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16.Summary/Notes

A steady barotropic primitive equation model is solved by spectral methods for the stationary waves forced by topography between 60°N and 60°S. The meridional transports of momentum by these waves are compared with those calculated earlier from observations. The cross equatorial exchange of momentum by these waves is negligible compared to that by observed standing waves and thus the present study suggests the importance of the thermally forced stationary waves. The role of friction in determining the tilt, amplitude and, as a result, the momentum transport is very crucial.

17.Remarks *Submitted for publication in monthly weather Review*

1720

ON THE MOMENTUM TRANSPORT BY THE STATIONARY WAVES FORCED BY THE TOPOGRAPHY

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1. INTRODUCTION

Stationary waves generated by the geographically fixed topography and slowly varying heat sources and sinks have a profound influence on the regional and global climate. Both osbservational and theoretical studies of these waves are abundant (see Saltzaman, 1968 for a review). Transports of momentum and heat by these waves are important in the global momentum and energy balances (Lorenz, 1967). Mak (1978) analysed the 500mb height data and found that the meridional transport of momentum by the planetary scale stationary waves in the Northern Hemisphere is comparable to that by the transient waves. He concludes that the stationary waves may play a more important role in the troposphere than past investigations have suggested. Recently, Lau (1979) reexamined the momentum and heat balances in the Northern Hemisphere troposphere. His results agree with those of Mak. Extensive statistics on the subject presented by Oort and Rasmusson (1971) give much smaller values for the meridional transports by standing waves. The differences may be due to the available data coverage. As far as the Southern Hemisphere is concerned, Obasi (1963) studied the poleward flux of relative angular momentum with the IGY (International Geophysical Year) data. His statistics showed the ineffectiveness of the standing eddies in the fulfilment of the angular momentum balance requirements of this Hemisphere. On the other hand, his study demonstrates that the principal agents in the interhemispheric exchange of this quantity are the standing eddies. However, such statistics obtained from more extensive data for the Southern Hemisphere trosposphere are non existent. The sparcity of data makes it difficult to get reliable statistics in this Hemisphere. A resonable idea regarding the importance of the momentum transports by the topographic waves could, however, be obtained theoretically, as attempted in the present study. Similar studies have been done for Northern Hemisphere (Egger, 1976).

Charney and Eliassen (1949) are the first to generate topographically forced waves by solving a one dimendional steady state potential vorticity equation. Their results for the middle latitudes in the Northern Hemisphere agree satisfactorily with the observed picture. An important outcome of their study is the role of friction in obtaining realistic results. Satyamurty et al. (1980) simulated numerically the orographic trough off the Andes with a limited area barotropic primitive equation model including topographic forcing. This trough typically has a northwest to southeast tilt which implies a southward transport of momentum. However, no such statistics are attempted in that study because of the area limitations. Realistic statistics could only be obtained with a more complete model. For this purpose a two dimensional barotropic steady state linearized primitive equation model is solved for the topographic waves at 500 mb between 60^{o}S and 60^{o}N.

2. MODEL EQUATIONS AND METHOD OF SOLUTION

The topographically forced waves do not present vertical phase shifts even in the presence of a vertically varying basic zonal flow (see Charney, 1973) and for their study a multilayer model is not essential. Thus, a primitive equation barotropic model with Rayleigh friction and topographic forcing is adapted in the present study. The basic state is assumed to be such that

$$1. \quad f\,\bar{u} + \frac{\bar{u}^2}{r}\tan\psi + g\frac{\partial\bar{Z}}{\partial y} = 0$$

where f is the coriolis parameter, r the mean radius of the earth, ψ the latitude, g the acceleration due to gravity, and u and Z are the zonal wind and the depth of the free surface, respectively. The overbar represents a basic state quantity which varies only in the meridional direction, y. With this, the shallow water equations with bottom topography (h) and Rayleigh friction are linearized to get

$$2. \quad (\bar{u}\frac{\partial}{\partial x} + D)\,u - (f - \frac{d\bar{u}}{dy} + \bar{u}\,\frac{\tan\psi}{r})\,v + g\frac{\partial Z}{\partial x} = 0$$

$$3. \quad (\bar{u}\frac{\partial}{\partial x} + D)\,v + (f + \frac{2\,\bar{u}\,\tan\psi}{r})\,u + g\frac{\partial Z}{\partial y} = 0$$

$$4. \quad \bar{u}\frac{\partial Z}{\partial x} + \left[\frac{d}{dy}(\bar{Z}-\bar{h}) + (\bar{Z}-\bar{h})\frac{\partial}{\partial y} - \frac{(\bar{Z}-\bar{h})}{r}\tan\psi\right]v$$

$$+ (\bar{Z}-\bar{h})\frac{\partial u}{\partial x} = \bar{u}\frac{\partial h}{\partial x}$$

The perturbation quantities u, v, Z, and h are functions of both the zonal (x) and meridional (y) coordinates; v is the meridional component of wind, h the bottom topography and D the coefficient of Rayleigh friction taken to be constant. The perturbation quantities are assumed to be sinusoidal in x, such that

$$5. \quad (u, v, Z, h) \cong \mathrm{Re} \sum_{k=1}^{15} (\hat{u}, \hat{v}, \hat{Z}, \hat{h}) \exp \left(i \frac{2\pi k}{L} x\right)$$

where a cap represents a complex amplitude of the corresponding quantity which is a function of y (or ψ) and k. $L = 2\pi r \cos \psi$ is the length of the latitude circle. Substitution of Equation 5 into Equations 2 -4 and elimination of $\hat{u}$ and $\hat{Z}$ give us

$$6. \quad a \frac{d^2 \hat{v}}{dy^2} + b \frac{d \hat{v}}{dy} + c \hat{v} = F$$

where

$$7. \quad \begin{cases} a = (i \mu \bar{u} + D) \\ b = \frac{a}{\bar{\phi}} \left(F \bar{u} + \frac{d\bar{\phi}}{dy} - \frac{\bar{\phi}}{r} \tan \psi\right) - i \mu \bar{u} \frac{\tan\psi}{r} \\ c = \frac{a}{\bar{\phi}} \left(\bar{u} \frac{dF}{dy} + F \frac{d\bar{u}}{dy} + \frac{d^2\bar{\phi}}{dy^2} - \frac{d\bar{\phi}}{dy} \frac{\tan\psi}{r}\right) - a \mu^2 + i \mu \frac{dF}{dy} \\ \quad - \left(F\bar{u} + \frac{d\bar{\phi}}{dy} - \bar{\phi} \frac{\tan\psi}{r}\right) \left(\frac{i\mu}{\bar{\phi}} \left[F + \frac{\bar{u}}{r} \tan\psi \right] + \frac{a}{\bar{\phi}^2} \frac{d\bar{\phi}}{dy}\right) \\ F = i \mu \frac{d}{dy} (\bar{u}\, a\, \hat{\phi} / \bar{\phi}) + \mu^2 \bar{u} \left(f + \frac{2\bar{u}}{r} \tan\psi\right) \hat{\phi} / \bar{\phi} \end{cases}$$

where

$$8.\ F \equiv (f + \frac{\bar{u}\tan\psi}{r} - \frac{d\bar{u}}{dy});\ \bar{\Phi} = g(\bar{Z} - \bar{h});\ \hat{\Phi} = g\hat{h};\ \mu = \frac{2\pi k}{L}$$

In arriving at Equation 6 the following assumptions are made

$$9.\quad \begin{aligned} &|\bar{u}\,(i\,\mu\,\bar{u} + D)| \ll |\mu\,\bar{\Phi}| \\ &|\frac{d}{dy}\,[\bar{u}\,(i\,\mu\,\bar{u} + D)\,]| \ll |\mu\,\frac{d\bar{\Phi}}{dy}| \end{aligned}$$

These assumptions are quite valid when we consider the mean depth of the fluid, $(\bar{Z} - \bar{h})$ to be $\gtrsim$ 3 km, $\bar{u} \lesssim$ 50m/sec and $D \lesssim 10^{-5}\text{sec}^{-1}$. Equation 6 is valid for all $k \neq 0$. It can be solved for $\hat{v}(k,Y)$ numerically if the forcing, $F(k,y)$ is known and two boundary conditions are specified. For this, the space between the two boundaries is divided into (N - 1) intervals of 5^0 latitude and the equation is applied at the interior points. The boundary conditions specified are $\hat{v} = 0$ for all k at the southern and northern boundaries i.e.,

$$10.\ \hat{v}_1 = \hat{v}_N = 0$$

Thus for each k we get (N - 2) coupled linear algebraic equations in as many unknowns $\hat{v}_2$, $\hat{v}_3$, ... $\hat{v}_{N-1}$. This set is solved by matrix inversion. Once v is obtained u and Z can be obtained from Equations 2 - 4.

The equation is solved at 500 mb between 60^{o}S and 60^{o}N latitude circles which form the southern and northern boundaries. The Fourier analysed topography at intervals of 5^{o} latitude for the first 15 harmonics is taken from Peixoto et al (1964). The model topography which is the sum of the first 15 harmonics is shown in Figure 1. The meridional profile of the mean zonal wind, $\overline{u}$ at 500 mb is taken from Newell et al.(1972). The unperturbed height of the free surface at the southern boundary is taken to be 5.500 m corresponding to 500 mb and at other latitudes it is obtained by integrating Equation 1. With these specifications, three cases, with $D = 10^{-5}$ sec^{-1}, $D = 5 \times 10^{-6}$ and $D = 10^{-6}$ sec^{-1}, are studied. The meridional transports of the zonal momentum by each of the 15 harmonics and their sum are calculated at all the latitude circles where u and v are solved for in the three cases.

3. RESULTS AND DISCUSSION

The heights of the free surface forced by the bottom topography in the three cases mentioned above are shown in Figures 2, 3 and 4. From these figures one can see the formation of very deep troughs or lows on the leeside of the Rockies and the Himalayas in the Northern Hemisphere. A moderately intense trough is generated off the Andes in the Southern Hemisphere. Besides, there are weaker troughs off South Africa and the Alps. Comparing Figures 2 - 4 it can be seen that the troughs are more intense when the friction is less and vice versa. Further, the troughs in the Northern Hemisphere have tilts from southwest to northeast and those in the Southern Hemisphere have tilts from northwest to southeast. These suggest poleward transport of momentum in mid latitudes of both Hemispheres. The position of the trough off the Andes agrees well with that obtained in the numerical integrations of Satyamurty et al. (loc. cit.). However, the intensity and the tilt of this trough are less than those obtained in the numerical integration. The reasons for this discrepancy could be: 1) the limited east west extend of the area of integration in Satyamurty et al. (loc. cit.); 2) the truncation of the higher harmonics (beyond the first 15) in the present study; and 3) the absence of non-linear interactions in the present study. The area limitation in the numerical integration effects the solution because the topography of South Africa and Australia does not come into picture and the cyclicity assumed makes the Andes repeat four times round the earth (see Satyamurty et al. (loc. cit.)). From the analysis of Peixoto and Saltzman (loc. cit.) it can be seen that in the Southern Hemisphere the amplitudes of the higher harmonics (13, 14, 15) are comparable to those of the lower harmonics (1, 2, 3) which suggests that the harmonics beyond 15 might also be of some importance. Now about the nonlinearity, it is difficult to guess how this effect alters the solution beyond cascading of energy as the friction does.

In Figures 5 and 6 the bottom topography and the 500 mb surface heights at 45^{o}N and 25^{o}S are given for the case $D = 5 \times 10^{-6}$ sec^{-1}. It can be seen that there are two troughs to the east of the Rocky mountains and Himalayas. The trough to the east of the Himalayas is more intense than the other one. Derome and Wiin-Nielson (1971) also found a similar difference of intensities in their study using a two-level quasi geostrophic model. From Figure 6 it can be seen that the amplitudes in the Southern Hemisphere are almost an order of magnitude smaller than those in the Northern Hemisphere (note the difference in scale between Figure 5 and 6 of the height profile) in spite of the fact that the mountain peaks are comparable in heights at the two latitudes. One of the causes of this discrepancy could be the differences in the zonal extent of mountains in the Northern and Southern Hemispheres (Batchelor, 1967 and Eliassen and Palm, 1961). The difference in the latitude also might play a role as noted by Frenzen (1955) in his experiment.

In Figure 7 the total meridional transport of momentum by all the 15 harmonics is shown for the 4 cases, viz., 1) without friction; 2) $D = 10^{-6}$ sec^{-1}; 3) $D = 5 \times 10^{-6}$ sec^{-1}; and 4) $D = 10^{-5}$ sec^{-1}. Results for the case 4 are not shown for the Southern Hemisphere because the transport is very small. Also shown in Figure 7 are the observed transports of Mak (1978) and Oort and Rasmusson (1971) for the Northern Hemisphere and those of Obasi (1963) for the Southern Hemisphere. As the present study is limited to the topographically forced waves alone, a direct comparison with the observational results where such waves forced by both topography and heat sources and sinks are included is perhaps not justified. However, it would be of interest to know the role of topographically induced waves in the momentum transport. From Figure 7, it can be seen that for the case of no friction and with $D = 10^{-6}$ sec^{-1}, the transport of momentum is unrealistially large. For the remaining two cases the calculated values compare reasonably well in the order of magnitude and direction with the observed ones, particularly with those of Oort and Rasmusson.

The peak value of the momentum transport increases and is shifted equatorward with the presence of a small friction ($D = 10^{-6}$ sec^{-1}) in relation to the no friction case. When the friction is increased (to $D = 5 \times 10^{-6}$ and 10^{-5} sec^{-1}) the position of the peak changes little while its magnitude decreases substantially. It is clear that the friction plays a complex role of changing the tilt and decreasing the amplitude of the stationary waves forced by topography. Introduction of a small friction changes the tilt substantially. It, perhaps, shifts the maximum amplitude region towards the equator while affecting its magnitude very little. This explains the larger peak of momentum transport in the case of $D = 10^{-6}$ sec^{-1} and its position equatorward of the one obtained in the no friction case. Sufficiently large friction ($D = 5 \times 10^{-6}$ and 10^{-5} sec^{-1}) decreases the amplitude very much while keeping the tilt more or less same as in the small friction case. Therefore the peaks in the momentum transport in these cases are well reduced. This peculiar effect of friction has not been brought out in the earlier studies.

4. CONCLUSIONS

The following conclusions can be drawn from the results of the present study:

1. The topographically induced waves in the Southern Hemisphere are very weak compared to those in the Northern Hemisphere. In general, friction reduces the intensity.

2. The position of the topographically induced trough in the Southern Hemisphere obtained in the present linear study agrees well with that obtained in the numerical integration, although the tilts do not agree well.

3. The meridional momentum transport by topographic eddies is poleward in the middle latidudes in both hemispheres. It is very large in the Northern Hemisphere compared to that in the Southern Hemisphere.

4. Interhemispheric exchange of momentum due to the topographic eddies is negligible compared to the observed exchange by standing waves as estimated by Obasi (1963). This suggests the importante of standing waves forced by heat sources and sinks.

5. The initial increase and equatorward shift of the peak momentum transport as the friction is increased from zero to $D = 10^{-5}\ sec^{-1}$ is explained as follows: Even a small friction affects the tilt and perhaps, the latitudinal position of the maximum amplitude. The amplitude decreases appreciably only if the friction is reasonably large.

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Legends for Figures

Fig. 1 - Model topography between 60^{o}N and 60^{o}S as obtained by the combination of the first 15 harmonics given by Peixoto and Saltzman (1964). The maximum height of Himalayas is 5630 m.

Fig. 2 - Computer output of 500 mb geopotential height pertubation field with the Rayleigh friction coefficient, $D = 10^{-5}$ sec^{-1}.

Fig. 3 - As in Fig. 2 except for $D = 5 \times 10^{-6}$ sec^{-1}.

Fig. 4 - As in Fig. 2 except for $D = 10^{-6}$ sec^{-1}.

Fig. 5 - Bottom topography (upper curve) and 500 mb geopotential perturbation(bottom curve) for $D = 5 \times 10^{-6}$ sec^{-1}. along the latitude circle 45^{o}N.

Fig. 6 - As in Fig. 5 except for the latitude circle 25^{o}S.

Fig. 7 - Latitudinal distribution of the momentum transport by the stationary eddies generated by topography.

REFERENCES

Batchelor, G.K., 1967: An introduction to fluid dynamics. Cambridge University Press, U.K., 573-577.

Charney, J.G., 1973: Planetary fluid dynamics, In: Dynamic Meteorology, (Ed) P. Morel,Reidel Publ. Co., 97-351.

Charney, J.G. and A. Eliassen, 1949: A numerical method for predicting the perturbations on the middle latitude westerlies. Tellus, 1, 38-54.

Derome, J. and A. Wiin Neilson, 1971: The response of a middle latitude model atmosphere to forcing by topography and stationary heat sources. Mon. Weath. Rev., 99, 564-576.

Egger, J., 1976: On the theory of the steady perturbations in the troposphere. Tellus, 28, 381-389.

Eliassen, A. and E. Palm, 1960: On the transfer of energy in stationary mountain waves. Geof. Publ., 22, 1-23.

Frenzen, P., 1955: Westerly flow past an obstacle in a rotating hemispherical shell. Bull. Am. Meteorol. Soc., 36, 204-210.

Lau, N-C., 1979: The observed structure of tropospheric stationary waves and the local balances of vorticity and heat. J. Atmos. Sci., 36, 996-1016.

Lorenz, E.N., 1967: The nature and theory of general circulation of the atmosphere. WMO No. 218, TP. 115, 161 pp.

Mak, M., 1978: On the observed momentum flux by stationary eddies. J. Atmos. Sci., 35, 340-346.

Newell, R.E., J.W. Kidson, D.G. Vincent and G.J. Boer, 1972: The general circulation of the tropical atmosphere and interactions with extratropical latitudes, Vol. I. The MIT Press, 258 pp.

Obasi, G.O.P., 1963: Poleward flux of atmospheric angular momentum in the Southern Hemisphere. J. Atmos. Sci., 20, 516-528.

Oort, A.H. and E.M. Rasmusson, 1971: Atmospheric circulation statistics, NOAA Prof. Pap 5, U.S. Dept. of Commerce, 323 pp.

Peixoto, J.P., B. Saltzman and S. Teweles, 1964: Harmonic analysis of the topography along parallels of the earth. J. Geophys. Res., 69, 1501-1505.

Saltzman, B., 1968: Surface boundary effects on the general circulation and macroclimate: A review of the theory of the quasi-stationary perturbations in the atmosphere. Met. Monographs, 8, American Met. Soc., Boston, 4-19.

Satyamurty, P., R.P. Santos and M.A.M. Lemes, 1980: On the stationary trough generated by the Andes. To appear in Mon. Wea. Rev.

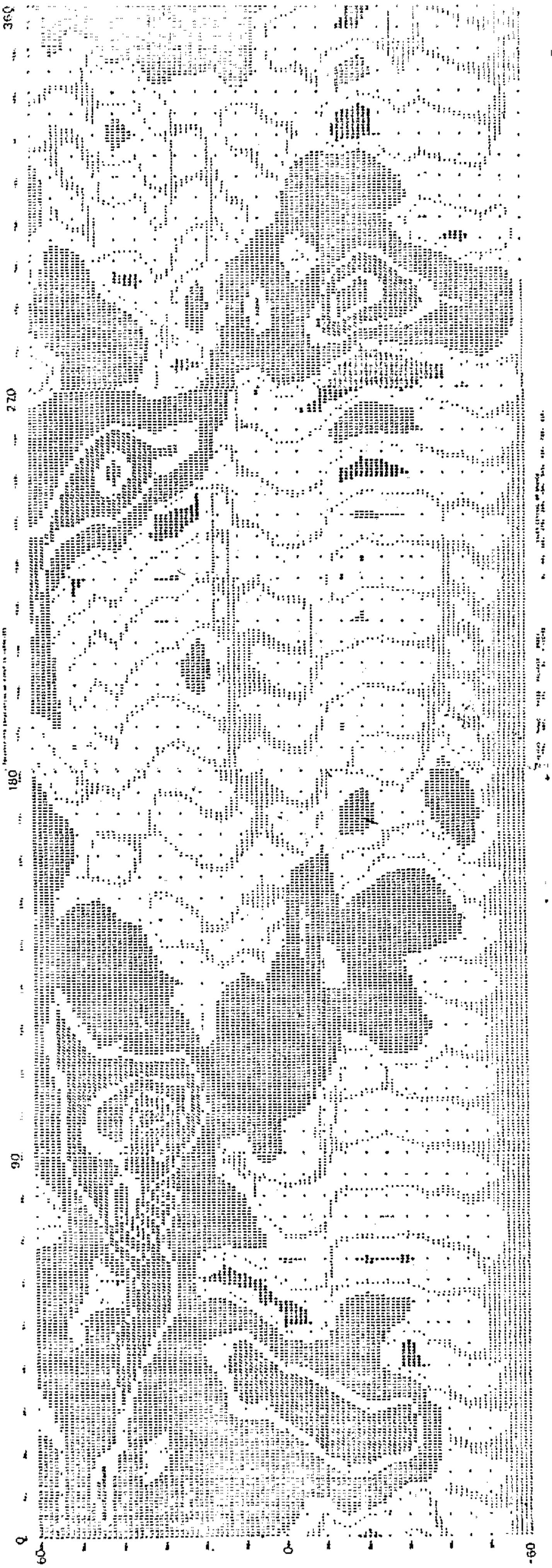

Fig. 1

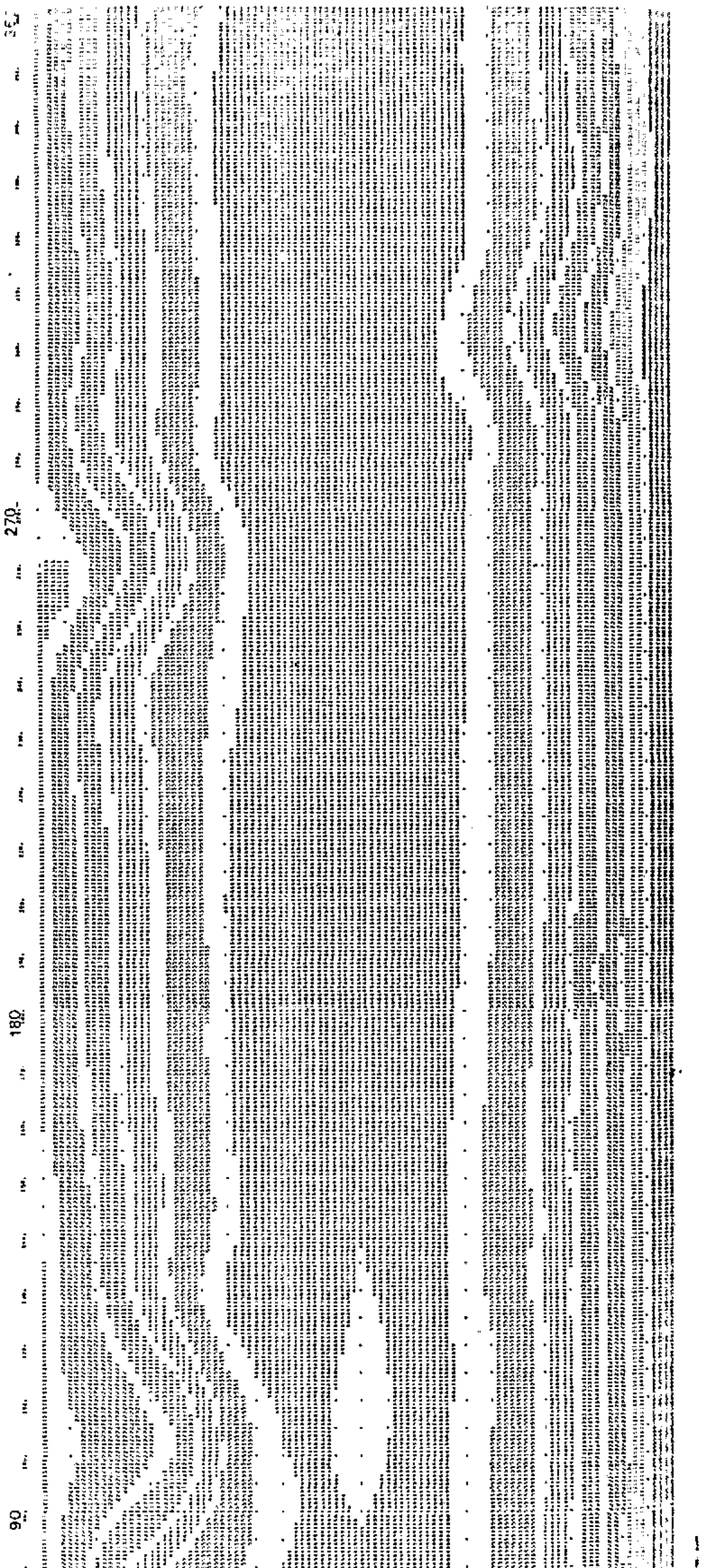

Fig. 2

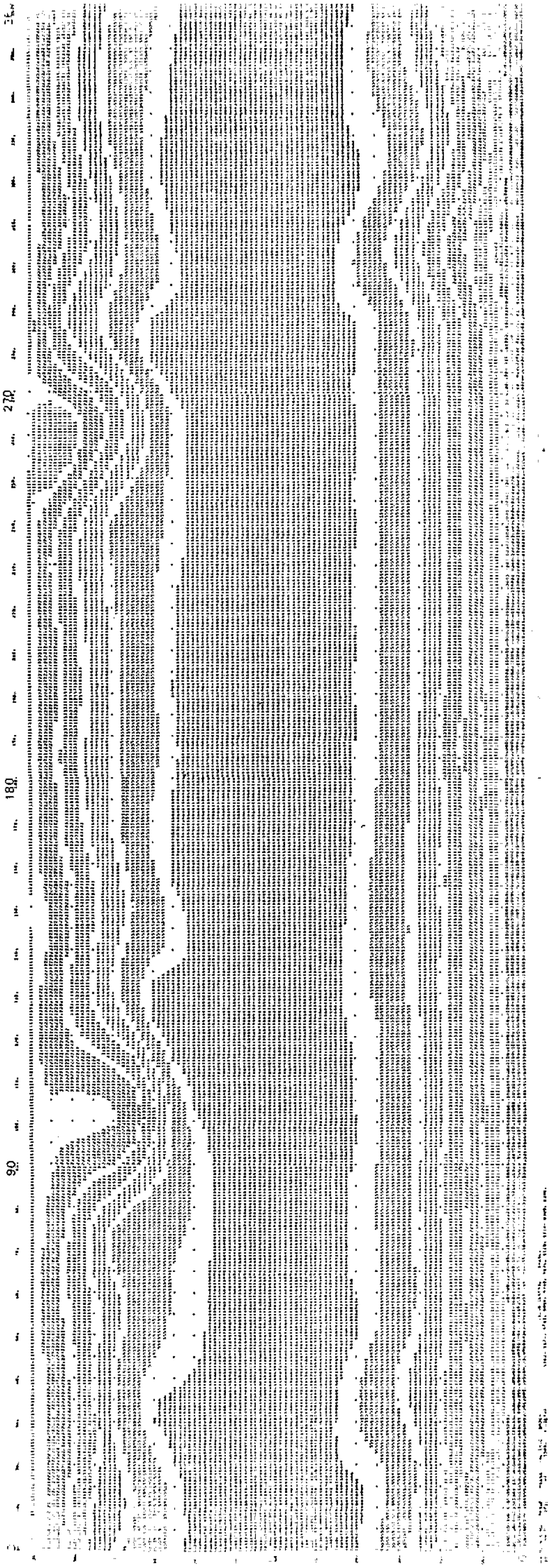

Fig. 3

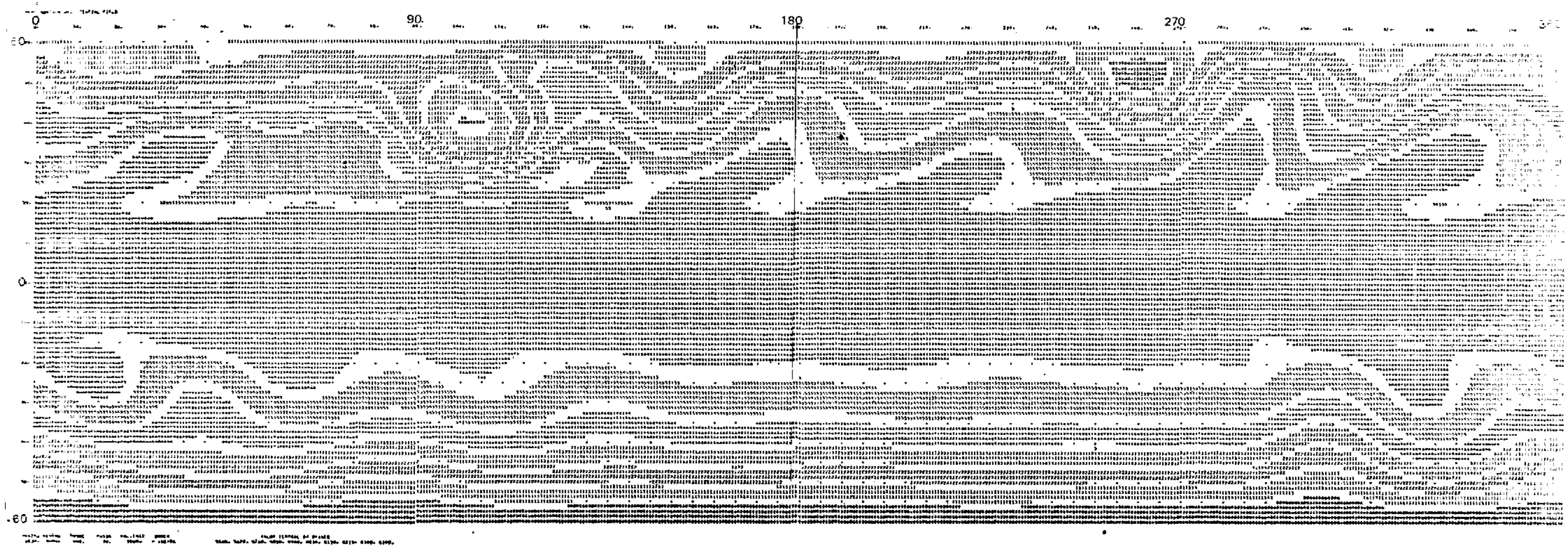

Fig. 4

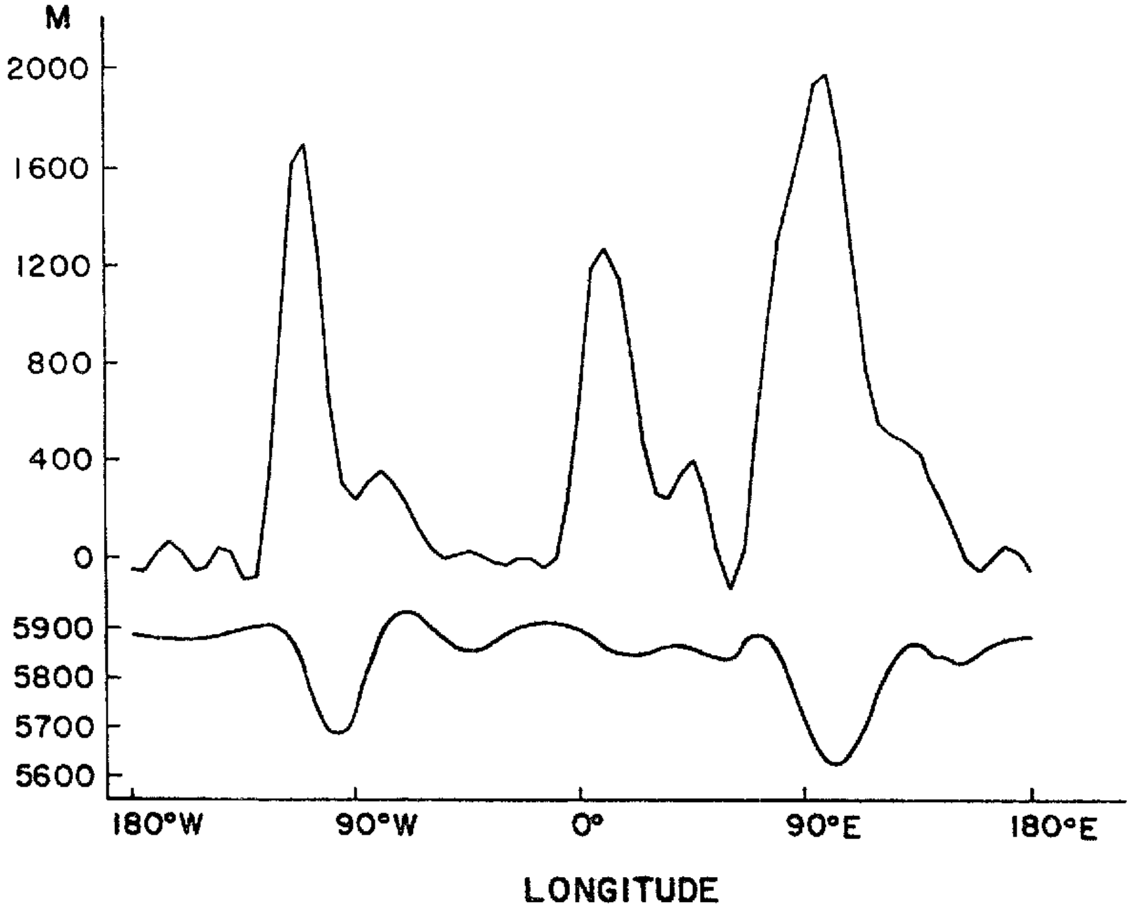
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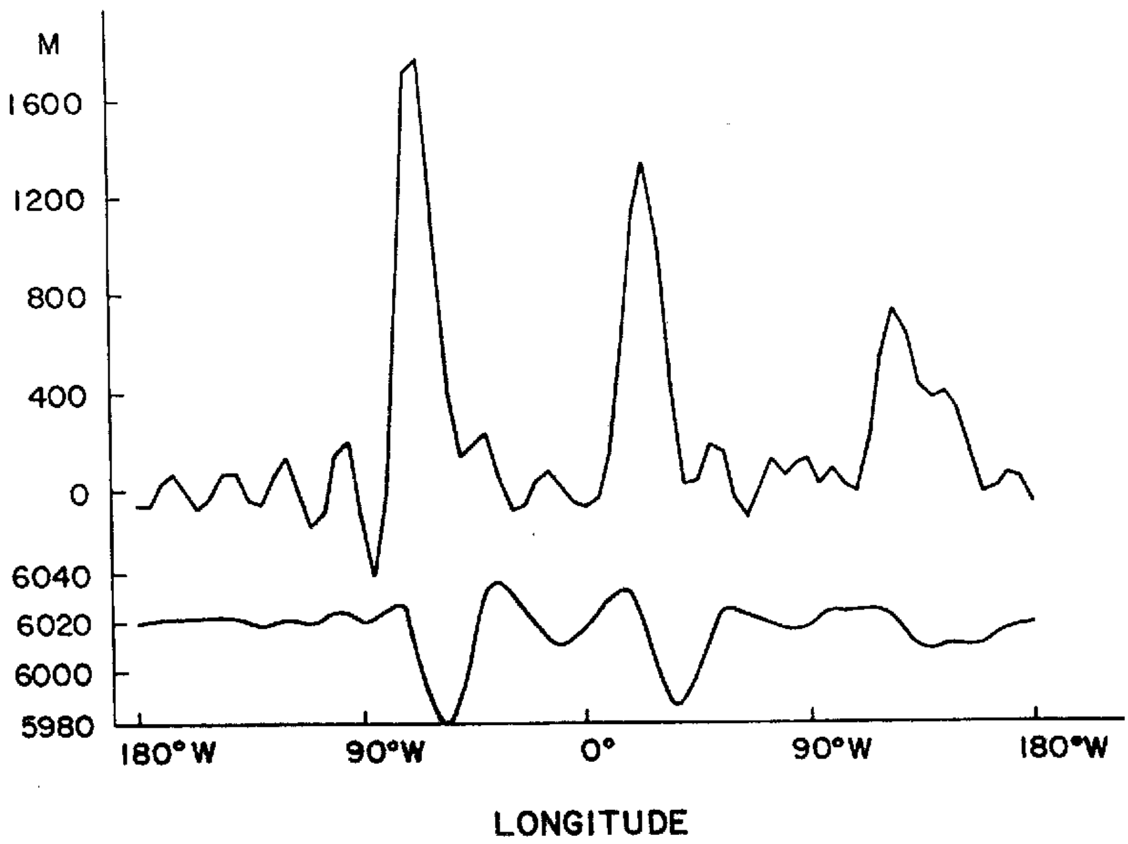
Fig. 5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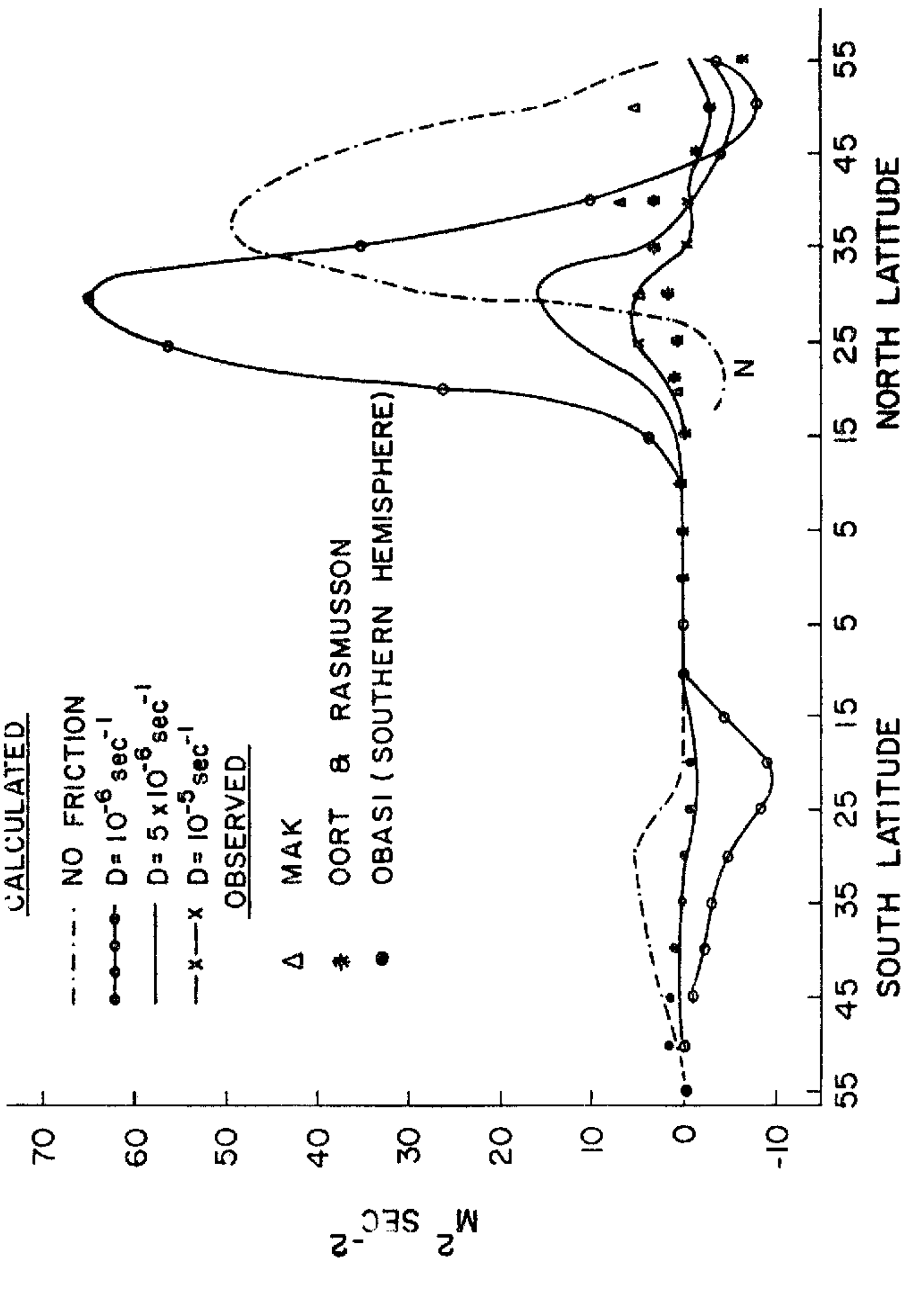
LONGITUDE

Fig. 6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D = 5 x 10-6 sec-1
D = 10-5 sec-1
OBSERVED
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OORT & RASMUSSON
OBASI (SOUTHERN HEMISPHERE)
N
M2 SEC-2
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SOUTH LATITUDE
NORTH LATITUDE

Fig. 7