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*José Augusto Bittencourt*

9. Authorship *Abraham C.-L. Chian*
*Charles F. Kennel*

13. Authorized by

*Marco Antonio Raupp*
*Director General*

Responsible author A. Chian

14. Abstract/Notes

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# RELATIVISTIC ELECTROMAGNETIC WAVES IN AN ELECTRON-ION PLASMA

by

Abraham C.-L. Chian* and Charles F. Kennel

Institute of Geophysics and Planetary Physics
University of California, Los Angeles
Los Angeles, CA 90024, USA

November 1982

*On leave from: Institute for Space Research (INPE/CNPq), C.P. 515,
12200-São José dos Campos, SP, Brazil

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Abraham C.-L. Chian* and Charles F. Kennel

Institute of Geophysics and Planetary Physics
University of California
Los Angeles, California 90024, USA

## ABSTRACT

High power laser beams can drive plasma particles to relativistic energies. An accurate description of strong waves requires the inclusion of ion dynamics in the analysis. The equations governing the propagation of relativistic electromagnetic waves in a cold electron-ion plasma can be reduced to two equations expressing conservation of energy-momentum of the system. The two conservation constants are functions of the plasma stream velocity, the wave velocity, the wave amplitude and the electron-ion mass ratio. The dynamic parameter, expressing electron-ion momentum conservation in the laboratory frame, can be regarded as an adjustable quantity, a suitable choice of which will yield self-consistent solutions when other plasma parameters have been specified. Circularly polarized electromagnetic waves and electrostatic plasma waves are used as illustrations.

## INTRODUCTION

Recently, there has been considerable interest in the subject of strong electromagnetic waves in plasma because of its applications to laser-plasma interaction and pulsar electrodynamics[(1-3)].

*On leave from: Institute for Space Research (INPE/CNPq), C.P. 515, 12200-São José dos Campos, SP, Brazil

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In the presence of an intense laser field, the plasma particles can acquire quiver oscillation energies exceeding particle rest mass energies such that relativistic effects of particle mass variation have to be taken into account[4]. Several studies have demonstrated that relativistic effects are of significant importance in such laser-plasma phenomena as resonant absorption[5], soliton formation[6], self-focusing[7], density profile modification[8] and production of large DC magnetic field[9]

In earlier works on strong traveling waves (see e.g. Ref. 10-12) only the electron motion was considered, whereas the motion of positive ions was ignored. The results obtained thus apply to comparatively small laser intensities for which the ion dynamics is negligible because of its large rest mass. Nevertheless, when a very intense laser field is present, ions can attain considerable quiver velocities. For instance, it has been suggested that the ion dynamics may be important in the later stages of the resonant absorption process[5]; in a recent study of the inverse Faraday effect[13] it was shown that the DC magnetic field induced by a circularly polarized laser can be greatly reduced by the effects of ion motion. Therefore, an accurate description of strong waves requires the inclusion of ion dynamics.

In the theory of relativistic electromagnetic waves in an electron-ion plasma a dynamic parameter, relating electron and ion momenta, necessarily appears. In previous papers (see e.g. Ref. 14), strong wave solutions were obtained by treating the dynamic parameter either as an arbitrary constant or as a plasma drift velocity. This leads to mathematically correct, but physically misleading solutions.

The purpose of this paper is to elucidate the role played by the dynamic parameter in the determination of self-consistent strong wave solutions. It is shown that, the problem can be reduced to two conservation relations for the energy-momentum of the system. In the laboratory frame, the dynamic parameter expresses conservation of electron-ion momenta. In addition, there is an amplitude parameter that expresses conservation of electromagnetic field-particle energy density. These two parameters are functions of the plasma stream velocity, the wave velocity, the wave amplitude and the electron-ion mass ratio. We will conclude that the dynamic parameter must be regarded as an adjustable quantity, equivalent to an eigenvalue of the system, a suitable choice of which will yield self-consistent wave solutions when the value of the amplitude parameter and other plasma parameters have been allocated.

CONSERVATION RELATIONS IN THE LABORATORY FRAME

Consider a cold two-fluid electron-ion plasma. The laboratory frame is chosen to be an arbitrary frame in which a traveling wave propagates with a constant velocity $c\hat{z}/n$ [where c = speed of light, n = index of refraction and $\hat{z} = (o,o,1)$], the plasma moves with a certain stream velocity $\vec{V}_s$ (where $-c < V_s < c$), and the solutions depend on space and time coordinates only through the combination $\theta = t - nz/c$.

The governing equations written in terms of the phase θ are the relativistic equations of motion

$$(1 - nv_{\alpha z}/c) \frac{d}{d\theta} (\gamma_\alpha \vec{v}_\alpha) = \frac{q_\alpha}{m_\alpha} \left( \vec{E} + \frac{\vec{v}_\alpha \times \vec{B}}{c} \right) \quad , \qquad (1)$$

the equations of continuity

$$\frac{dN_\alpha}{d\theta} - \frac{n}{c} \frac{d}{d\theta} (N_\alpha v_{\alpha z}) = 0 \quad , \qquad (2)$$

and Maxwell's equations

$$-\frac{n}{c} \frac{dE_z}{d\theta} = 4\pi e(N_i - N_e) \quad , \qquad (3)$$

$$\frac{dB_z}{d\theta} = 0 \quad , \qquad (4)$$

$$n\hat{z} \times \frac{d\vec{E}}{d\theta} = \frac{d\vec{B}}{d\theta} \quad , \qquad (5)$$

$$-n\hat{z} \times \frac{d\vec{B}}{d\theta} = \frac{d\vec{E}}{d\theta} + 4\pi e(N_i \vec{v}_i - N_e \vec{v}_e) \quad , \qquad (6)$$

where $\gamma_\alpha = (1 - v_\alpha^2/c^2)^{-1/2}$ and $\alpha = (e,i)$.

Conditions describing the average particle number density and flux can be obtained by taking the phase-average of Eqs. (3) and (6), which gives

$$\langle N_e \rangle = \langle N_i \rangle \quad , \qquad (7)$$

$$\langle N_e \vec{v}_e \rangle = \langle N_i \vec{v}_i \rangle \quad , \qquad (8)$$

where the angular bracket denotes averaging over one period in $\theta$. In nonlinear theory, it is convenient to define the plasma stream velocity as the ratio of the average particle flux to the average number density[1-15]. It follows from Eqs. (7) and (8) that the electron and ion streaming velocities are equal

$$\vec{V}_s = \frac{\langle N_e \vec{v}_e \rangle}{\langle N_e \rangle} = \frac{\langle N_i \vec{v}_i \rangle}{\langle N_i \rangle} \quad . \tag{9}$$

A stationary plasma corresponds to the particular case in which $\langle N_\alpha \vec{v}_\alpha \rangle = 0$.

It is easy to show that $\vec{V}_s$, as defined by Eq. (9), Lorentz-transforms like any velocity. Suppose there are two frames S' and S, where S has a velocity $V\hat{z}$ relative to S'. Then, the averaged particle flux and number densities in two frames are related by

$$\langle N\vec{v}_\perp \rangle = \langle N'\vec{v}_\perp' \rangle \ , \ \langle Nv_z \rangle = \Gamma(\langle N'v_z' \rangle - V\langle N' \rangle) \ ; \tag{10}$$

$$\langle N \rangle = \Gamma(\langle N' \rangle - V\langle N'v_z' \rangle/c^2) \quad , \tag{11}$$

where $\perp = (x,y)$ and $\Gamma = (1 - V^2/c^2)^{-\frac{1}{2}}$. Note that phase averaging is a Lorentz-invariant operation. Dividing Eq. (10) by Eq. (11) gives

$$\vec{V}_{s\perp} = \frac{\vec{V}'_{s\perp}}{\Gamma(1 - VV'_{sz}/c^2)} \quad , \quad V_{sz} = \frac{V'_{sz} - V}{1 - VV'_{sz}/c^2} \quad , \tag{12}$$

which indeed satisfies the Lorentz transformation for velocities. This suggests that if in S' the plasma has a certain stream velocity $\vec{V}_s'$, then an observer in S that has a velocity $\vec{V}_s'$ relative to S' will "see" a stationary plasma with $\vec{V}_s = 0$. Hence, the nonlinear dispersion relation for a streaming plasma can be obtained from the nonlinear dispersion relation for a stationary plasma by a Lorentz transformation[12,15,16].

The behavior of electron and ion number densities follows upon integrating Eq. (2), giving

$$N_\alpha = \frac{N^*}{1 - nv_{\alpha z}/c} \tag{13}$$

where $N^* = \langle N_\alpha \rangle \ (1 - nV_{sz}/c)$.

An integration of Eq. (5) gives

$$\vec{B} = n\hat{z} \times \vec{E} + \langle\vec{B}\rangle \quad , \tag{14}$$

where $\langle\vec{B}\rangle$ denotes the magnetostatic field. In this paper an unmagnetized plasma is considered so $\langle\vec{B}\rangle = 0$.

A conservation relation for electron and ion momenta can be obtained from the transverse and longitudinal components of Eq. (1), namely[17],

$$\vec{u}_{i\perp} + \mu\vec{u}_{e\perp} = \vec{D}_{\perp} \quad , \tag{15}$$

$$(u_{iz} - n\gamma_i) + \mu(u_{ez} - n\gamma_e) = D_z \quad , \tag{16}$$

where $\vec{u} = \gamma\vec{v}/c$, $\mu = m_e/m_i$ and $\vec{D}$ is a constant vector. Eqs. (15) and (16) can be combined into a single equation

$$\vec{u}_i + \mu\vec{u}_e - n(\gamma_i + \mu\gamma_e)\hat{z} = \vec{D} \tag{17}$$

Thus, the dynamic parameter $\vec{D}$ expresses conservation of electron-ion momenta. In the absence of a wave, $\vec{v}_e = \vec{v}_i = \vec{V}_s$, Eq. (17) becomes

$$(1 + \mu)\ (1 - V_s^2/c^2)^{-\frac{1}{2}}\ \vec{V}_s - n(1 + \mu)(1 - V_s^2/c^2)^{-\frac{1}{2}}\hat{z} = \vec{D}, \tag{18}$$

which shows that $\vec{D}$ is a function of $\vec{V}_{s\perp}$ n and $\mu$. It will be seen later that for large wave amplitudes $\vec{D}$ also depends on the wave intensity. In linear theory, $\vec{v}_\alpha = \vec{V}_s + \vec{v}_{p\alpha}$ ( where $\vec{v}_{p\alpha}$ denotes small perturbations), Eq. (17) reduces to

$$\vec{v}_{ip} + \mu\vec{v}_{ep} = 0 \quad . \tag{19}$$

Hence, in this limit, $\vec{D}$ is still given by Eq. (18) because the average values of the perturbed particle velocities are zero.

An energy conservation relation can be obtained as follows. A scalar product of Eq. (1) with $\vec{v}_\alpha/c^2$ gives

$$m_e c^2 \frac{d\gamma_e}{d\theta} = -\frac{eN_e}{N^*}\vec{E}\cdot\vec{v}_e \quad , \tag{20}$$

$$m_i c^2 \frac{d\gamma_i}{d\theta} = \frac{eN_i}{N^*}\vec{E}\cdot\vec{v}_i \quad , \tag{21}$$

where Eq. (13) has been applied. Adding Eqs. (20) and (21), then making use of Eqs. (5) and (6), it yields

$$m_e c^2 \frac{d\gamma_e}{d\theta} + m_i c^2 \frac{d\gamma_i}{d\theta} = - \frac{1}{8\pi N^*} \frac{d}{d\theta} (E^2 + B^2) \quad . \qquad (22)$$

An integration of Eq. (22) then yields a conservation relation for the electromagnetic field-particle energy density

$$\frac{E^2 + B^2}{8\pi} + N^* m_e \gamma_e c^2 + N^* m_i \gamma_i c^2 = N^* m_e c^2 W \quad , \qquad (23)$$

where W is a scalar constant. The energy constant W can be considered an amplitude parameter that characterizes the magnitude of wave intensity, as will be shown later. In the absence of a wave, Eq. (23) reduces to

$$W = (1 + 1/\mu)\ (1 - V_s^2/c^2)^{-\frac{1}{2}} \quad . \qquad (24)$$

W must exceed the above value for a wave solution to exist.

## CIRCULARLY POLARIZED ELECTROMAGNETIC WAVES

Purely transverse, circularly polarized electromagnetic waves have been studied extensively (see e.g. Ref. 1, 17-19). For the case of an unmagnetized electron-ion plasma streaming in the wave direction, the dispersion relation in the laboratory frame is

$$n^2 = 1 - \left[ \frac{\omega_{pe}^2}{(1 + \nu^2)^{\frac{1}{2}}} + \frac{\omega_{pi}^2}{(1 + \mu^2\nu^2)^{\frac{1}{2}}} \right] \frac{(1 - V_s^2/c^2)^{\frac{1}{2}}}{\omega^2} \quad , \qquad (25)$$

where $\omega_{p\alpha}^2 = 4\pi\langle N_\alpha\rangle e^2/m_\alpha$ and $\nu = eE_{max}/m_e\omega c$ is an invariant parameter that measures the wave amplitude. Eq. (25) relates the wave velocity, the plasma stream velocity, the electron-ion mass ratio and the wave amplitude. The aim of this paper is to demonstrate that similar dispersion relations for other strong waves can also be obtained through a proper treatment of the conservation parameters W and $\vec{D}$.

## ELECTROSTATIC PLASMA WAVES

As an illustration, consider the case of electrostatic plasma waves. For longitudinal waves, $\vec{u}_\alpha = (0,0,u_\alpha)$ and the two conservation relations Eqs. (17) and (23) reduce, respectively, to

$$(u_i - n\gamma_i) + \mu(u_e - n\gamma_e) = D \quad , \qquad (26)$$

$$\frac{1}{2}\left(\frac{du_e}{d\tau}\right)^2 = \frac{W - \gamma_e - \gamma_i/\mu}{(1 - nu_e/\gamma_e)^2} \quad , \qquad (27)$$

where $\tau^2 = \omega_{pe}{}^2\theta^2(1 - nV_s/c)$. Evidently, Eq. (27) indicates that an oscillatory solution exists, with a period of oscillation given by

$$P = \frac{\sqrt{2}}{\omega_{pe}} \int_{u_1}^{u_2} \frac{1 - nu_e/\gamma_e}{(W - \gamma_e - \gamma_i/\mu)^{\frac{1}{2}}}\, du_e \quad , \qquad (28)$$

where the turning points $u_{1,2}$ are determined by the equation $\gamma_e + \gamma_i/\mu = W$. Note that $\gamma_i$ is related to $u_e$ and $\gamma_e$ through Eq. (26). The general dispersion relation (28) recovers the electron plasma result [12] if the ion term $\gamma_i/\mu$ is ignored.

Before discussing the general solutions of Eq. (28), consider first the phenomenon of wavebreaking which may occur for subluminous waves and is relevant to the laser-plasma interaction[5]. A condition for the occurrence of wavebreaking in the laboratory frame can be obtained as follows. First notice from Eq. (13) that, for a stationary plasma, a traveling wave solution exists (i.e. $0<N_\alpha<\infty$) only if the condition

$$v_\alpha < c/n \qquad (29)$$

is satisfied. Upon transformation from the stationary plasma case to the streaming plasma case[12] the condition (29) then becomes

$$\begin{cases} v_c < v_\alpha < c/n & \text{if} \quad V_s < c/n \\ c/n < v_\alpha < v_c & \text{if} \quad V_s > c/n \end{cases} \quad , \qquad (30)$$

where

$$\frac{v_c}{c} = \frac{1 + V_s{}^2/c^2 - 2nV_s/c}{2V_s/c - n(1 + V_s{}^2/c^2)}$$

Hence, for subluminous longitudinal waves, the electron and ion velocities are subject to the conditions (29) and (30), whose violation implies the occurrence of wavebreaking. A graphical display of Eq. (30) in Fig. 1 shows that, for a given wave velocity,

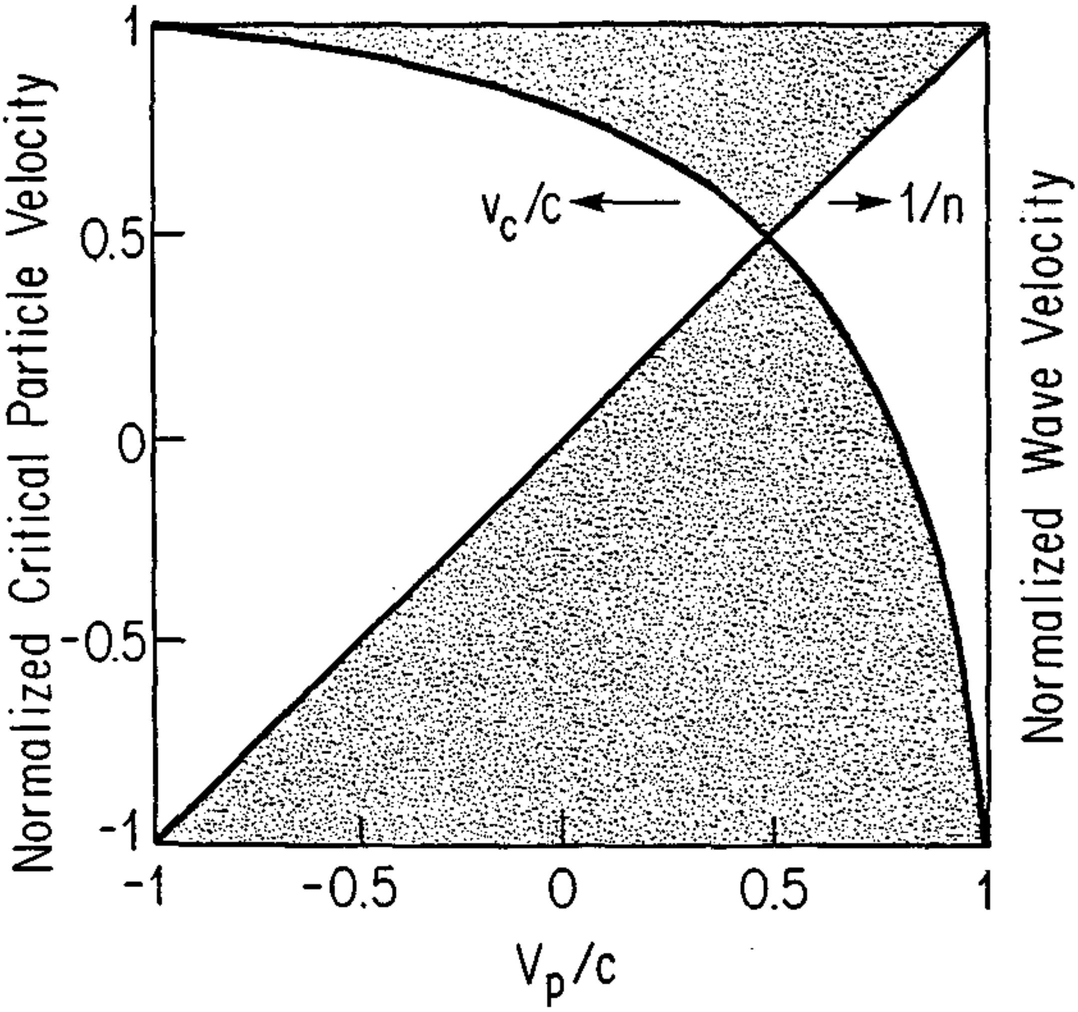


Figure 1. Variation of $v_c/c$ and $1/n$ with normalized wave velocity for $V_s/c = 0.5$. Shaded region indicates wavebreaking.

the range of $v_\alpha/c$ permitted for traveling wave solutions is bounded by the $1/n$ curve and the $v_c/c$ curve. The shaded region represents the domain in which wavebreaking takes place.

The problem of wavebreaking can further be clarified by examining the conservation relations in the wave frame, for which there is no time dependence. In the wave frame, the governing equations become

$$v_\alpha \frac{d}{dz} (\gamma_\alpha v_\alpha) = \frac{q_\alpha}{m_\alpha} E \quad , \tag{31}$$

$$N_e v_e + N_i v_i \equiv N_o V_o = \text{constant} \quad , \tag{32}$$

$$\frac{dE}{dz} = 4\pi e(N_i - N_e) \qquad (33)$$

The above system of equations can be combined to give two conservation relations[20]

$$m_e\gamma_e c^2 + m_i\gamma_i c^2 = m_e c^2 D_1 \quad , \qquad (34)$$

$$N_o V_o m_e \gamma_e v_e + N_o V_o m_i \gamma_i v_i - \frac{E^2}{8\pi} = N_o V_o m_e c W_1 \ . \qquad (35)$$

Eq. (34) corresponds to Eq. (26) in the laboratory frame with $D_I = -(1 - 1/n^2)^{-\frac{1}{2}}D/n$. Therefore, the dynamic parameter expresses

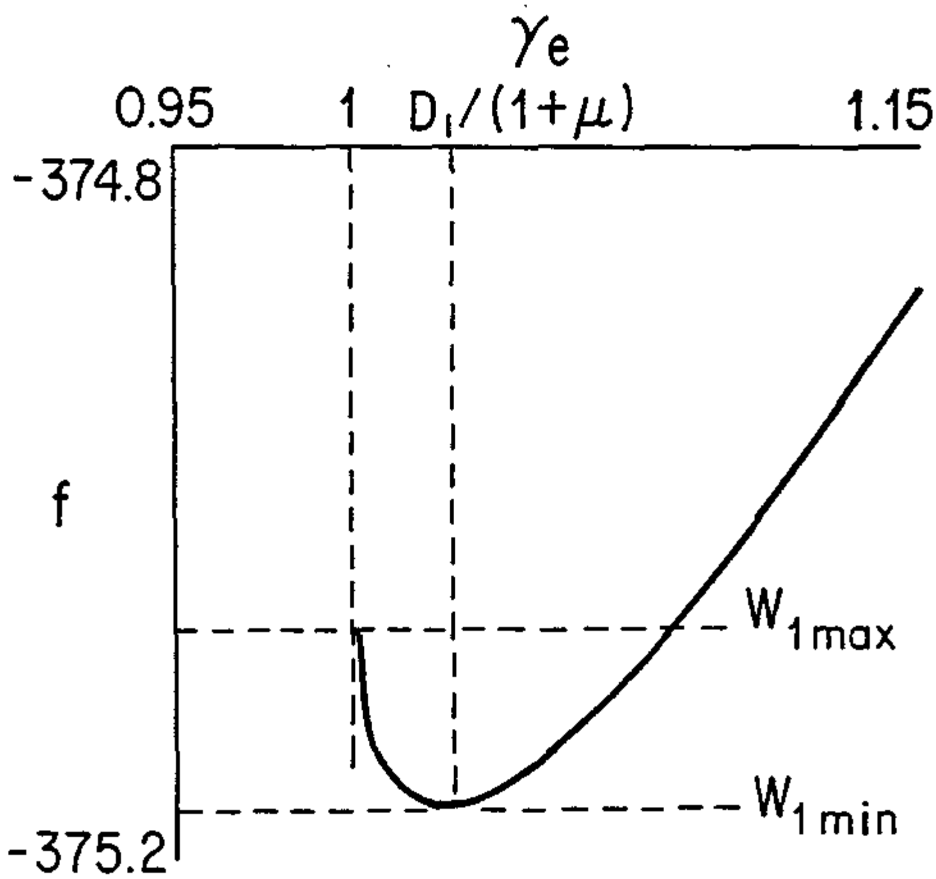


Figure 2. Variation of $f$ with $\gamma_e$ for $D_1 = 1.021$ ( = equilibrium value of $D_1$, $n = 5$, $\mu = 1/1837$, $V_s = 0$).

conservation of electron-ion energy in the wave frame. Eq. (35) corresponds to Eq. (27) in the laboratory frame and can be rewritten as

$$\frac{1}{2}\left(\frac{d\gamma_e}{d\tau}\right)^2 = W_1 - f(\gamma_e) \quad , \tag{36}$$

with

$$f(\gamma_e) = -(\gamma_e{}^2 - 1)^{\frac{1}{2}} - \frac{1}{\mu}\left[(\mu\gamma_e - D_1)^2 - 1\right]^{\frac{1}{2}} \quad , \tag{37}$$

where $\tau_1 = z\omega_{pel}(|V_o|/c^3)^{\frac{1}{2}}$, $\omega_{pel} = 4\pi N_o e^2/m_e$ and $W_1 = -(1 - 1/n^2)^{\frac{1}{2}}W$. Hence, the amplitude parameter expresses conservation of electromagnetic field-particle momentum density in the wave frame. A typical plot of $f$ as a function of $\gamma_e$ is displayed in Fig. 2,

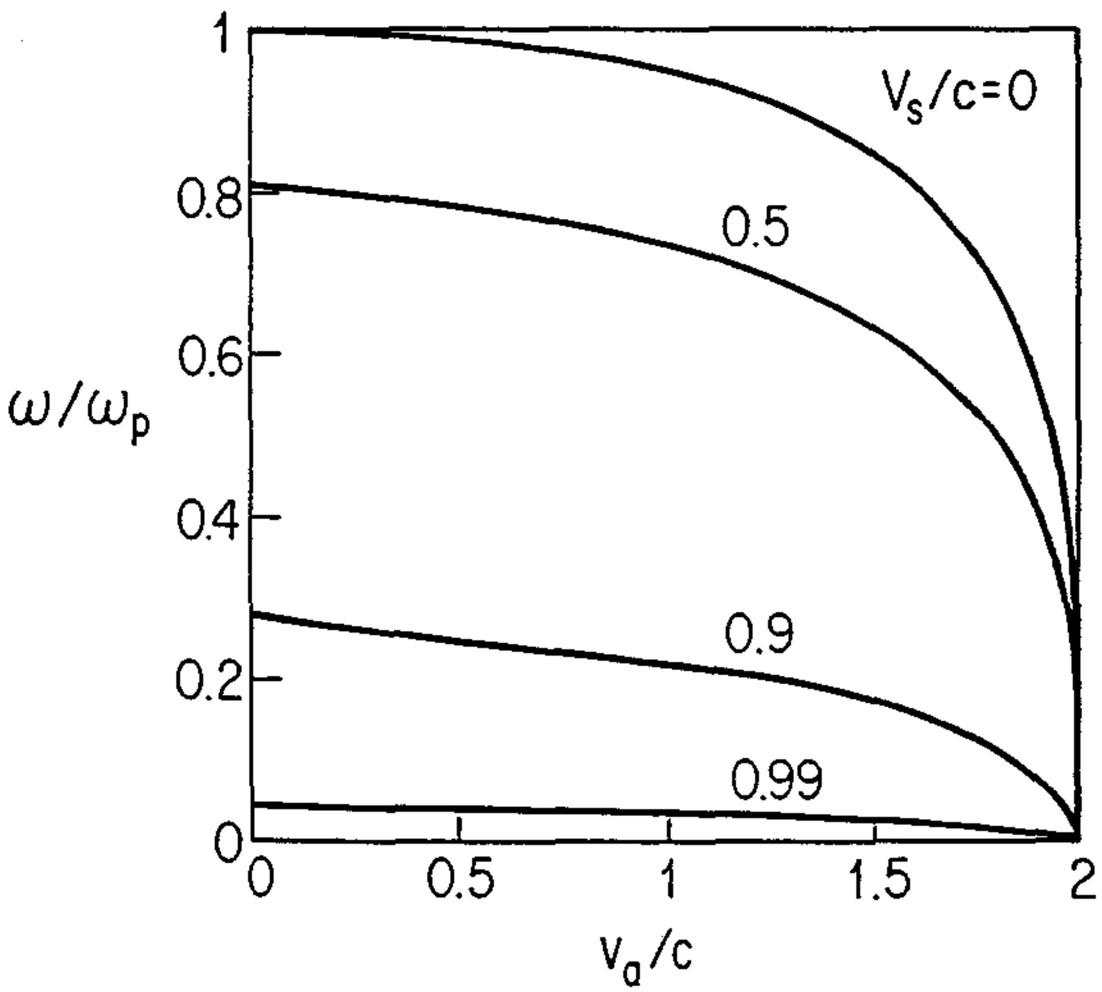


Figure 3. Variation of $\omega/\omega_p$ with $v_a/c$ for indicated values of $V_s/c$; n = 0.

which shows that oscillatory solutions exist only if $W_1$ lies in the interval

$$W_{min} < W < W_{max} \tag{38}$$

where $W_{min} = -[D_1{}^2 - (1 + \mu)^2]^{\frac{1}{2}}/\mu$ and $W_{max} = -[(\mu - D_1)^2 - 1]^{\frac{1}{2}}/\mu$. It is easy to see from Fig. 2 that wavebreaking occurs if $W_1$ exceeds $W_{max}$. Furthermore, it shows that $W_1$ determines the amplitude of the wave oscillations.

The general properties of electrostatic plasma waves in the laboratory frame can be computed from Eq. (28). When evaluating a particular solution it is necessary to decide what values should be assigned to the two conservation parameters W and D. Our wave frame analysis indicates that W can be considered as an amplitude parameter. To obtain a specific solution with given plasma stream velocity, wave velocity, the electron-ion mass ratio and the wave amplitude, the dynamic parameter D must be adjusted accordingly so as to render self-consistency to the solution. Fig. 3 shows the variation of the cut-off frequency $\omega/\omega_p$ (where $\omega_p{}^2 = \omega_{pe}{}^2 + \omega_{pi}{}^2$ and $n = 0$) with $v_\alpha/c$ (where $v_\alpha = |v_1 - v_2|$, $v_{1,2}$ being the turning points of $v_e$ oscillations) for different values of plasma stream velocity. The computations are carried out for a hydrogen plasma ($\mu = 1/1837$). In Fig. 4, the variation of the self-consistent dynamic parameter D with $u_\alpha$ (where $u_\alpha = |u_1 - u_2|$) for a stationary plasma is displayed for different values of n. It confirms that the dynamic parameter is a function of wave amplitude. Note that for small wave amplitudes, D stays very close to the value given by Eq. (18).

## CONSERVATION RELATIONS IN THE SPACE-INDEPENDENT FRAME

For superluminous electromagnetic waves the analysis can be simplified by referring to the space-independent frame which has a velocity $nc\hat{z}$ with respect to the laboratory frame. The basic equations in this frame are

$$\frac{d(\gamma_\alpha \vec{v}_\alpha)}{d\tau} = \frac{q_\alpha}{m_\alpha} \vec{E} \quad , \tag{39}$$

$$N_e = N_i \equiv N = \text{constant} \quad , \tag{40}$$

$$\vec{B} = \text{constant} \equiv 0 \quad , \tag{41}$$

$$\frac{d\vec{E}}{d\tau} + 4\pi Ne(\vec{v}_i - \vec{v}_e) = 0 \quad . \tag{42}$$

The above system of equations can be combined to yield two conser-

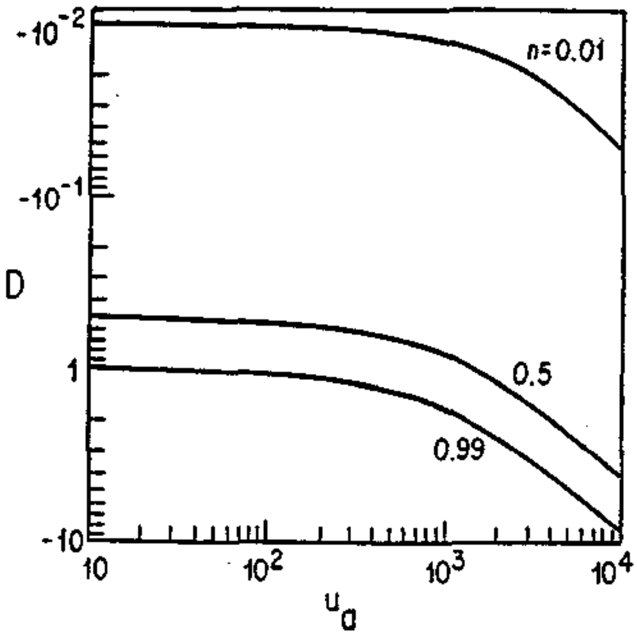


Figure 4. Variation of D with $u_a$ for indicated values of n; $V_s = 0$.

vation relations[18]

$$\begin{cases} m_e \gamma_e \vec{v}_e + m_i \gamma_i \vec{v}_i = m_e c\, \vec{D}_2 \quad , & (43) \\ \dfrac{E^2}{8\pi} + Nm_e \gamma_e c^2 + Nm_i \gamma_i c^2 = Nm_e c^2 W_2 \quad . & (44) \end{cases}$$

Eq. (43) corresponds to Eq. (17) in the laboratory frame. Upon transforming to the laboratory frame, the transverse components of Eq. (43) gives Eq. (15) with $\vec{D}_{2\perp} = \vec{D}_\perp$, while the longitudinal component of Eq. (43) gives Eq. (16) with $D_{2z} = (1 - n^2)^{-1/2} D_z$. Hence the dynamic parameter expresses conservation of electron–ion momentum in the space-independent frame. Eq. (44) corresponds to Eq. (25) in the laboratory frame and can be rewritten as

$$\frac{1}{2} \left( \frac{d\vec{u}_e}{d\tau_2} \right)^2 + \gamma_e + \gamma_i/\mu = W_2 \quad , \qquad (45)$$

where $\tau_2 = \omega_{pe2}t$, $\omega_{pe2}^2 = 4\pi Ne^2/m_e$ and $W_2 = (1 - n^2)^{\frac{1}{2}}W$. Therefore, the amplitude parameter expresses conservation of electromagnetic field-particle energy density in the space-independent frame. For the special case of $\vec{D}_2 = D_2\hat{z}$, $W_2$ must exceed $[D_2{}^2 + (1 + \mu)^2]^{\frac{1}{2}}/\mu$ for a solution to exist[18]. An elegant formulation of the energy-momentum conservation relations in terms of the Maxwell's stress tensor in the space-independent frame can be found in Ref. 21.

The two energy-momentum relations (43) and (45) can be used to study all possible wave solutions. As in the case of electrostatic plasma oscillations, a proper handling of the conservation constants, $\vec{D}_2$ and $W_2$, is essential for obtaining self-consistent solutions.

The case of circularly polarized electromagnetic waves serves to demonstrate the correct behavior of the two conservation constants. For simplicity, consider the plasma to be stationary in the laboratory frame (i.e., $\vec{V}_s = 0$). For circularly polarized waves, $\gamma_e$ and $\gamma_i$ are constants given, in the space-independent frame, by[18]

$$\gamma_e = \left(\frac{1 + \nu^2}{1 - n^2}\right)^{\frac{1}{2}} \quad , \quad \gamma_i = \left(\frac{1 + \mu^2\nu^2}{1 - n^2}\right)^{\frac{1}{2}} \tag{46}$$

where $\nu = eE_{max}/m_e\omega c$ as defined in Eq. (25). Substitution of Eq. (46) into Eqs. (43) and (45), respectively, yields

$$\begin{cases} \vec{D}_2 = -n(1 - n^2)^{-\frac{1}{2}}[(1 + \mu^2\nu^2)^{\frac{1}{2}} + \mu(1 + \nu^2)^{\frac{1}{2}}]\hat{z} \quad , & (47) \\ W_2 = \nu^2/2 + (1 - n^2)^{-\frac{1}{2}}[(1 + \nu^2)^{\frac{1}{2}} + (1 + \mu^2\nu^2)^{\frac{1}{2}}/\mu] \ . & (48) \end{cases}$$

Note that $\vec{D}_{2\perp} = 0$ and $(d\vec{u}_e/d\tau_2)^2 = \nu^2$ for circularly polarized waves. Eqs. (47) and (48) show clearly that the energy-momentum conservation parameters are functions of the wave velocity, the wave amplitude and the electron-ion mass ratio. In addition, from Eqs. (47) and (48) one obtains the following relation

$$\vec{D}_2 = \mu n(\nu^2/2 - W_2)\hat{z} \quad , \tag{49}$$

which shows explicitly that for a given amplitude parameter $W_2$ there is only one value of the dynamic parameter $\vec{D}_2$ that satisfies self-consistent solutions.

## CONCLUSION

It has been shown that the equations governing the propagation of relativistic electromagnetic waves in an electron-ion plasma can be reduced to two energy-momentum conservation relations. The two conservation constants, the amplitude parameter and the dynamic parameter, respectively, are functions of the plasma stream velocity, the wave velocity, the electron-ion mass ratio and the wave intensity. In order to obtain self-consistent solutions the dynamic parameter must be chosen appropriately when other parameters have been specified.

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RELATIVISTIC ELECTROMAGNETIC WAVES IN AN ELECTRON-ION PLASMA

**IDENTIFICAÇÃO**

**AUTOR(ES)**

Abraham C.-L. Chian
Charles F. Kennel

**ORIENTADOR**

**CO-ORIENTADOR**

**DISS. OU TESE**

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# University of Illinois at Urbana-Champaign

**Fusion Studies Laboratory**

NUCLEAR ENGINEERING PROGRAM
214 Nuclear Engineering Laboratory
103 S. Goodwin Avenue
Urbana, Illinois 61801
(217) 333-3772

August 3, 1982

Nelson de Jesus Parada, Director General
INPE
C.P. 515
12200 - Sao Jose dos Campos - SP
BRAZIL

Reference: "Relativistic Electromagnetic Waves..."; Log No. 163A

Dear Dr. Parada:

I am pleased to inform you that the Program Committee for the Sixth International Workshop on Laser Interaction and Related Plasma Phenomena have recommended your submission for presentation at the Workshop. They also recommend that the full-length manuscript be considered for publication in the book-type proceedings published by Plenum Press.

Your abstract will be included in the abstract booklet to be distributed at the meeting; a preliminary program indicating the time and session for your oral presentation will be mailed to you shortly. Instructions for the preparation of the full manuscript will also be sent to you in the near future. However, to help you begin planning for this full length manuscript, I would note that a camera-ready format is required with a maximum of 15 pages in the book-type proceedings, equivalent to roughly 45 pages (8 1/2" x 11" page) double-spaced, typewritten copy, including figures, tables and references. The completed manuscript is to be hand carried to the workshop.

For your convenience, an additional registration form is enclosed. You should complete it and also make appropriate room reservations at your earliest convenience.

I am looking forward to seeing you in Monterey in October. We hope to organize the sessions in such a way that the exchange of new ideas can be encouraged in the spirit of a true workshop.

Sincerely,

George H. Miley
Co-Workshop Director

GHM:je

# University of Illinois at Urbana-Champaign

**Fusion Studies Laboratory**

NUCLEAR ENGINEERING PROGRAM
214 Nuclear Engineering Laboratory
103 S. Goodwin Avenue
Urbana, Illinois 61801
(217) 333-3772


July 16, 1982

Nelson de Jesus Parada, Director General
INPE
C.P. 515
12200 - Sao Jose dos Campos - SP
Brazil

Dear Dr. Parada:

The abstract titled "Relativistic Electromagnetic Waves in an Electron-Ion Plasma" submitted to the Sixth Workshop on Laser Interaction and Related Plasma Phenomena has been received. It has been assigned Log No. 163A, which should be used for identification in any further correspondence. Notification of the decision of the program committee on this will be sent to you approximately 1 August. Your interest in the workshop is appreciated.

Sincerely,

George H. Miley
Workshop Director

GHM:cs

DAD
DTE

São José dos Campos, July 07, 1982

Ref.: 30.100.000.1056-82

Prof. George H. Miley
Fusion Studies Laboratory
University of Illinois
214 Nuclear Engineering Laboratory
103 South Goodwin Avenue
Urbana, Illinois 61801
USA

Dear Prof. Miley:

I am submiting a paper by Abraham C.-L. Chian of our Institute entitled "Relativistic Electromagnetic Waves in an Election-Ion Plasma" to be presented in the Sixth International Workshop on Laser Interaction and Related Phenomena to be held in Monterey in October 1982.

With best regards

Yours sincerely,

original assinado por
NELSON DE JESUS PARADA
Nelson de Jesus Parada
Director General

Mail correspondence to:

INPE
C.P. 515
12200 - São José dos Campos - SP
Brasil

ACLC/zl

# RELATIVISTIC ELECTROMAGNETIC WAVES IN AN ELECTRON-ION PLASMA

Abraham C.-L. Chian

Instituto de Pesquisas Espaciais - INPE
Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq
C.P. 515 - 12200 São José dos Campos, SP, Brazil

Strong electromagnetic waves capable of driving plasma particles to relativistic energies are of considerable interest due to applications for laser fusion. Recent works[1,2] have demonstrated the importance of including ion dynamics in the theoretical analysis of relativistic waves. In the model of electron-ion plasma a dynamic parameter, relating electron and ion velocities, necessarily appears. Previous authors obtained relativistic wave solutions by considering the dynamic parameter either as an arbitrary constant or as a drift velocity of the plasma. This leads to mathematically correct, nonetheless unphysical, solutions.

The purpose of this paper is to elucidate the role played by the dynamic parameter in the determination of self-consistent relativistic wave solutions. By relating the momentum conservation equation in the space-independent frame to that in the laboratory frame, it is shown that the dynamic parameter is a function not only of the plasma stream velocity as considered by previous authors, but also of the wave velocity, the wave amplitude and the electron-ion mass ratio. It is concluded that the dynamic parameter must be regarded as an adjustable quantity, equivalent to an eigenvalue of the system, a suitable choice of which will yield self-consistent solutions when the values of other parameters have been allocated. The relevance of this result to laser fusion will be discussed.

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