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RESUMO - NOTAS / ABSTRACT - NOTES

This work describes the performance of filtering and smoothing techniques applied to orbit determination of earth satellites. A (forward pass) Kalman filter along with an adaptive procedure for estimating the dynamic noise level which prevents the divergence of estimates due to inaccurate modelling of the orbital motion is implemented in the UD factorization form . The backward smoother is a numerically efficient version of the Rauch-Tung-Striebel (RTS) smoother. This smoother, developed in the UD form, has proven to be economical, compact and competitive for computer implementation. Digital simulations are performed for 2 situations of orbit determination: i) short arc orbit determination with favorable tracking geometry; ii) long arc orbit determination with existing tracking stations. Tests containing many levels of modelling degradation are carried out. The forward pass adaptive UD filter behaves so as to deal with these modelling nuisances and does not allow the divergence phenomenon to occur. In no real time operations, the backward UD smoother is used to improve the accuracy of the estimates and of the covariances resulting from the filtering phase. The results show, in the main, that the UD smoother can enhance the accuracy of the forward pass filter, sometimes by an order of magnitude. For post-flight analysis, the UD smoother is a useful tool when one aims at reconstituting the entire trajectory of the orbital motion covered by the tracking data.

OBSERVAÇÕES/REMARKS

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UD FILTERING AND SMOOTHING APPLIED TO ORBIT DETERMINATION

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1 - Introduction

This work describes the performance of filtering and smoothing techniques applied to orbit determination of earth satellites (Kuga, 1989a). A (forward pass) Kalman filter, along with an adaptive procedure for estimating the dynamic noise level which prevents the divergence of estimates due to inaccurate modelling of the orbital motion (Rios Neto and Kuga, 1985), is implemented in the UD factorization form (Bierman, 1977). The backward smoother is a numerically efficient version of the Rauch-Tung-Striebel (RTS) smoother (Bierman, 1983). This smoother, developed in the UD form, has proven to be economical, compact and competitive for computer implementation. Digital simulations are performed for 2 situations of orbit determination: i) short arc orbit determination with favorable tracking geometry; ii) long arc orbit determination with existing tracking stations. Tests containing many levels of modelling degradation are carried out. The forward pass adaptive UD filter behaves so as to deal with these modelling nuisances and does not allow the divergence phenomenon to occur. In no real time operations, the backward UD smoother is used to improve the accuracy of the estimates and of the covariances resulting from the filtering phase. The results show, in the main, that the UD smoother can enhance the accuracy of the forward pass filter, sometimes by an order of magnitude. For post-flight analysis, the UD smoother is a useful tool when one aims at reconstituting the entire trajectory of the orbital motion covered by the tracking data.

2 - UD forward-pass filtering

Let the discrete process be modelled by:

$$x_{k+1} = \phi_k x_k + G_k w_k \tag{2.1}$$

and the discrete observations by:

$$y_k = H_k x_k + v_k \tag{2.2}$$

where x is the n-dimensional state, ϕ is the nxn transition matrix, G is the nxp matrix of state noise inclusion, y is the m-dimensional vector of observations, H is the mxn matrix relating observations to state, and w_k and v_k are p and m dimensional white noise sequences defined by:

$$w_k = N(0,Q_k), \quad v_k = N(0,R_k) \tag{2.3}$$

Without loss of generality one considers Q and R as diagonal matrices. Besides, one assumes the usual hypotheses of uncorrelation:

$$E[w_k x_o^T] = 0, \quad E[v_k x_o^T] = 0, \quad E[v_k w_k^T] = 0 \tag{2.4}$$

The conventional Kalman filter consists of cycles of time updating and measurement updating, given by (Bierman, 1977):

<u>time update cycle</u>:

$$\hat{x}_{k+1/k} = \phi_k \hat{x}_{k/k} \tag{2.5}$$

$$P_{k+1/k} = \phi_k P_{k/k} \phi_k^T + G_k Q_k G_k^T \tag{2.6}$$

<u>measurement update cycle</u>:

$$K_k = P_{k/k-1} H_k^T \, (H_k P_{k/k-1} H_k^T + R_k)^{-1} \tag{2.7}$$

$$x_{k/k} = x_{k/k-1} + K_k (y_k - H_k x_{k/k-1}) \tag{2.8}$$

$$P_{k/k} = (I - K_k H_k) P_{k/k-1} \tag{2.9}$$

Equation 2.9 is known to present numerical problems. Therefore, one uses the so-called stable UD algorithms to code the forward pass filter.

2.1 - UD filter implementation

The UD algorithms implement the Kalman filter with much better numerical properties, being less prone to errors due to limitation of the computer word length. The three main algorithms used here were:

1 - UD Gram-Schmidt algorithm (Thornton and Bierman, 1977) which implements the equation:

$$\tilde{P}_{k+1} = \phi_k P_{k/k} \phi_k^T \tag{2.10}$$

giving the UD factors corresponding to P.

2 - UD rank 1 algorithm (Bierman, 1977; Thornton and Bierman, 1980) which includes one state noise component at a time, and produces the UD factors corresponding to:

$$P_1 = \tilde{P}_{k+1} \quad (2.11)$$

$$P_{i+1} = P_i + q_i g_i g_i^T \quad i=1,\ldots,p \quad (2.12)$$

where q_i are the elements of the diagonal state noise covariance matrix $Q = \text{diag.}(q_i)$, and g_i are n-dimensional column vectors which form the nxp matrix $G_k = (g_1,\ldots,g_i,\ldots,g_p)$. At the end:

$$P_{k+1/k} = P_{p+1} \quad (2.13)$$

These two algorithms mechanize the covariance time update described by Equation 2.6, in the UD factorization form.

3 - UD measurement update (Bierman 1977), which implements the equation:

$$P_{k/k} = (I-K_k H_k)P_{k/k-1} \quad (2.14)$$

and gives the UD factors corresponding to $P_{k/k}$.

Here, one should emphasize a few points:

i) Usually the covariance time update is performed by algorithm 1 only. Nevertheless, since it was included an on-line adaptive estimation (see Section 2.2) of the state noise level, the time update cycle was achieved through algorithms 1-2 shown above.

ii) Also, one should compute and store some variables to be used in the smoothing pass:

$$\lambda_i = q_i/(1+q_i v_i^T g_i), \; i=1,\ldots,p \quad (2.15)$$

$$v_i = P_i^{-1} g_i \quad (2.16)$$

which will be computed during the loop of algorithm 2 in Equation 2.12. The variables to be stored are $\hat{x}_{k/k-1}$, λ_i, v_i, i=1,...,p; and eventually ϕ_{k-1} and $\hat{x}_{k/k}$ if the process is time variant and or non-linear. In Equation 2.16, v_i is easily computed from the U_i, D_i factors corresponding to P_i, through the following sequence of computations:

$$L_i = U_i D_i$$

$$\tilde{v}_i = L_i^{-1} g_i$$

$$v_i = U^{-T} \tilde{v}_i$$

where the inversions of L and U^T are never explicitly needed, because they are triangular matrices.

2.2 - Adaptive state noise estimation

An adaptive technique (Rios Neto and Kuga, 1985) was used to estimate on-line the level of noise necessary to prevent divergence from occurring. By an imposition of consistency between the occurred residuals and their statistics (Jazwinski, 1970), and introducing the concept of true residual, one arrives at an equation named pseudo-measurement equation of the form:

$$z_k = M_k q_k + \eta_k \tag{2.17}$$

where q_k is the vector containing the elements of the diagonal of matrix Q, the state noise covariance matrix. Recall that Q increases the covariance by means of Equation 2.6 and does not allow the filter gain to become too small. Otherwise, the filter would follow the process model, which could be innaccurate, causing the divergence; or in other words, the filter would learn well the wrong model. The variables of this equation are:

scalar pseudo-measurement realization, z_k, and M_k vector:

$$z_k = r_{k+1/k}^2 + R_{k+1} - H_{k+1} \phi_k P_{k/k} \phi_k^T H_{k+1}^T \tag{2.18}$$

$$M_k = [(H_k g_1)^2 : \ldots : (H_k g_r)^2] \tag{2.19}$$

where $r_{k+1/k}$ is the predicted residual (innovation) given by:

$$r_{k+1/k} = y_k - H_k x_{k+1/k} \tag{2.20}$$

pseudo-noise statistics:

$$\eta_k = N(0, 4r_{k+1/k}^2 R_{k+1} + 2R_{k+1}^2) \tag{2.21}$$

The algebraic development is shown in Rios Neto and Kuga (1985). A secondary filter is built to process this pseudo-measurement equation. In this, the UD measurement update algorithm is used to obtain the estimate of q, processing the m pseudo-measurements sequentially. After all the pseudo-measurements are processed, the noise vector q_k is added to the covariance matrix through the Recursion 2.12. Thus, between computations of Equations 2.11 and 2.12, the secondary filter generates the q estimates by means of this adaptive technique.

3 - UD backward pass smoothing

Once the terminal estimates $x_{N/N}$, $U_{N/N}$ and $D_{N/N}$ are obtained by the forward-pass UD Kalman filter, and the variables λ_i, v_i, i=1,..,p and $\hat{x}(k/k-1)$ for each step (time) k=0,1,...,N are stored, the smoothing phase may begin.

3.1 RTS equations

The smoother used is a computationally efficient version of the Rauch-Tung-Striebel (RTS) fixed interval smoother. The basic RTS equations (Rauch et al., 1965) are:

$$C_k = P_{k/k}\phi_k^T P_{k+1/k}^{-1} \tag{3.1}$$

$$x_{k/N} = x_{k/k} + C_k(x_{k+1/N} - x_{k+1/k}) \tag{3.2}$$

$$P_{k/N} = P_{k/k} + C_k(P_{k+1/N} - P_{k+1/k})C_k^T \tag{3.3}$$

where k = N,...,0, C_k is the smoother gain, $x_{k/N}$ is the smoothed estimate, and $P_{k/N}$ is the smoothed covariance.

3.2 - RTS-Bierman equations

Bierman (1983) created a recursive implementation of the RTS smoother, which is more efficient computationally and eliminates numerical shortcomings of the convencional RTS equations 3.1 and 3.3. At each step k, the Bierman smoother computes a recursive estimate and covariance:

initial conditions:

$$x_{p+1} = x_{k+1/N} \tag{3.4}$$

$$P_{p+1} = P_{k+1/N} \tag{3.5}$$

for i=p,...,1

$$\delta_i = \lambda_i v_i^T(\hat{x}_{k+1/k} - x_{i+1}) \quad (3.6)$$

$$x_i = x_{i+1} + g_i \delta_i \ , \quad (3.7)$$

$$P_i = (I-\lambda_i g_i v_i^T)P_{i+1}(I-\lambda_i g_i v_i^T)^T + \lambda_i g_i g_i^T \quad (3.8)$$

finally:

$$\hat{x}_{k/N} = \phi_k^{-1} x_1 \quad (3.9)$$

$$P_{k/N} = \phi_k^{-1} P_1 \phi_k^{-T} \quad (3.10)$$

The proofs of equivalence with the RTS equations were not given by Bierman (1983), but were shown in Kuga (1989a).

3.3 - UD RTS-Bierman implementation

The RTS-Bierman smoother is adequate for mechanization in terms of UD factorization, and the resulting UD smoother is explicitly shown in Kuga (1989a). Basically, Equation 3.8 is solved through the UD measurement update algorithm in conjunction with the UD rank 1 algorithm. Equation 3.8 is shown to be equivalent to solving the following problem (Thornton and Bierman, 1980; Kuga, 1989b):

$$\tilde{P} = (I-\tilde{K}\tilde{H})P + (\hat{K}-\tilde{K})\alpha(\hat{K}-\tilde{K}) \quad (3.11)$$

where the temporal indexes are not written for the sake of notational simplicity. The following holds:

$$\tilde{H} = \lambda v^T, \ \tilde{R} = \lambda, \ \hat{K} = g, \quad (3.12)$$

where $\tilde{K}$ and α are intermediate computed Kalman gain and weighted residual variance corresponding to the application of the UD measurement update to the first term of Equation 3.11. Next, the UD rank 1 algorithm incorporates the effect of the second term of Equation 3.11. After cycling of Recursion 3.8 p-times, one has the resulting U_1 and D_1. Then, the UD factors corresponding to Equation 3.10 are produced by the UD Gram-Schmidt algorithm. In addition, in non-linear problems, one must take into account the fact that the computation of the time-updated estimate is not accomplished through:

$$x_{k+1/k} = \phi_k x_{k/k} \quad (3.13)$$

but (as in the extended Kalman filter) by an integration:

$$\hat{x}_{k+1/k} = \int_k^{k+1} f(\bar{x},t)\, dt \; . \quad (3.14)$$

with initial condition $\bar{x}_k = \hat{x}_{k/k}$, where f is a non-linear vectorial function of state and the independent term t. Hence, one should store not only $\hat{x}_{k+1/k}$ but also $\hat{x}_{k/k}$ if one wants to avoid computing this integration again. For then, the smoothed estimate may be computed by:

$$\hat{x}_{k/N} = \phi_k^{-1}(x_1 - \hat{x}_{k+1/N}) + \hat{x}_{k/k} \quad (3.15)$$

which is simply a correction due to the assumption of linear process.

4 - Results of orbit determination

The adaptive filter and smoother were applied to orbit determination of earth satellites. Simulated data were generated in order to compare the performance of these techniques. The orbit simulator accounted for geopotential perturbations (up to the 6th zonal, and 4th tesseral and sectoral coefficients), atmospheric drag, radiation pressure, and Sun and Moon gravitational effects. In addition, a practical test with real tracking data is also shown.

4.1 - Filtering results

A typical case of divergence is shown in Figure 4.1, for a short arc test with three simultaneous tracking stations measuring a simulated polar orbit (700km, circular). The UD filter models only the keplerian motion, which is a poor approximation of the real orbit. Figure 4.2 shows the results using the adaptive scheme of noise estimation. The divergence does not occur and the filter behaves as expected. For a test of long arc (almost 2 revolutions), the orbit of the Giotto satellite (Halley's comet encounter mission) during the transfer orbit phase was simulated. In this case, the UD adaptive filter only produced acceptable results when the model included at least the J_2 zonal coefficient. The keplerian model yielded bad estimates because the time interval between measurements was, in general, greater than 5 minutes, which contributed to the accumulation of modelling errors. Figure 4.3 shows the results of the filter. One should point out that, when the conventional Kalman filter formulation was used, numerical problems such as negative diagonal elements in the covariance matrix, overflows, and negative residual variances arose. With the UD adaptive algorithms, this kind of problems have never happened.

4.2 - Smoothing results

Given the terminal estimates of the forward filter, x(N/N), U(N/N), D(N/N) and the variables stored in the filtering pass (Section 3), the same cases were submitted to the proposed smoother. Figure 4.4 shows the superimposed results of the filter and the smoother, for the short arc test case. Figure 4.5 shows the same type of plot for the long arc test case (Giotto satellite). One notes that the smoother in general improves the estimates of the filtering phase, sometimes by an order of magnitude. In reconstitution of trajectories this is a tool which should be considered. The overhead of the proposed UD smoother was insignificant compared to the CPU time expended by the UD filter. The UD algorithms are not the major burden indeed. Actually, linearization, numerical integration, and partial derivatives computations are the expensive part of the filtering phase.

4.3 - Real tracking data

The UD filter and smoother were applied to the real tracking data of the Giotto satellite, which were supplied by ESOC (European Space Operation Center). As in the former test of long arc, the filter used the J_2 orbit model only. Figure 4.6 shows the filtered and smoothed covariances behavior. Figure 4.7 shows the residuals history. The decrease of the residuals magnitude after application of the smoothing is remarkable.

5 - Conclusion

This work showed the results obtained through application of techniques of filtering and smoothing to orbit determination. Divergence of estimates may occur when a poor orbit model is used and, in this case, some compensation method such as the proposed one should be used. The smoother use is highly recommended when no real time constraints are imposed, and when one needs a precision reconstitution of trajectory. Besides, the use of UD algorithms takes care of many numerical problems which might occur due to bad initialization or ill-conditioned systems. In many instances, due to the particular form of the UD factors, they may become more compact, economic, and efficiently implemented than the conventional methods.

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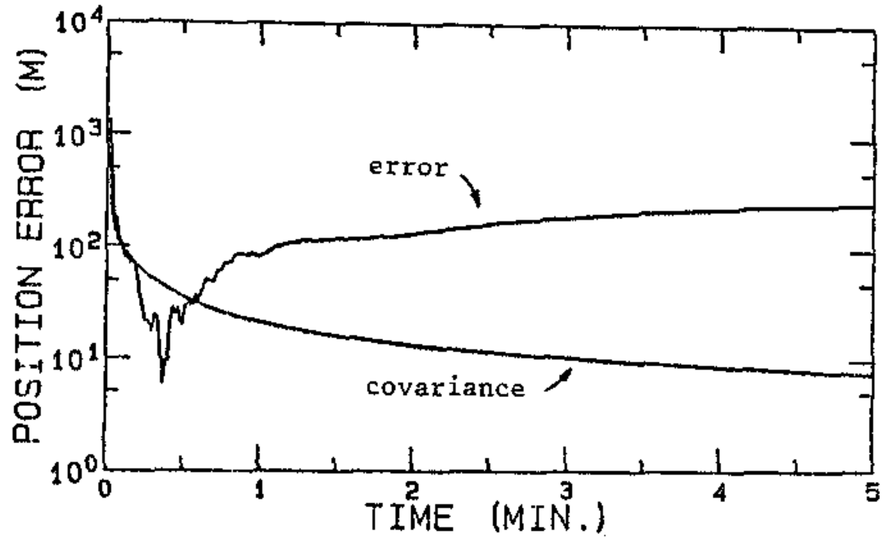


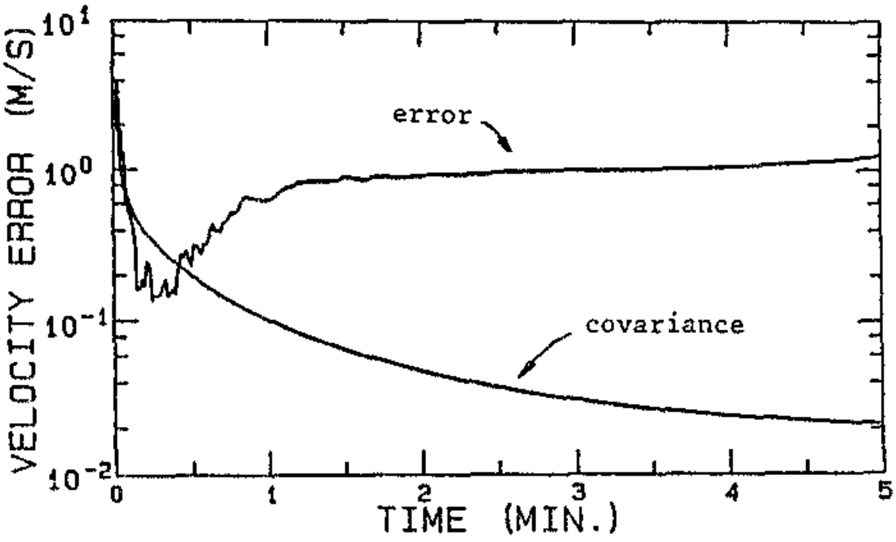


Fig. 4.1 - Divergence in position and velocity

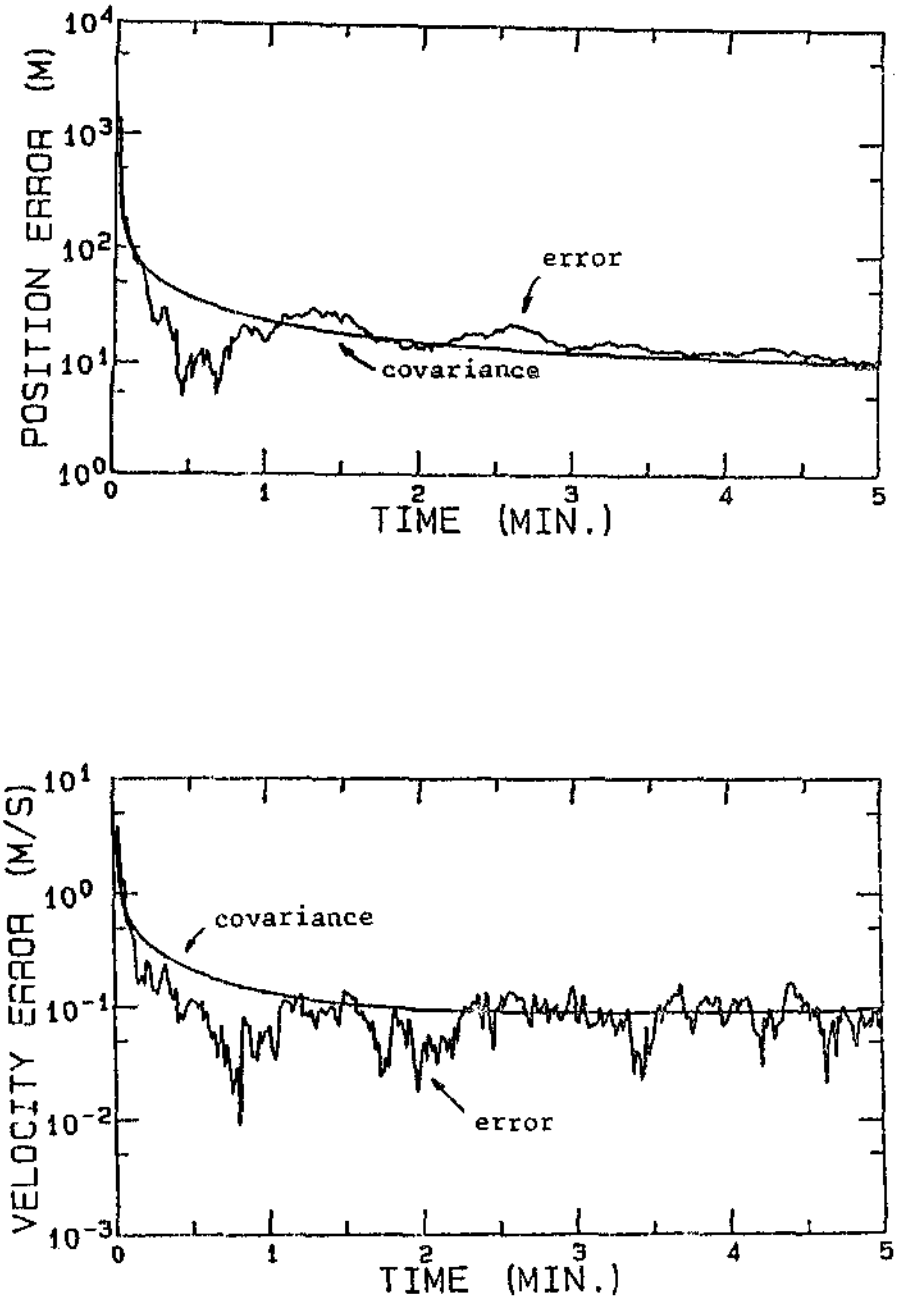
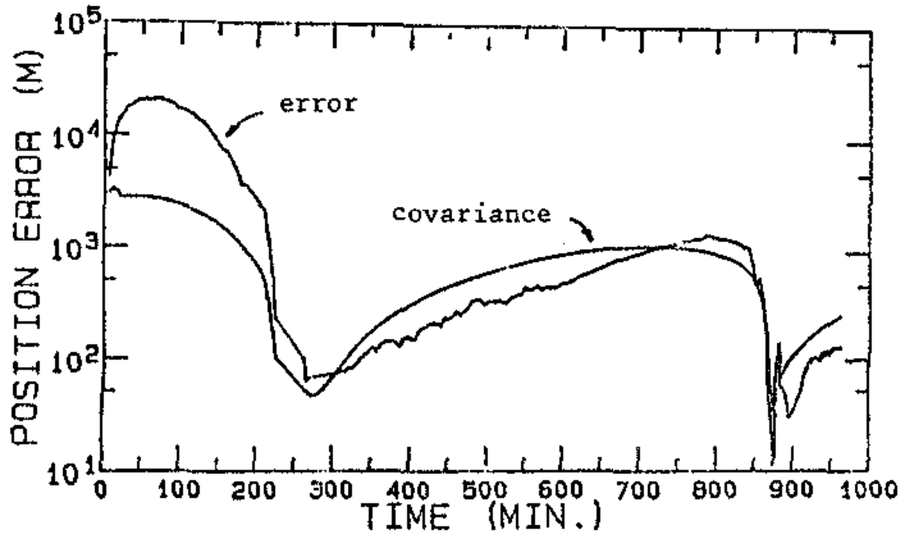


Fig. 4.2 - UD adaptive filter results for short arc case



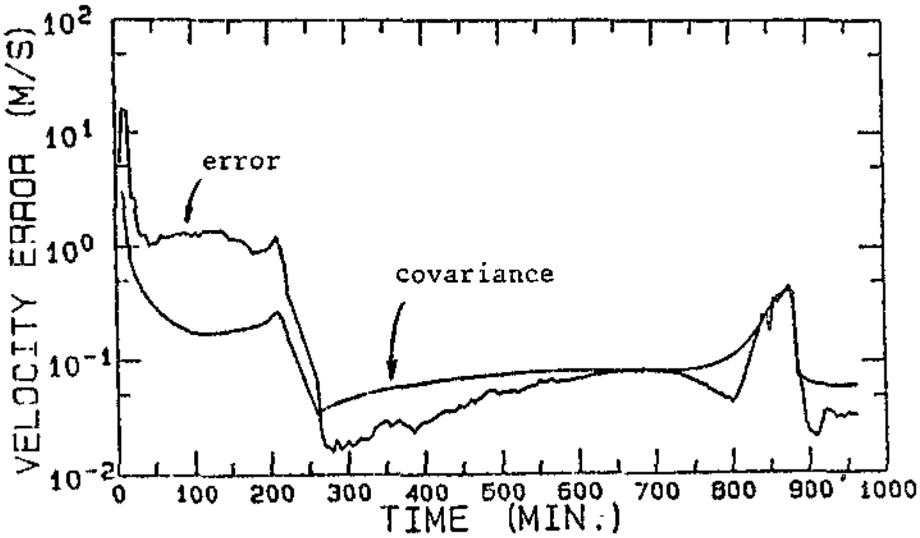


Fig. 4.3 - UD adaptive filter results for long arc case

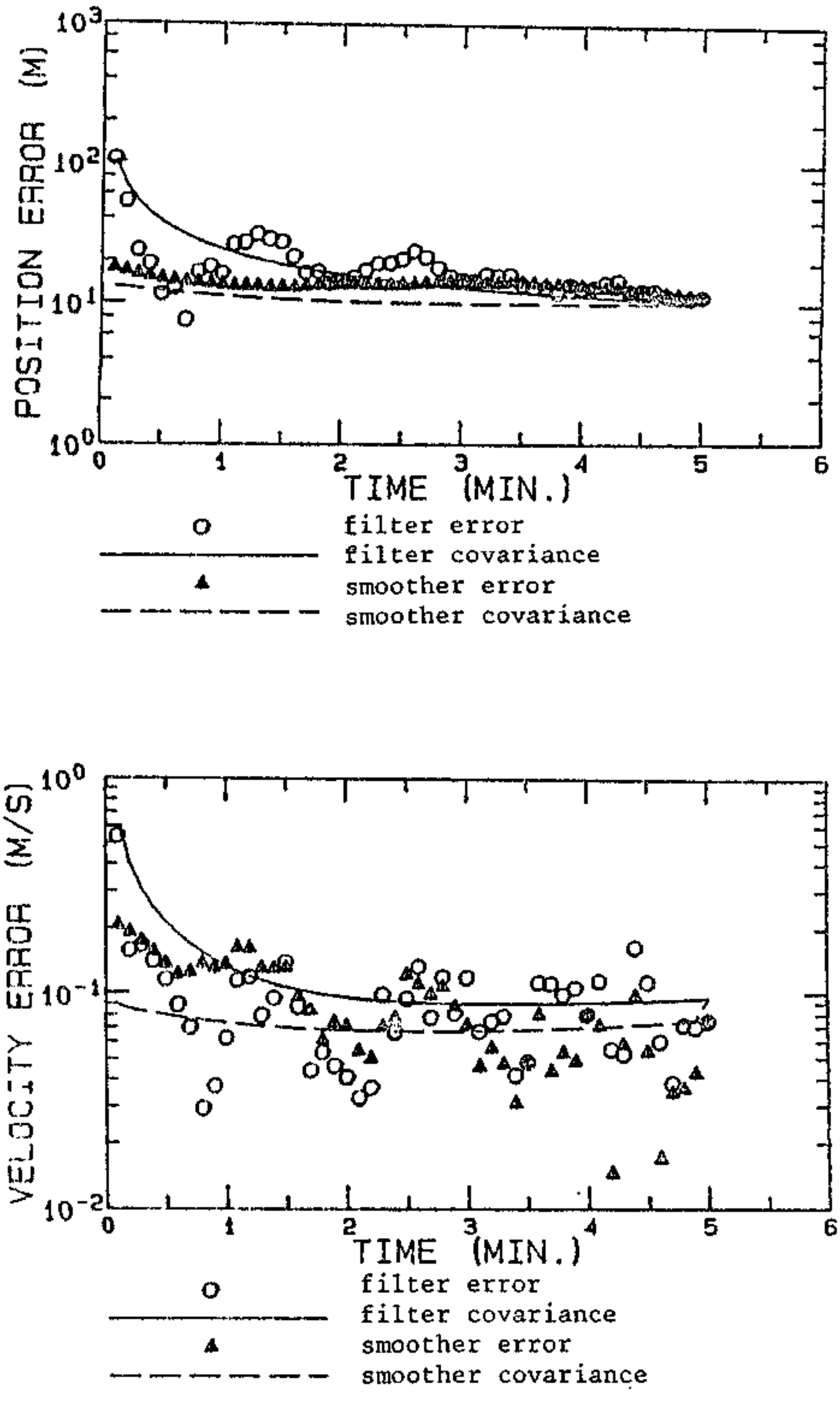


Fig. 4.4 - UD filter and smoother results for short arc case

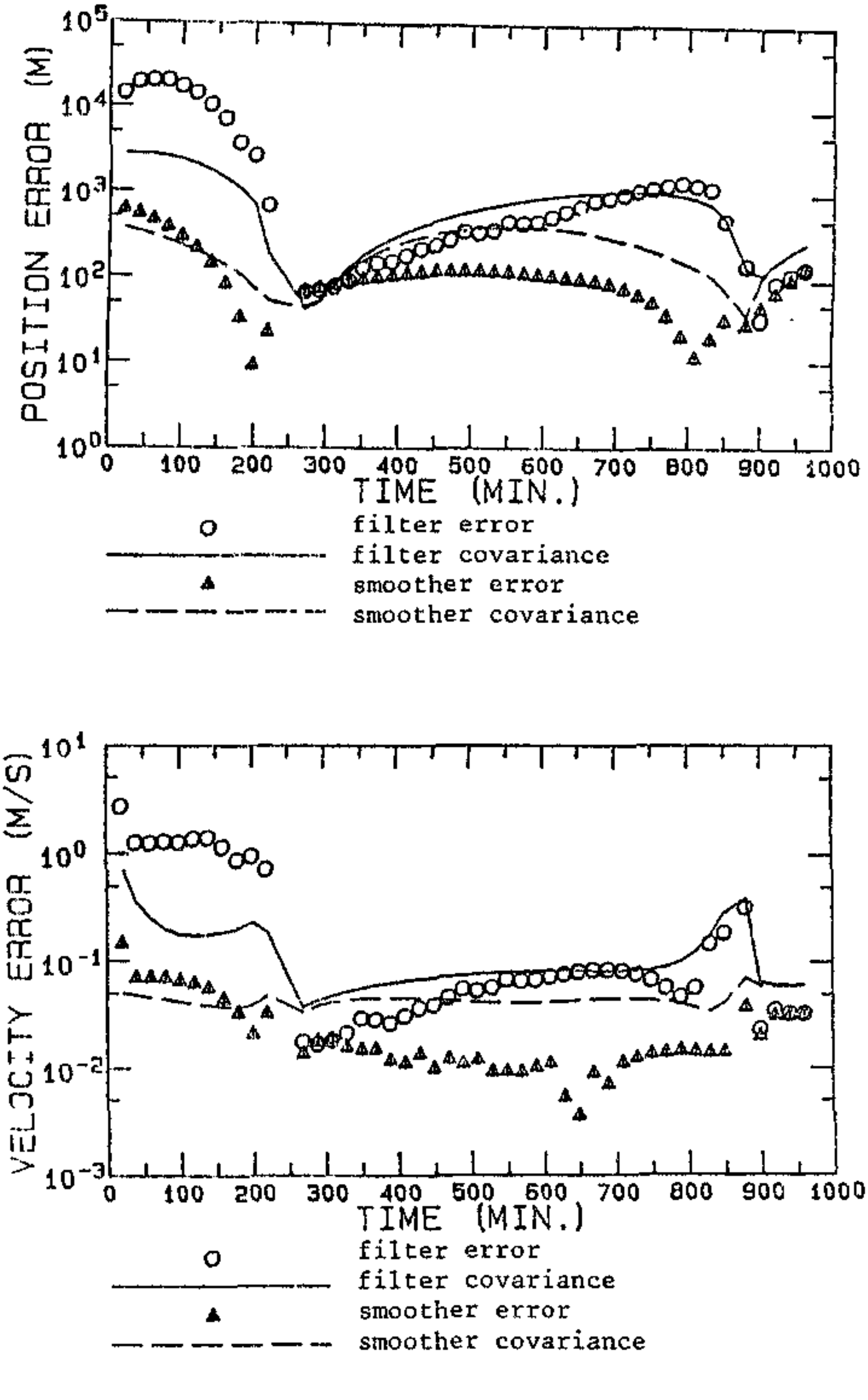


Fig. 4.5 - UD filter and smoother results for long arc case

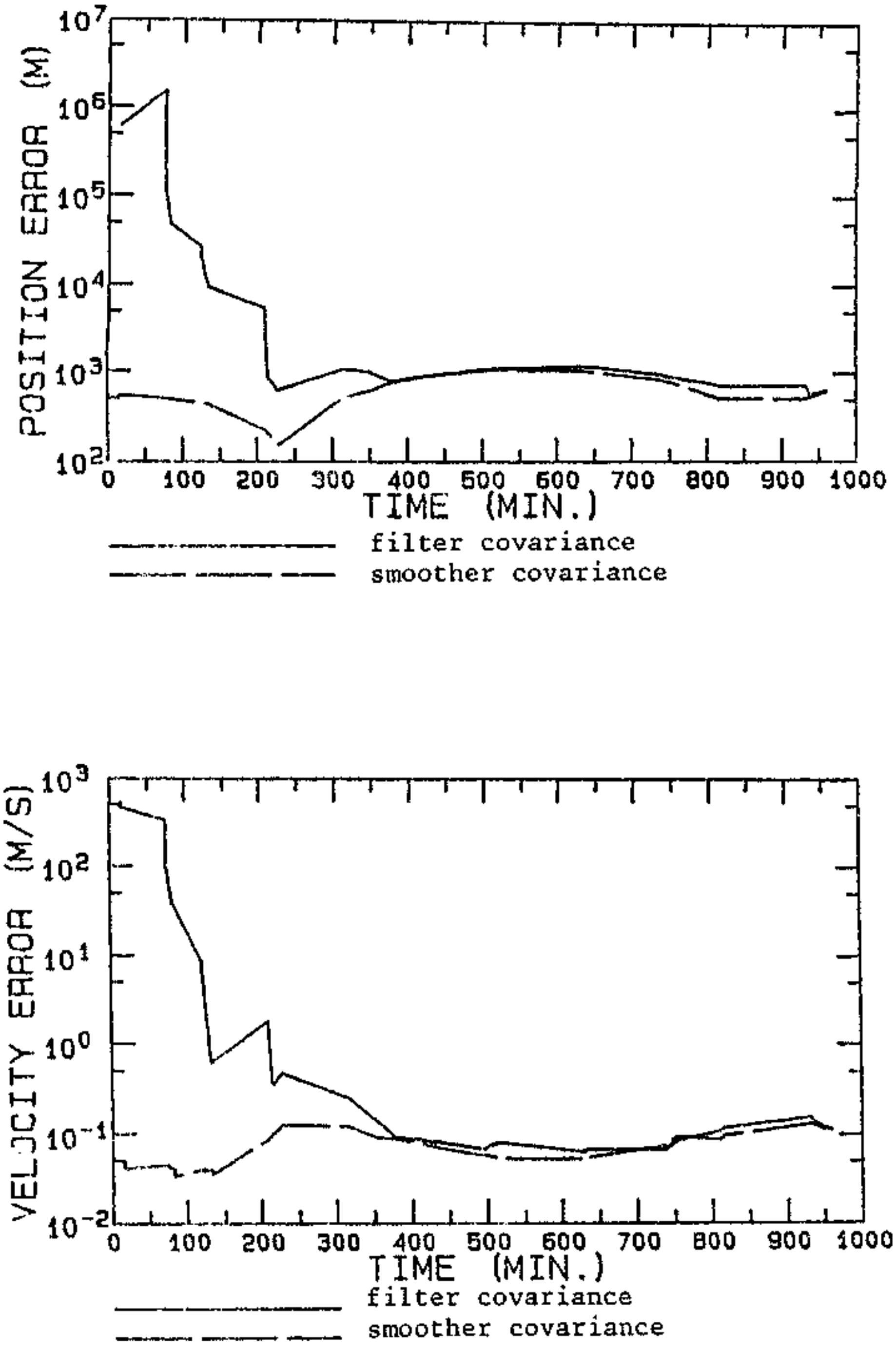
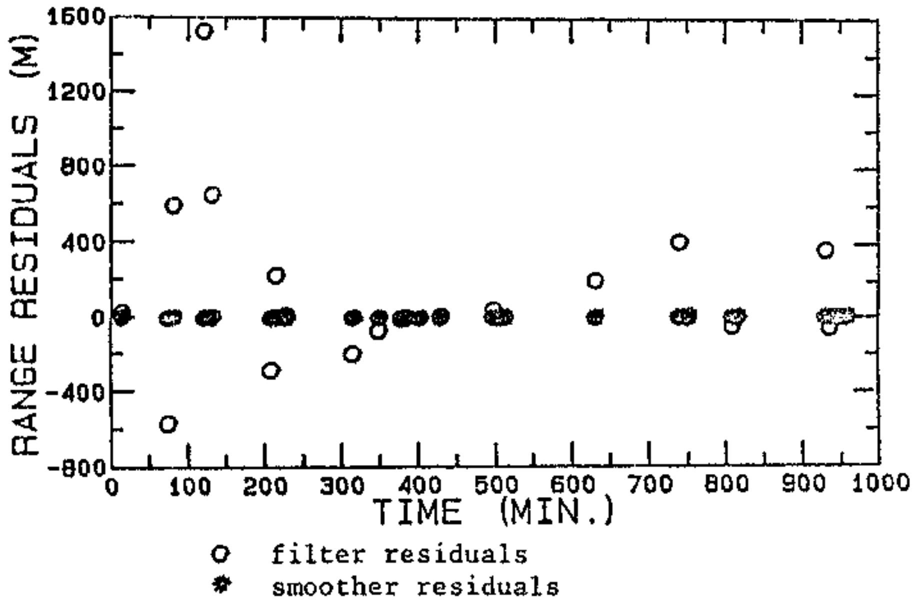


Fig. 4.6 - UD filter and smoother covariances for real case



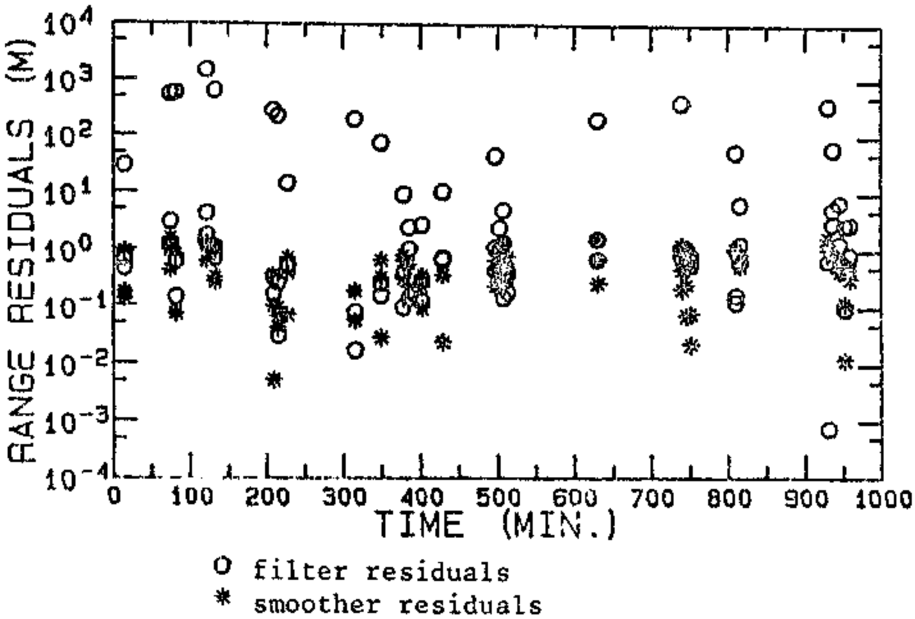


Fig. 4.7 - UD filter and smoother residuals for real case

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TÍTULO

UD FILTERING AND SMOOTHING APPLIED TO ORBIT DETERMINATION

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