensional values and U is four-dimensional, whereas $(\beta^T,\omega^T)^T$ is seven dimensional, while τ is three-dimensional. Then a control law design in (x_1, x_2, u) space cannot be globally transformed back into the (β,ω,τ) -space. Invertibility is obtained however, by requiring that the attitude trajectory in terms of the Euler relative quaternion should be restricted to the upper half unit sphere in R^4. So the projection map:

$$(\beta_0,\beta_1,\beta_2,\beta_3)^T =: \beta \rightarrow \gamma := \hat{\beta} := (\beta_1,\beta_2,\beta_3)^T \qquad (19)$$

takes β with $\beta_0 > 0$ onto the unit ball $\gamma^T\gamma < 1$ in R^3. Conversely, each γ corresponds to a unique quaternion in the upper unit sphere $\{\beta^T\beta = 1,\ \beta_0 > 0\}$ in R^4 by the lifting

$$(\gamma_1,\ \gamma_2,\ \gamma_3)^T =: \gamma \rightarrow (\gamma_0, \gamma_1, \gamma_2, \gamma_3)^T \qquad (20)$$

where $\gamma_0 := \sqrt{1-\gamma^T\gamma}$

Since a unit relative quaternion β and its negative $-\beta$ correspond to the same attitude orientation the restriction on allowable trajectories is only that the orientation corresponding to $\beta_0 = 0$ (i.e. $|\phi| = \pi$) must be avoided, so that $-\pi < \phi < \pi$.

The transformations (19) and (20) change (3) into the reduced kinematic equation

$$\dot{\gamma} = \frac{1}{2}\,[\gamma_0(\omega-\Omega) + \gamma\times(\omega+\Omega)] \qquad (21)$$

Then the reduced state transformations (15) and (16) become

$$x_1 = X_1(\gamma) = \gamma \qquad (22)$$

$$x_2 = X_2(\gamma,\omega) = \frac{1}{2}[\gamma_0(\omega-\Omega) + \gamma\times(\omega+\Omega)] \qquad (23)$$

and the input transformation becomes

$$u = U(\gamma,\omega,\tau) = \frac{1}{2}(\gamma_0\dot{\omega} + \hat{\beta}\times\dot{\omega}) + 2[\omega\Omega^T + \Omega\omega^T]\gamma$$

$$- [(\omega+\Omega)^T(\omega+\Omega)]\,\gamma + 2\gamma_0(\omega\times\Omega) \qquad (24)$$

Similarly to the case treated by Dwyer (1984), the equivalent linear system given by (6) and (7) can be implemented by using a set of three double integrators in parallel.

5. INVERSE TRANSFORMATIONS

By letting

$$\Gamma(\gamma)\omega := \frac{1}{2}\,(\gamma_0\omega + \gamma\times\omega) \qquad (25)$$

(21) and (24) can be rewritten:

$$\dot{\gamma} = \Gamma(\gamma)\omega - \frac{1}{2}\,[\gamma_0\Omega - \gamma\times\Omega] \qquad (26)$$

and

$$u + [(\omega+\Omega)^T(\omega+\Omega)]\gamma - 2\gamma_0(\omega\times\Omega) - 2[\omega\Omega^T + \Omega\omega^T]\gamma =$$

$$= \Gamma(\gamma)\dot{\omega} \qquad (27)$$

Under the conditions previously stated (see Dwyer (1984)) there is an inverse operator

$$\Gamma(\gamma)^{-1}\,\mu = (\frac{2}{\gamma_0})\,(\gamma_0^2\,\mu + \gamma\gamma^T\mu - \gamma_0\gamma\times\mu) \qquad (28)$$

for any $\mu \in R^3$. Then one can state that.

$$\omega = \Gamma^{-1}(\gamma)\ [\dot{\gamma} + \frac{1}{2}(\gamma_0\Omega - \gamma\times\Omega)] \qquad (29)$$

which can be simplified to:

$$\omega = \Gamma^{-1}(\gamma)\dot{\gamma} + \gamma_0\ \Gamma^{-1}(\Omega) - \Omega \qquad (30)$$

Therefore the inverse state transformation are given by

$$\gamma = x_1 \qquad (31)$$

and

$$\omega = (\frac{2}{x_0})\ (x_0^2 x_2 + x_1 x_1^T x_2 - x_0 x_1\times x_2) +$$

$$+\ 2(x_0^2\Omega + x_1 x_1^T\Omega - x_0 x_1\times\Omega) - \Omega \qquad (32)$$

with $x_0 := \sqrt{1-x_1^T x_1}$

The inverse input transformation is similarly obtained from (4), (27) and (28) and has the form:

$$\tau = \omega\times I^O\omega + (\frac{2}{\gamma_0})\ I^O(\gamma_0^2\mu + \gamma\gamma^T\mu - \gamma_0\gamma\times\mu) \qquad (33)$$

with

$$\mu := u + [(\omega+\Omega)^T(\omega+\Omega)]\gamma - 2\gamma_0(\omega\times\Omega) -$$

$$- 2[\omega\Omega^T + \Omega\omega^T]\gamma \qquad (34)$$

The expressions (32) and (34) coincide with the expressions obtained by Dwyer (1984) when Ω is made equal to zero.

6. LINEAR FEEBACK LAWS IN (x,u) SPACE

Since the evolution equations of the new state vector that are given by equations (6) and (7) are linear, it is a natural step to

The optimal transformed states and inputs are:

$$x_1^*(t) = 2(1-\frac{t}{t_f})^2(\frac{1}{2} + \frac{t}{t_f})\gamma(0) \qquad (43)$$

$$x_2^*(t) = -\frac{6}{t_f}(1-\frac{t}{t_f})\frac{t}{t_f}\gamma(0) \qquad (44)$$

$$u^*(t) = \frac{12}{t_f^2}(\frac{t}{t_f} - \frac{1}{2})\gamma(0) \qquad (45)$$

The optimal torque are, therefore, given by:

$$\tau^*(t) = \Omega x I^o \Omega + \frac{24}{t_f^2}(\frac{t}{t_f} - \frac{1}{2})I^o\gamma(0) \qquad (46)$$

The optimal attitude and rate variables $\gamma^*(t)$ and $\omega^*(t)$ are generated via (22) and (32).

8. CONCLUSIONS

By using the relative Euler quaternion for representing attitude of a rotating body, state and input transformations were constructed in order to obtain linear system equations that are equivalent to the kinematic and dynamic equations of motion. Since the linear system is completely controllable, it is possible to apply multivariable linear system theory to controller design. Two of such designs were described: one in which the behaviour of the relative quaternion is specified and the other where a minimization of a quadratic performance index is accomplished. In both cases, control laws not requiring too much computation were obtained. Since these laws are also only function of the relative quaternion and the body inertial velocity, measured in body axes, they are easily implemented in processors to be used on board of satellites. By using the measurements of inertial sensors (earth sensors, sun sensors, etc) the processor calculates an estimator of the relative quaternion. An estimate of the inertial angular velocity can be implement with gyroscopes measurements. Using both estimates one obtains the required torque value to be supplied by the satellite propulsion system.

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SATELLITE ATTITUDE CONTROL ANALYSIS AND DESIGN USING LINEARIZING TRANSFORMATIONS

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Abstract

The problem of maneuvering a satellite relative to a desired noninertial reference is dealt by the use of linearizing transformations and the concept of relative quaternion. The rigid body attitude equations are transformed into an equivalent linear form which considers the fact that the control purpose is to align the satellite with a moving reference frame (e.g., and Earth resource satellite). The equations so obtained are used to design control strategies for stabilizing the satellite attitude by linear techniques. This approach is more useful for algorithm design for on-board processors than the technique using inertial quaternion since the laws obtained are function of the noninertial sensor outputs (sun sensors, earth sensors, etc) and of the inertial sensor (gyroscopes) outputs.

Keywords: Linearizing transformation; Attitude control; Relative quaternions; On-board processors.

1. INTRODUCTION

Linearizing transformation were used by Dwyer (1984) to transform the rigid body attitude control problem with external torques into an equivalent linear form implementable by three double integrators. Using this approach is possible to perform regulator design and to generate optimal commands for fast slewing maneuvers. This formulation has, however, the incovenience that the control laws obtained are function of a symmetric Euler (unit) quaternion and its derivative. These parameters describe the attitude of the rigid body with respect to an inertial reference frame. For most of the satellites, one is interested that their attitude should follow a reference frame that is rotating with respect to an inertial reference frame. In the particular case of remote sensing and geostationary satellites this frame is the so called orbital reference frame and has an inertial angular velocity equal to the orbital velocity. For this reason the sensors being used for attitude control provide measurements that are processed for obtaining an estimate of the satellite attitude with respect to the moving frame. Such estimate should be used for constructing the control laws that will keep the satellite attitude as near as possible nominal value.

Taking into consideration the above reasoning, this paper rederives the state equations for multiaxis attitude control using the notion of relative quaternion. It was defined by Mayo (1979) and is a symmetric Euler (unit) quaternion that can represent the attitude of a rigid body with respect to a rotating reference frame whose inertial rate is known. With this new set of equations is then possible to design control laws that will comply with the requirements of actual satellites.

Conditions for calculating the desired torque for a linear quaternion behaviour are computed.

2. RELATIVE QUATERNION EQUATIONS

For obtaining the linearized equations, one needs to know the relative quaternion strapdown equation. Using the same notation as in Mayo (1979) one defines three reference frames, as follows:

a - an inertial reference frame

b - an R reference frame that is rotating with a constant angular rate with respect to the inertial frame

c - a B reference frame fixed to the satellite body

Then the strapdown equation is given by:

$$\dot{\beta}=\frac{1}{2}\begin{bmatrix} 0 & -(\omega_x-\Omega_x) & -(\omega_y-\Omega_y) & -(\omega_z-\Omega_z) \\ (\omega_x-\Omega_x) & 0 & (\omega_z+\Omega_z) & -(\omega_y+\Omega_y) \\ (\omega_y-\Omega_y) & -(\omega_z+\Omega_z) & 0 & (\omega_x+\Omega_x) \\ (\omega_z-\Omega_z) & (\omega_y+\Omega_y) & -(\omega_x+\Omega_x) & 0 \end{bmatrix}\beta \quad (1)$$

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SATELLITE ATTITUDE CONTROL ANALYSIS AND DESIGN USING LINEARIZING TRANSFORMATIONS

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Equation (17) can be reduced to:

$$U(\beta,\omega,\tau) = \frac{1}{2}\ \mathrm{col}(-\bar{\beta}^T\dot{\omega}, \beta_0\dot{\omega} + \bar{\beta}\,x\,\dot{\omega}) -$$

$$-\frac{1}{4}\ \mathrm{col}(-\beta_0(\omega-\Omega)^T(\omega-\Omega) - 2\omega^T(\bar{\beta}\,x\,\Omega), \qquad (18)$$

$$2[\omega\Omega^T + \Omega\omega^T]\bar{\beta} - [(\omega+\Omega)^T(\omega+\Omega)]\bar{\beta} + 2\beta_0(\omega\,x\,\Omega))$$

4. REDUCED STATE EQUATIONS USING RELATIVE QUATERNIONS

The transformations above are not invertible, since X has eight dimensional values and U is four-dimensional, whereas $(\beta^T,\omega^T)^T$ is seven dimensional, while τ is three-dimensional. Then a control law design in (x_1, x_2, u) space cannot be globally transformed back into the (β,ω,τ) -space. Invertibility is obtained however, by requiring that the attitude trajectory in terms of the Euler relative quaternion should be restricted to the upper half unit sphere in R^4. So the projection map:

$$(\beta_0,\beta_1,\beta_2,\beta_3)^T =: \beta \to \gamma := \hat{\beta} := (\beta_1,\beta_2,\beta_3)^T \qquad (19)$$

takes β with $\beta_0 > 0$ onto the unit ball $\gamma^T\gamma < 1$ in R^3. Conversely, each γ corresponds to a unique quaternion in the upper unit sphere $\{\beta^T\beta = 1,\ \beta_0 > 0\}$ in R^4 by the lifting

$$(\gamma_1,\ \gamma_2,\ \gamma_3)^T =: \gamma \to (\gamma_0,\gamma_1,\gamma_2,\gamma_3)^T \qquad (20)$$

where $\gamma_0 := \sqrt{1-\gamma^T\gamma}$

Since a unit relative quaternion β and its negative $-\beta$ correspond to the same attitude orientation the restriction on allowable trajectories is only that the orientation corresponding to $\beta_0 = 0$ (i.e. $|\phi| = \pi$) must be avoided, so that $-\pi < \phi < \pi$.

The transformations (19) and (20) change (3) into the reduced kinematic equation

$$\dot{\gamma} = \frac{1}{2}\ [\gamma_0(\omega-\Omega) + \gamma\ x\ (\omega+\Omega)] \qquad (21)$$

Then the reduced state transformations (15) and (16) become

$$x_1 = X_1(\gamma) = \gamma \qquad (22)$$

$$x_2 = X_2(\gamma,\omega) = \frac{1}{2}[\gamma_0(\omega-\Omega) + \gamma\,x\,(\omega+\Omega)] \qquad (23)$$

and the input transformation becomes

$$u = U(\gamma,\omega,\tau) = \frac{1}{2}(\gamma_0\dot{\omega} + \bar{\beta}\,x\,\dot{\omega}) + 2[\omega\Omega^T + \Omega\omega^T]\gamma$$

$$-[(\omega+\Omega)^T(\omega+\Omega)]\ \gamma + 2\gamma_0(\omega\,x\,\Omega) \qquad (24)$$

Similarly to the case treated by Dwyer (1984), the equivalent linear system given by (6) and (7) can be implemented by using a set of three double integrators in parallel.

5. INVERSE TRANSFORMATIONS

By letting

$$\Gamma(\gamma)\omega := \frac{1}{2}\ (\gamma_0\omega + \gamma\,x\,\omega) \qquad (25)$$

(21) and (24) can be rewritten:

$$\dot{\gamma} = \Gamma(\gamma)\omega - \frac{1}{2}\ [\gamma_0\Omega - \gamma\,x\,\Omega] \qquad (26)$$

and

$$u + [(\omega+\Omega)^T(\omega+\Omega)]\gamma - 2\gamma_0(\omega x\Omega) - 2[\omega\Omega^T + \Omega\omega^T]\gamma =$$

$$= \Gamma(\gamma)\dot{\omega} \qquad (27)$$

Under the conditions previously stated (see Dwyer (1984)) there is an inverse operator

$$\Gamma(\gamma)^{-1}\ \mu = (\frac{2}{\gamma_0})\ (\gamma_0^2\ \mu + \gamma\gamma^T\mu - \gamma_0\gamma x\mu) \qquad (28)$$

for any $\mu \in R^3$. Then one can state that.

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which can be simplified to:

$$\omega = \Gamma^{-1}(\gamma)\dot{\gamma} + \gamma_0\ \Gamma^{-1}(\Omega) - \Omega \qquad (30)$$

Therefore the inverse state transformation are given by

$$\gamma = x_1 \qquad (31)$$

and

$$\omega = (\frac{2}{x_0})\ (x_0^2 x_2 + x_1 x_1^T x_2 - x_0 x_1 x\ x_2) +$$

$$+\ 2(x_0^2\Omega + x_1 x_1^T\Omega - x_0 x_1\,x\,\Omega) - \Omega \qquad (32)$$

with $x_0 := \sqrt{1-x_1^T x_1}$

The inverse input transformation is similarly obtained from (4), (27) and (28) and has the form:

$$\tau = \omega\ x\ I^o\omega + (\frac{2}{\gamma_0})\ I^o(\gamma_0^2\mu + \gamma\gamma^T\mu - \gamma_0\gamma\,x\,\mu) \qquad (33)$$

with

$$\mu := u + [(\omega+\Omega)^T(\omega+\Omega)]\gamma - 2\gamma_0(\omega\,x\Omega) -$$

$$-\ 2[\omega\Omega^T + \Omega\omega^T]\gamma \qquad (34)$$

The expressions (32) and (34) coincide with the expressions obtained by Dwyer (1984) when Ω is made equal to zero.

6. LINEAR FEEBACK LAWS IN (x,u) SPACE

Since the evolution. equations of the new state vector that are given by equations (6) and (7) are linear, it is a natural step to where:

- $\beta:=col(\beta_0,\beta_1,\beta_2,\beta_3)$ is the symmetric Euler (unit) relative quaternion with $\beta_0=\cos(\phi/2)$ and $\beta_j = e_j \sin(\phi/2)$ for $j = 1,2,3$, $e = col(e_1,e_2,e_3)$ being an arbitrary unit axis and ϕ is an arbitrary rotation about e;
- $\omega:=col(\omega_x,\omega_y,\omega_z)$ = inertial rate of B frame in B frame coordinates;
- $\Omega:=col(\Omega_x,\Omega_y,\Omega_z)$ = inertial rate of R space in R space coordinates.

Equation (1) is deduced in Mayo (1979) and can be written in an more compact form as:

$$\dot{\beta} = \frac{1}{2} col(-(\omega-\Omega)^T\bar{\beta},\ \beta_0(\omega-\Omega) + \bar{\beta}\, x(\omega+\Omega)) \quad (2)$$

where x is the vector product and $\bar{\beta}:= col\ (\beta_1, \beta_2, \beta_3)$.

3. STATE EQUATIONS FOR MULTIAXIS ATTITUDE CONTROL USING RELATIVE QUATERNIONS

Using (2) the state equations that describe an rigid body satellite are given by:

$$\dot{\beta} = \frac{1}{2} col(-(\omega-\Omega)^T\bar{\beta},\ \beta_0(\omega-\Omega) + \bar{\beta}x(\omega+\Omega)) \quad (3)$$

and

$$I^o\dot{\omega} = \tau - \omega\, x\, I^o\, \omega \quad (4)$$

where I^o is the matrix of principal moments of inertia, τ is an applied external torque and the B reference frame should coincide with the one consisting of the principal axes of inertia.

The general method of linearizing transformations described in Hunt and others (1983) consists in the construction of memoryless state transformations $x=X(\beta,\omega)$ and input transformations $u = U(\beta,\omega,\tau)$, such that the evolution equation of the new state vector is of the form

$$\dot{x} = Ax + Bu . \quad (5)$$

with A and B such that the system is realized by parallel channels of integrators (the number of integrators per channel being the controlability indexes, with as many channels as the dimension of u). The more restricted approach in Meyer (1981) can be interpreted as requiring that x should be factored as $x = col\ (x_1,x_2)$ with

$$\dot{x}_1 = x_2 \quad (6)$$

$$\dot{x}_2 = u \quad (7)$$

In this case, one has

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

Defining $X =: col\ (X_1,X_2)$ where $x_j = X_j(\beta,\omega)$, one obtains

$$\dot{X}_1 = (\partial/\partial\beta)X_1(\beta,\omega)\dot{\beta} + (\partial/\partial\omega)X_1(\beta,\omega)\dot{\omega} \quad (8)$$

$$\dot{X}_2 = (\partial/\partial\beta)X_2(\beta,\omega)\dot{\beta} + (\partial/\partial\omega)X_2(\beta,\omega)\dot{\omega} \quad (9)$$

By using (3) and (4) one finds

$$\dot{X}_1 = [(\partial/\partial\beta)X_1]\frac{1}{2}col(-(\omega-\Omega)^T\bar{\beta},\beta_0(\omega-\Omega)+\bar{\beta}x(\omega+\Omega)) + [(\partial/\partial\omega)X_1](I^o)^{-1}(\tau-\omega x I^o\omega). \quad (10)$$

$$\dot{X}_2 = [(\partial/\partial\beta)\ X_2]\ \frac{1}{2}\ col\ (-(\omega-\Omega)^T\bar{\beta},\ \beta_0(\omega-\Omega) + \bar{\beta}\, x\,(\omega+\Omega)) + [(\partial/\partial\omega)X_2](I^o)^{-1}(\tau-\omega x I^o\omega) \quad (11)$$

The expressions (10) and (11) are then compared to (6) and (7) to imply the conditions:

$$(\partial/\partial\omega)\ X_1(\beta,\omega) = 0$$

for $\dot{X}_1$ to be independent of τ and

$$(\partial/\partial\omega)\ X_2(\beta,\omega) \neq 0$$

for U to depend on τ.

The required state and input transformations shall have the following form:

$$x_1 = X_1(\beta) \quad (12)$$

$$x_2 = X_2(\beta,\omega) = [(\partial/\partial\beta)X_1]\dot{\beta} \quad (13)$$

$$u = U(\beta,\omega,\tau) = [(\partial/\partial\beta)\ X_2]\dot{\beta} + [(\partial/\partial\omega)X_2]\dot{\omega} \quad (14)$$

The simplest choice for X_1 is

$$X_1(\beta) = \beta \quad (15)$$

which with (3), (12) and (13) yields:

$$X_2(\beta,\omega) = \frac{1}{2} col(-(\omega-\Omega)^T\bar{\beta},\beta_0(\omega-\Omega)+\bar{\beta}x(\omega+\Omega)) \quad (16)$$

Equation (16) with (3), (13) and (14) gives in turn

$$U(\beta,\omega,\tau) = \frac{1}{2}\ col\ (-\bar{\beta}^T\dot{\omega},\ \beta_0\dot{\omega} + \bar{\beta}x\dot{\omega}) - \frac{1}{4}\ col(-\beta_0(\omega-\Omega)^T(\omega-\Omega)-(\omega-\Omega)^T[\bar{\beta}x\ (\omega+\Omega)],\ -(\omega-\Omega)(\omega-\Omega)^T\bar{\beta} - \beta_0(\omega+\Omega)x(\omega-\Omega)-(\omega+\Omega)x[\bar{\beta}x(\omega+\Omega)]) \quad (17)$$

see how τ for linear state feedback will look like.

(6) and (7) describe a system that is completely controllable, By rearranging the state vector is possible to find its controller canonical form:

$$\begin{bmatrix} \dot{x}_{21} \\ \dot{x}_{11} \\ \dot{x}_{22} \\ \dot{x}_{12} \\ \dot{x}_{23} \\ \dot{x}_{13} \end{bmatrix} = \left[\begin{array}{cc|cc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} x_{21} \\ x_{11} \\ x_{22} \\ x_{12} \\ x_{23} \\ x_{13} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} u \qquad (35)$$

where $x_1 := \mathrm{col}(x_{11}, x_{12}, x_{13})$ and $x_2 := \mathrm{col}(x_{21}, x_{22}, x_{23})$.

For allocating all system poles it can be proved that one only has to define a gain matrix K'of the form below to be used in the state feedback

$$K' := \left[\begin{array}{cc|cc|cc} k_{11} & k_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{23} & k_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{35} & k_{36} \end{array}\right] \qquad (36)$$

In the origal system of equations (6) and (7)K ' corresponds to a gain matrix K of the form

$$k := \left[\begin{array}{ccc|ccc} k_{12} & 0 & 0 & k_{11} & 0 & 0 \\ 0 & k_{24} & 0 & 0 & k_{23} & 0 \\ 0 & 0 & k_{36} & 0 & 0 & k_{35} \end{array}\right] := [K_1 \quad K_2] \qquad (37)$$

K_1 and K_2 being diagonal matrices.

In the particular case when the designer wants all relative quaternions to have the same dynamical behaviour the matrices to be chosen are of the form $K_1 = a_1 I$ and $K_2 = a_2 I$. Applying (33) to $u = -a_1 x_1 - a_2 x_2$, one finds:

$$\tau = \omega \times I^o \omega - I^o \Gamma^{-1}(\gamma)\, \varepsilon + I^o\left(\frac{2}{\gamma_0}\right)(\omega+\Omega)^T(\omega+\Omega)\gamma$$

$$+ a_1 I^o\left(\frac{2}{\gamma_0}\right)\gamma + a_2 I^o \omega \qquad (38)$$

where $\varepsilon := 2[\omega\Omega^T + \Omega\omega^T]\gamma + 2\gamma_0(\omega \times \Omega)$

(38) permits to calculate the torque value given that the behavior of γ is described by two given modes.

7. COMMAND GENERATION

By using linearizing transformations, on-line command generation can be performed without too much computation effort. This situation is ideal for designing control laws to be used in on-board processors, based on microprocessors.

The memoryless prelinearization permits generation of an optimal command signal through the minimization of a quadratic performance index, based on the linear system given by (4) and (5) as it was previously noted by Dwyer (1984). In the case of a finite time maneuver with the purpose of orient a rigid body along a determined moving reference frame, one should have $\beta(t_f) = \mathrm{col}\,(\pm 1,0,0,0)$, where t_f stands for the terminal time. This condition will imply $\gamma(t_f) = 0$ and $\gamma_0 = 1$. If the final rate is to remain unchanged after t_f one should also have $\omega(t_f) = \Omega$. From (22), (23), (33) and (34), it follows that

$$x_1(t_f) = x_2(t_f) = 0 \qquad (39)$$

$$\Omega \times I^o \Omega + 2\, I^o u(t_f) = \tau(t_f) \qquad (40)$$

Therefore, a quadratic performance index of the form

$$I(t - t_0, x_1(t_0), x_2(t_0)) = (p_1/2) x_1(t_f)^T x_1(t_f) +$$

$$+ (P_2/2) x_2(t_f)^T x_2(t_f) + \frac{1}{2}\int_{t_0}^{t_f} u(t)^T R\, u(t)\,dt \qquad (41)$$

will yield a linear feedback law in (x,u)-space, provided that the constraint x_0 is also verified.

Since the condition given by (39) and the performance index (41) are the same that appear in Dwyer (1984) one can use directly the results already obtained there. Then the optimal transformed command signal u* for zero terminal error is given by:

$$u^*(t) = -\{6/(t_f - t)^2\} x_1^*(t) + \{4/(t_f - t)\} x_2^*(t) \qquad (42)$$

By using (42), it is also possible to obtain control laws for a maneuver similar to the rest-to-rest maneuver already tackled by Dwyer. This maneuver consists in aligning a rigid body with an initial attitude given by $\gamma(0)$ and initial angular velocity

$$\omega(0) = (1 - \gamma(0)^T \gamma(0))\Omega + 2\gamma(0)\gamma(0)\Omega - 2\gamma(0) \times \Omega.$$

The initial conditions to be used in (6) and (7) are $x_1(0) = \gamma(0)$ and, by using (23), $x_2(0) = 0$.

The optimal transformed states and inputs are:

$$x_1^*(t) = 2(1-\frac{t}{t_f})^2(\frac{1}{2} + \frac{t}{t_f})\gamma(0) \quad (43)$$

$$x_2^*(t) = -\frac{6}{t_f}(1-\frac{t}{t_f})\frac{t}{t_f}\gamma(0) \quad (44)$$

$$u^*(t) = \frac{12}{t_f^2}(\frac{t}{t_f} - \frac{1}{2})\gamma(0) \quad (45)$$

The optimal torque are, therefore, given by:

$$\tau^*(t) = \Omega x\, I^o\Omega + \frac{24}{t_f^2}(\frac{t}{t_f} - \frac{1}{2})I^o\gamma(0) \quad (46)$$

The optimal attitude and rate variables $\gamma^*(t)$ and $\omega^*(t)$ are generated via (22) and (32).

8. CONCLUSIONS

By using the relative Euler quaternion for representing attitude of a rotating body, state and input transformations were constructed in order to obtain linear system equations that are equivalent to the kinematic and dynamic equations of motion. Since the linear system is completely controllable, it is possible to apply multivariable linear system theory to controller design. Two of such designs were described: one in which the behaviour of the relative quaternion is specified and the other where a minimization of a quadratic performance index is accomplished. In both cases, control laws not requiring too much computation were obtained. Since these laws are also only function of the relative quaternion and the body inertial velocity, measured in body axes, they are easily implemented in processors to be used on board of satellites. By using the measurements of inertial sensors (earth sensors, sun sensors, etc) the processor calculates an estimator of the relative quaternion. An estimate of the inertial angular velocity can be implement with gyroscopes measurements. Using both estimates one obtains the required torque value to be supplied by the satellite propulsion system.

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Satellite Attitude Control Analysis and Design using Linearizing Transformations

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