Dealing with covering problems in fuzzy rule systems by similarity-based extrapolation

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Abstract - In the present paper we propose a solution for overcoming possible lacks of covering of the input space in fuzzy rule bases. Our approach is based on similarity-based reasoning and considers a kind of extrapolative inference rule which enlarges the range of applicability of a fuzzy rule by replacing conditions in the premise of the form "X is A" by "X is *approximately_A*", where *approximately_A* is the image of A by a suitable fuzzy similarity relation.

I. INTRODUCTION

The term *fuzzy rule based system*, FRS for short, is a common designation to systems that make use of a knowledge base KB consisting of a set of rules of the kind "If X is A_i then Y is B_i", where both the A_i's and B_i's are fuzzy sets [2], [7], together with an approximate inference mechanism. Ideally, such systems should be capable to produce, for any given input A', a corresponding global output B' that is useful, in the sense that B' is neither the empty set nor the whole universe of discourse of the output variable.

In [3], fuzzy rules are basically classified as conjunctive or implicative-based, depending on the kind of if-then operator employed to define, from the fuzzy sets appearing in the premise and in the conclusion, the fuzzy relation induced by each rule. If we think of a fuzzy rule-based system as modeling an imprecise description of a graph, the two models of rules respectively correspond to two possible ways of specifying an imprecise graph: either as a disjunction of fuzzy points $\cup_{i\in I} A_i \times B_i$, or as a conjunction of fuzzy implications $\cap_{i\in I} A_i \to B_i$. Therefore, conjunctive rule-based systems, widely used in real-world applications, use a t-norm (i.e. a conjunction operator), such as min or product, in order to implement the if-then operator and a t-conorm (i.e. a disjunction operator), usually the max operator, to aggregate the output issued by the rules fired by a given input. On the other hand, implicative-based systems use truly implication operators (as opposed to conjunction operators) to implement the if-then operator and a t-norm, usually the min operator, to aggregate the outputs. Also in [3], the authors provide a typology for conjunction and implication-based rules based on their different semantics.

Here we adopt the following basic terminology, regarding some characteristics that a fuzzy rule-based system may present:

- A FRS *covers* the input space if for each precise input $X = x_0$ there is at least one fuzzy rule "If X is A_i then Y is B_i" for which $A_i(x_0) > 0$. In implicative systems, this amounts to guarantee, under general conditions, that outputs will never be the whole universe of Y, whereas in conjunctive this amounts to guarantee that we never get an empty output.
- A FRS is (potentially) *inconsistent* if there exists an input such that the rules fired by that input produce completely "conflicting" outputs (this corresponds to the notion of *coherence* in [4], [5]). An implicative system may easily get into inconsistency problems as soon as an input can simultaneously fire rules with incompatible conclusions (i.e. with empty intersection). In contrast, due to their different nature, conjunctive systems are never inconsistent from a logical point of view, however, it can be argued that the inference of non-convex outputs can be considered as a kind of inconsistency (see e.g. [5], [8]).

According to this terminology, a FRS may present at least two different kinds of defficiencies; one regarding the covering of the input space, and the other concerning the consistency. Although similar, solutions to these problems are of different nature. Namely, in what regards consistency in implication-based systems, given an inconsistent FRS, the task is to find another consistent (and less informative) FRS' as close as possible to FRS. On the other hand, solving the covering problem consists of completing knowledge: given a FRS in which the input

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space is not fully covered, the task is to find a FRS', as close as possible to FRS, that covers all the input space. When both problems are present in a given application they can be solved by first addressing the covering problem, resulting in a new (and possibly inconsistent) FRS', and then restoring the consistency of FRS', if needed.

In a companion paper [6] we present an approach to restore consistency for a class of implication-based fuzzy rules known as *gradual rules* [3], corresponding to statements of the form "the more X is A_i, the more Y is B_i", using the characterization of consistency conditions given by Dubois, Prade and Ughetto in [4], [5]. Our approach involves the use of similarity relations to resolve potential inconsistencies among rules by weakening them. Namely, the solution proposed amounts to applying a weakening inference rule of the form

$$\frac{\text{If } X \text{ is } A \text{ then } Y \text{ is } B}{\text{If } X \text{ is } A \text{ then } Y \text{ is ``}\textit{approximately\_B}\text{''}} : [WK]$$

to some of (or all) the rules in a rule base KB and replacing the original ones by the new derived weaker rules. Obviously, such a transformation is sound in the sense that, from a logical point of view, the new rule base KB^* can be considered as a logical consequence of the original one, i.e. we could write $KB \models KB^*$, since KB^* is less informative than KB.

In the present paper we propose a solution for the covering problem, also based on similarity reasoning. Namely, when the rule base does not fully cover the input space what we propose is to consider a kind of extrapolative inference rule of the form

$$\frac{\text{If } X \text{ is } A \text{ then } Y \text{ is } B}{\text{If } X \text{ is ``}\textit{approximately\_A}\text{'' then } Y \text{ is } B} : [EX]$$

or, alternatively, an even intuitively better rule of the form

$$\frac{\text{If } X \text{ is } A \text{ then } Y \text{ is } B}{\begin{array}{l}\text{If } X \text{ is ``}\textit{approximately\_A}\text{''} \\ \text{then } Y \text{ is ``}\textit{approximately\_B}\text{''}\end{array}} : [EX'].$$

Such inference rules are indeed extrapolative mechanisms since they enlarge the range of applicability of the original rule, and of course, they are not logically sound rules in contrast to the above inference rule $[WK]$. So, in this case, the new rule base KB^*, obtained from a original KB by applying the inference rules $[EX]$ or $[EX']$ to some of the rules in KB, is indeed more informative than KB, and thus it cannot be just a logical consequence of KB, but of KB together with some extra domain knowledge D. Then we could write $KB \not\models KB^*$ but $\{KB, D\} \models KB^*$.

The paper is organized as follows. In Section II we describe how to extrapolate a set of rules in order to cover a given set of uncovered input points. This mechanism is used in Section III to outline a method to solve the covering problem for a FRS. The extrapolation process is applied over specific subsets of rules, which can be different depending on the set of input points to be covered. How to select these subset of rules is described in Section IV.

II. COVERING BY SIMILARITY-BASED RULE SET EXTRAPOLATION

Assume we have a set of rules

$$K = \{R_j : \text{If } X \text{ is } A_j \text{ then } Y \text{ is } B_j\}_{j \in J},$$

where, in the general case, $X = (X_1, \ldots, X_n)$ is a multidimensional variable taking values on a domain $\Omega = \Omega_1 \times \ldots \times \Omega_n$ and $A_j = (A_{j1}, \ldots, A_{jn})$, with A_{ji} being a fuzzy sets of Ω_i.

The set of points of the input domain Ω uncovered by K is the set

$$\mathcal{U}_K = \{(w_1, \ldots, w_n) \mid \forall j \in J, \exists i : 1 \leq i \leq n, A_{ji}(w_i) = 0\}.$$

Let us consider a set of uncovered points $\emptyset \neq \mathbf{U} \subseteq \mathcal{U}_K$. To solve the covering problem of K relative to the region $\mathbf{U}$ means to produce another rule set K^*, by applying the extrapolative mechanism [EX'] to the rules in K, such that it covers $\mathbf{U}$ and which is as *close* to K as possible, i.e. KB^* contains as less extra knowledge as possible in relation to KB.

What the inference rule [EX'] does is to transform a fuzzy rule by relaxing the premise conditions of the kind "X is A" into conditions "X is *approximately_A*". Within our similarity-based approach, if A is a fuzzy set defined on a domain Ω, we interpret *approximately_A* as the image of A by some similarity relation S on Ω, $S \circ A$, defined as the the sup-min composition

$$(S \circ A)(w) = \sup_{w' \in \Omega} \min(S(w, w'), A(w')).$$

Then, the above extrapolation rule [EX'] can be rewritten as

$$\frac{\text{If } X \text{ is } A \text{ then } Y \text{ is } B}{\text{If } X \text{ is } S_X \circ A \text{ then } Y \text{ is } S_Y \circ B} : [EX']$$

for some suitable similarity relations S_X and S_Y on the input and output domains respectively.

The set of similarity relations on a domain Ω form a lattice (not linearly ordered) with respect to the pointwise ordering (or fuzzy-set inclusion) relationship. The top of the lattice is the similarity $S_\top$ which makes all the elements in the domain maximally similar: $S_\top(v, v') = 1$ for all $v, v' \in U_Y$. The bottom of the lattice $S_\bot$ is the classical identity relation: $S_\perp(v,v') = 1$ if $v = v'$, $S_\perp(v,v') = 0$, otherwise. The higher a similarity is placed in the lattice (i.e. the bigger are their values), the less discriminating it is.

From a knowledge representation point of view, it is clear that the bigger is a similarity relation S the more imprecise is the set $A_j^* = S \circ A_j$, and the stronger is a rule

$$R_j^*\text{: "If } X \text{ is } A_j^* \text{ then } Y \text{ is } B_j\text{".}$$

Thus, since we are interested in introducing as less additional knowledge as possible when replacing the rules, we are interested in using a similarity S as small as possible. On the other hand, the bigger the similarity S, the larger the region covered by the rule R_j^*, so the bigger is S the better is KB^* from the covering problem point of view. Notice that if $S \leq S'$, then if R_j^* covers a region, so will $R_j^{*'}$. Moreover, the trivial solutions do not help at all: if we take $S = S_\top$, R_j^* will for sure cover all the domain but it will be completely useless; if we take the $S = S_\perp$, of course, there is no information gain since $R_j^* = R_j$, but the covering problem will remain. Therefore, optimal solutions would be the smallest S's for which R_j^* covers an uncovered region.

Therefore, the above aim of finding a K^* as close as possible to K and covering a region $\mathbf{U}$, is to be understood as finding the smallest similarity relations S_X^j, S_Y^j such that

$$K^* = \{R_j^* : \text{If } X \text{ is } S_X^j \circ A_j \text{ then } Y \text{ is } S_Y^j \circ B_j\}_{j \in J}$$

covers a given uncovered region $\mathbf{U}$, i.e. such that $\mathbf{U}$ be included in the support of *some* $S_X^j \circ A_j$'s (at least one!).

As far as we were only concerned with the covering problem, to simplify the problem by taking similarities S_Y^j's to be just the classical identity relation, so from now on we shall only be concerned with similarities on the input domain. Nevertheless the above covering problem is still non-trivial. As a fisrt step, that will become clear in the next sections, we restrict ourselves now to the following *extrapolation* problem:

- given am uncovered region $\mathbf{U}$, find a smallest similarity relation S_X on Ω such that

  $$K^* = \{R_j^* : \text{If } X \text{ is } S_X \circ A_j \text{ then } Y \text{ is } B_j\}_{j \in J}$$

  fully covers $\mathbf{U}$ in the sense that $\mathbf{U}$ be included in the support of *each* $S_X \circ A_j$'s

Moreover, to be pragmatic, we shall address this problem with respect to what we call *covering nested families* of similarity relations. By this we mean a parametric family $\mathcal{S} = \{S_0, S_{+\infty}\} \cup \{S_\lambda\}_{\lambda \in I \subseteq (0,+\infty)}$ of similarity relations on Ω such that:

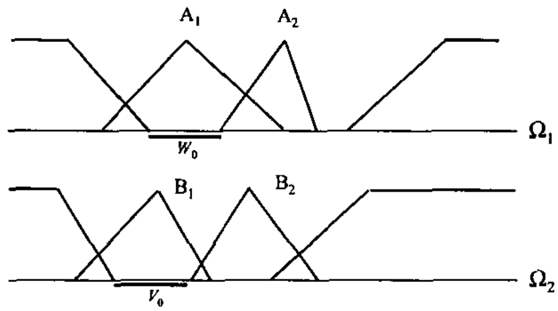

Fig. 1. Partition of input spaces.

(i) $S_0 = S^\perp$,
(ii) $S_{+\infty} = S^\top$, and
(iii) $\lambda < \lambda'$, then $S_\lambda \prec S_{\lambda'}$.

Here $S \prec S'$ means $S(x,y) \leq S'(x,y)$ for all $x, y \in U_Y$ and $S(x_0,y_0) < S'(x_0,y_0)$ for some $x_0, y_0 \in U_Y$.

Therefore, given a parametric family $\{S\}_\lambda$ our extrapolation problem is reduced to find the smallest λ such that $\mathbf{U}_k \subseteq$ *support*$(S_\lambda \circ A_j)$ for all $j \in J$.

Example. For an easier understanding, let us consider the following example. Let K be composed of the following two rules:

R_1: If X_1 *is* A_1 and X_2 *is* B_2 then Y *is* C_1
R_2: If X_1 *is* A_2 and X_2 *is* B_1 then Y *is* C_2

where the domains of variables X_1 and X_2 are $\Omega_1 = \Omega_2 = \mathbb{R}$, and the fuzzy sets involved are depicted in Figure 1. This set of rules has obvious problems of covering, in particular, consider the uncovered region $\mathbf{U} = (W_0, V_0)$ also shown in Figure 1. So, let us solve the extrapolation problem of K relative to the region $\mathbf{U}$. For this consider the following simple parametric family $\mathcal{S}$ of similarity relations: for each $\lambda > 0$ define

$$S_\lambda(x,y) = \mu_\lambda(|x-y|),$$

where

$$\mu_\lambda(z) = \max(1 - \lambda^{-1} \cdot z, 0).$$

One can easily check that if A is a trapezoidal fuzzy number $[a,b,c,d]$, then $S_\lambda \circ B_i$ is again a trapezoidal fuzzy number defined by the 4-tuple $[a-\lambda, b, c, d+\lambda]$.

Then the approach consists of replacing rules R_1 and R_2 respectively by the new rules

R_1^*: If X_1 *is* A_1^* and X_2 *is* B_1^* then Y *is* C_1
R_2^*: If X_1 *is* A_2^* and X_2 *is* B_2^* then Y *is* C_2

where $A_i^* = S_{\lambda_0} \circ A_i$ and $B_i^* = S_{\delta_0} \circ B_i$, for $i = 1, 2$, with

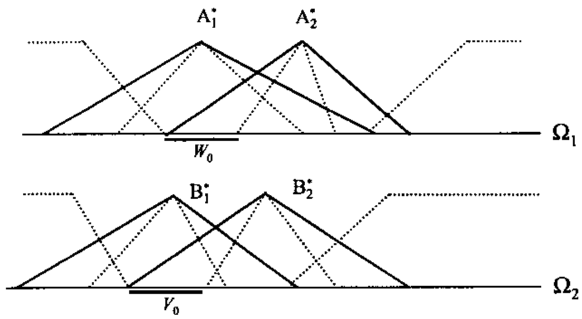

Fig. 2. Extrapolated terms of the input spaces covering the region (W_0, V_0).

$$\lambda_0 = \inf\{\lambda \mid S_\lambda \circ A_1 \supseteq W_0, S_\lambda \circ A_2 \supseteq W_0\},$$
$$\delta_0 = \inf\{\delta \mid S_\delta \circ B_1 \supseteq V_0, S_\delta \circ B_2 \supseteq V_0\},$$

That is, we enlarge the fuzzy sets A_i's and B_i's as much as necessary for their supports to cover the regions W_0 and V_0 respectively. As it can be easily checked, in this case, we get $\lambda_0 = length(W_0)$ and $\delta_0 = length(V_0)$, and the produced extrapolated fuzzy sets are the ones shown in Figure 2.

III. THE COVERING PROBLEM

The covering problem in a FRS, i.e., what do we do when there does not exist any rule in the rule base KB of FRS whose premise addresses a given input vector X^*, may occur in at least two deffective situations. In a first case, there may be "holes" in the input space of a given linguistic variable, i.e., regions not covered by the union of the supports of all the terms associated with that variable [1]. The other case occurs when, even if there are no holes in the partitions, for a given input vector X^* there exists no rule in the KB whose premise covers it. In this work we mainly address this latter problem since the first one can be considered as a particular case of the second one.

A related problem concerns how should a solution to the covering problem affect outputs of inputs already covered by the original KB. On the one hand, when a general solution is employed, e.g. by stretching (extrapolating) all the terms so that all possible inputs are covered, it is possible that the new KB produces, for a given input vector covered by rules in the original KB, a distinct output than the one produced by the original KB. To avoid this side effect, if $\mathcal{U}_{KB} = \cup_{k=1,m}\mathbf{U}_k$ is the total uncovered region by KB, a local solution can be used in which the original KB is employed for the regions already covered by it and a new specific KB_i is created for each uncovered region $\mathbf{U}_i$ of the input space.

[1] A term is considered to be associated with a linguistic variable when there exists at least one rule in the KB in which that term is employed as the value of that variable.

In the remaining of t his section we outline a method to the covering problem following this local approach, present a simple framework on which it can be applied and show how to determine the uncovered regions considering that framework.

A. *Method proposed*

Let us suppose the premises of the rules of the original KB involve n variables $X_1, \ldots, X_n$, each variable X_k taking values in a domain Ω_k and let $\mathcal{U}_{KB}$ be the total set of uncovered points by KB. Notice that $\mathcal{U}_{KB}$ can be put as

$$\mathcal{U}_{KB} = \cup_{i=1,m}\mathbf{U}_i$$

where each $\mathbf{U}_i$ is a cartesian product of uncovered subregions in each subdomain Ω_k. Let us denote an uncovered region $\mathbf{U}_i$ by a tuple $(U_i^1, ..., U_i^n)$, where each U_i^k is an interval in Ω_k, such that there is no rule that covers all the U_i^k's simultaneously.

The method consists in the following two main steps:

i) Given an uncovered multidimensional region $\mathbf{U}_i$ in the input space, find a specific rule sub-base KB_i whose range of applicability is the "closest" one to $\mathbf{U}_i$.

ii) Extrapolate the rules in KB_i by means of a suitable similarity relation(s) in such a way that the "extrapolated" rule base KB_i^* yields a valid output for any input inside the region $\mathbf{U}_i$.

In Section IV we describe how to deal with i), while the process of extrapolation addressed in ii) is just the one described in the previous section.

B. *Working framework*

Before detailing the method in the next sections we introduce some of the definitions we shall use in the remaining of this document.

- Let T be a fuzzy set in Ω with membership function $\mu_T(.)$. The *support* and the *core* of a term T are respectively defined as $supp(T) = \{\omega \mid \mu_T(\omega) > 0\}$ and $core(T) = \{\omega \mid \mu_T(\omega) = 1\}$. Let $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be 2 intervals in Ω. Then $hull(I_1, I_2) = [min(a_1, a_2), max(b_1, b_2)]$ is the convex hull of I_1 and I_2.
- A fuzzy set T on the real numbers scale is said to be a *fuzzy number* if it is normalized (i.e., $\exists \omega \in \Omega, \mu_T(\omega) = 1$), unimodal, upper semi-continuous, and has a bounded support [7].

For the sake of clarity, in this work we propose implementations of the method in the context of a very simple framework. More complex implementations may however be designed for any larger framework.

- The terms associated with a linguistic variable are distinct fuzzy numbers in *consecutive order*. We say that a set of terms $\{T_1, \ldots, T_n\}$ is in consecutive order when for all i, if $supp(T_i) \cap supp(T_{i-1}) \neq \emptyset$ and $supp(D_i) \cap supp(T_{i+1}) \neq \emptyset$, then $supp(T_i) \cap supp(T_j) = \emptyset, \forall j \notin \{i-1, i+1\}$.
- If T and T' are consecutive terms associated with a linguistic *input variable* defined on Ω then, $\mu_T(\omega) + \mu_{T'}(\omega) \leq 1$ for all $\omega \in supp(T) \cap supp(T')$. This amounts to say that the terms of an input variable may at most form a Ruspini's fuzzy partition of the domain of the variable.

C. *Determination of uncovered regions*

Considering our simple working framework, each possibly uncovered region on the domain of variable X_k can be described as:

- $F_k^{>i} = [supp(T_{i-1}) \cap supp(T_i)] \cup [hull(supp(T_{i-1}), supp(T_i)) - supp(T_{i-1}) - supp(T_i)]$,
- $F_k^{<i>} = supp(T_i) - supp(T_{i-1}) - supp(T_{i+1})$,

where each $T_j, j \in \{i-1, i, i+1\}$, is a term associated with variable X_k. When $F_k^{>i} \neq supp(T_{i-1}) \cap supp(T_i) = \emptyset$ then $F_k^{>i}$ is said to be a *hole* in the partition of the domain Ω_k. For example, region W_0 in Figure 1 corresponds to $F_k^{<1>}$ and region W_1 in Figure 3 corresponds to a hole $F_k^{>2}$.

A multidimensional region $\mathbf{U} = (..., U^k, ...)$, $U^k \in \{F_k^{>i}, F_k^{<i>}\}$ will be said to be *uncovered* in the following cases:

- For some k, U^k is a hole in the partition of domain Ω_k.
- There does not exist a rule

  R_j : If X_1 is T_{j1} and ...X_n is T_{jn} then Y is D_j

  such that $U^k \subseteq supp(T_{jk})$, for $k = 1, n$.

IV. DETERMINATION OF RULES CLOSE TO UNCOVERED REGIONS

In this section, given a rule base KB, we describe first how to determine the set of rules KB_i that are most closely related to each uncovered region U_i, according to a metric-like criterium, in the particular case in which the partitions contain no holes. Then we propose a similarity-based to solve the problem of partitions presenting holes.

A. *Creation of KB_i's for non-hole uncovered regions*

In the case in which the fuzzy terms related to each input variable are such that if T and T' are consecutive terms then $0 < \mu_T(\omega) + \mu_{T'}(\omega) \leq 1$ for all $\omega \in supp(T) \cap supp(T')$, (i.e. the partition has no "holes"), a reasonable closeness measure consists in taking the highest number of variables X_k covered simultaneously by each rule in KB. More formally, let $\mathcal{U} = \{\mathbf{U_1}, ..., \mathbf{U_p}\}$ be the set of uncovered multidimensional regions. We define

$$c_i = max_{j=1,n} c_{ij}$$

where c_{ij} denotes the number of intervals from $\mathbf{U_i}$ covered by terms in rule R_j. Then we define $KB_i = \{R_j \mid c_{ij} = c_i\}$ as the set of rules most closely related to the uncovered region $\mathbf{U_i}$.

For instance, let us suppose we have the set of rules:

R_1 : If X_1 is A_1 and X_2 is B_1 and X_3 is C_1 then Y is D_1
R_2 : If X_1 is A_2 and X_2 is B_2 and X_3 is C_2 then Y is D_2
R_3 : If X_1 is A_3 and X_2 is B_2 and X_3 is C_2 then Y is D_3

and a region $\mathbf{U_i}$ such that U_i^1 is covered by A_1 but not from $A_2 \cup A_3$, U_i^2 is covered by B_2 but not from B_1, and U_i^3 is covered by C_2 but not from C_1. Then we have $c_{i1} = 1$, and $c_{i2} = d_{i3} = 2$. Therefore, $c_i = 2$ and $KB_i = \{R_2, R_3\}$.

If there are holes in the partitions of the input space, the above approach can be inadequate, particularly when an uncovered region $\mathbf{U_i}$ is composed solely of holes. In that case, we would get as KB_i the whole rule base KB. An alternative approach to deal with this situation uses similarity relations and is presented in Section V.

B. *Dealing with holes in the input partitions*

Similarity relations can be used to deal with input partitions having holes by ranking rules according to how close they are (under a suitable metric notion) to an uncovered region U under concern. Then we can extrapolate that (those) rule(s) which are closer.

For instance, assume the input space partitions are now the ones depicted in Figure 3 and consider the uncovered region $\mathbf{U} = (W_1, V_1)$. Let $\mathcal{S}^1$ and $\mathcal{S}^2$ be two covering nested families of similarity relations on Ω_1 and Ω_2 respectively, for each rule R_j in the KB, we compute, as above,

$$\lambda_j = \inf\{\lambda \mid supp(S^1_\lambda \circ A_j) \supseteq W_1\},$$
$$\delta_j = \inf\{\delta \mid supp(S^2_\delta \circ B_j) \supseteq V_1\},$$

Intuitively, the larger the λ_j and δ_j, the more we need to enlarge the fuzzy sets A_j and B_j to cover the regions

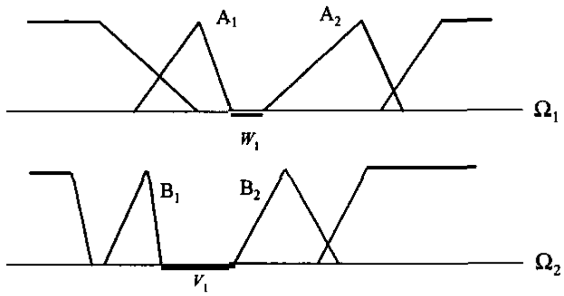

Fig. 3. Partition of input spaces.

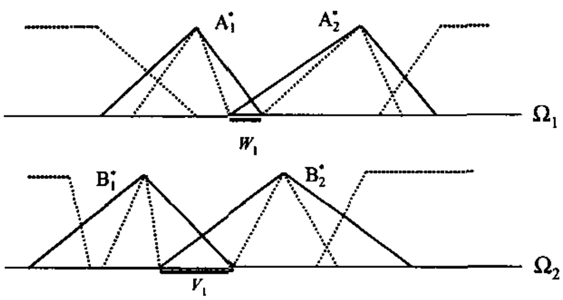

Fig. 4. Extrapolated terms of the input spaces covering the region (W_1, V_1).

W_1 and V_1 respectively. Therefore we can define a distance measure between the region **U** and the rule R_j as

$$d(\mathbf{U}, R_j) = h(\langle \lambda_j, \delta_j \rangle)$$

for some suitable combination function h of the scalar vector $\langle \lambda_j, \delta_j \rangle$, the choice of which may depend on the problem context. Obvious candidates are aggregation functions, like weighted means or more sophisticated operators, or even a vector ranking method based on well-known criteria like Pareto (partial order), lexi-min (total order) or similar.

Then, we get a ranking of the closest rules to region **U**, from where we select a set of rules to be finally extrapolated. Again, we can consider different criteria to select this set, for instance, just the optimal ones, or the k-closest ones for some natural k to be specified.

Following with the previous example, and assuming the rules R_1, R_2 are in the rule base, then it is clear that both have equal distance to (W_1, V_1) since $\lambda_1 = \lambda_2 = length(W_1)$ and $\delta_1 = \delta_2 = length(V_1)$. In that case, the extrapolated fuzzy sets would be the ones depicted in Figure 4.

V. FUTURE WORK

When for a given input not only at least one but two fuzzy gradual rules are required to be applied, then the covering problem in that case is obviously related to interpolation methods in sparse rule bases. Interpolation mechanisms in fuzzy rule systems have already received considerable attention in the literature (see for instance [1] for a comparative survey and the references listed there). We plan to check how our approach works in an interpolation problem setting and compare it with other existing approaches.

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