

OPTIMAL RENDEZVOUS WITH CONSTRAINTS ON ORBITS

Vasile Istratie

National Institute for Aerospace Research  
77538 Bucharest, Romania

This work studies the optimum rendezvous with minimum fuel consumption two space vehicles on circular and elliptical orbits, the surveyor vehicle being equipped with a low thrust installation, their motion equations being written in the three dimensional space, in relative motion, the space origin being the target vehicle. The equations system (the general motion equations) contains 6 differential equations of the state variables: coordinates (x, z, y) and velocities (Vx, Vz, Vy), with initial and final conditions, for circular orbits, 7 differential equations of the state variables: coordinates (x, z, y), velocities (Vx, Vz, Vy) and phi - elliptical anomaly, or elliptical orbits, with initial conditions and final only for coordinates and velocities.

The variational problem is of Lagrange type and the optimum controls to be determined being the acceleration due to thrust ax, az, ay, related to the minimum fuel consumption,  $J = \int_0^{t_f} (a_x^2 + a_z^2 + a_y^2) dt$ , (t\_f - the final time) This

controls are impose at constraints  $|a_x| \leq a, |a_z| \leq a, |a_y| \leq a$ , where for orbits in around Earth,  $a = 10^{-3} g$ , g - the gravity acceleration. By means of the Legendre-Clebsch condition it is demonstrated that the formulated optimization problem is, indeed, a maximum problem. The final time is considered to be imposed. The optimization of this problem is solved applying the Pontryagin maximum principle. So, the above defined problem of optimal control is transformed in a well-known into a two point boundary problem. The nonlinear differential equations of the extremals where any kind of approximation was eliminated are precisely integrated by a numerical method, shooting. So, this method becomes applicable for any kind of elliptical and circular orbit of the target.

The calculations were performed both for circular (e = 0) and elliptical (e = .05) orbits around the Earth, e - being eccentricity of target orbit. The calculations performed using based on the non-linear theory presented in this work show that the differences between the results obtained for circular orbits and the elliptical ones are small if the eccentricity of them are also small.

The initial value of the target real anomaly does not significantly influence the results even in case the target reaches the proximity of the orbit apogee.

COMPARISON OF THERMOSPHERIC DENSITY MODELS BY SATELLITE ORBITAL DECAY ANALYSIS

C. Pardini & L. Anselmo

CNUE/CNR, Area della Ricerca di Pisa, Via Alfieri 1,  
Loc. San Cataldo, 56010 Ghezzano - Pisa - Italy

Over the years, the air density at satellite altitudes was accurately represented by various atmospheric density models like Jacchia-Roberts 1971 [JR71], Thermosphere Density Model 1988 [TD88] and Mass Spectrometer Incoherent Scatter models [MSIS86 and MSIS90].

The aim of the semi-empirical models is to represent a large quantity of data collected by various techniques and to describe thermospheric parameter variations. Those of Jacchia are based mainly on total density data derived from satellite orbital decay observations. The Mass Spectrometer Incoherent Scatter models are derived from combining satellite in-situ composition measurements and thermospheric temperatures inferred from ground-based incoherent scatter radars. A common feature of all these models is the use of relatively simplified physical concepts to describe the atmospheric density variations as a function of altitude, solar and geomagnetic activity, latitude, longitude, local time and day of the year. But significant differences exist between models, they being constructed using different data, covering different time periods and geographic areas. Another intrinsic limitation of each model includes the use of the 10.7 cm solar flux (F10.7) and the geomagnetic planetary indices (Kp, Ap) as proxy indicators to represent solar and geophysical influences on the atmosphere.

Reducing semi-empirical model errors has proven to be very difficult, but the accuracy of each model can be evaluated through the analysis of the orbital decay of satellites with known dimensions, mass, attitude and orbital state evolution. Following this approach, we performed a comparison of semi-empirical air density models as a function of altitude and variable environmental conditions. The satellite sample and the time interval examined encompassed a broad range of altitudes (200-1500 km) and solar activity levels (1987-2000).

MSIS-86 and MSISE-90, practically identical above 200 km, resulted to be the best models to compute the air density below 400 km, in low solar activity conditions. JR-71 seemed to be more precise at greater altitudes and/or solar fluxes. TD-88 gave quite mixed performances, but generally closer to the JR-71 results. The intrinsic accuracy of JR-71, MSIS-86 and MSISE-90 was generally better than 20%, and sometimes close to 10%. This picture resulted progressively degraded at altitudes greater than 400 km.

An Evaluation of the Jacchia and MSIS 90 Atmospheric models with CBERS data.

Polamraju Rajendra Prasad, Hélio Koiti Kinga  
Space Mechanics and Control Division  
Instituto Nacional de Pesquisas Espaciais - INPE  
Av. dos Astronautas, 1758-São José dos Campos-SP  
CEP-1227-010, BRAZIL

Abstract

"CBERS" (China Brazil Earth Resources Satellite) was put into orbit on 14<sup>th</sup> of October 1999 at 03:28:52 UTC. The most difficult satellite perturbations to model are those arising from surface forces: these include air drag and radiation pressure forces. The sensitivity to drag is a consequence of the stringent ground track control requirement. The atmospheric drag is the major non-gravitational perturbation affecting the control of ground track. Atmospheric drag causes continuous decay in the orbit semi-major axis. The decay rate is the function of satellite physical parameters and atmospheric density. As accuracy requirements increase, greater reliance is placed on empirical techniques. Nonetheless the different drag models have little impact on the results during the low solar activity period. During the initial operational phase of CBERS, the solar activity has been at its peak. This phenomenon has provided fairly a realistic opportunity to carryout an evaluation of atmospheric density models. In this study the emphasis is put on two commonly used atmospheric density models viz. Jacchia and Mass Spectrometer Incoherent Scatter (MSIS). The evaluation is carried out based on the decay histories of CBERS to assess the modeling of the total atmospheric density. Density models used for orbit propagation are usually derived empirically from actual flight data. One of the problems with analyzing the CBERS orbit solution results is the very sparse tracking, which makes it difficult to discriminate precisely between the two different models. Brief synopsis of some of the models is presented along with some of the density tables and orbit solutions of CBERS along with some of the plots. The study indicates that the drag estimation for polar orbits is relatively precise using MSIS model. MSIS-90 is analyzed to be a better compromise in terms of accuracy, flexibility and computational aspects. The analysis would be useful in mitigating the impact of solar activity maximum on orbit prediction and orbit maintenance.

AN ANALYTICAL DESCRIPTION OF THE CLOSE APPROACH MANEUVER IN THREE DIMENSIONS

Antonio Fernando Bertachini de Almeida Prado  
Instituto Nacional de Pesquisas Espaciais - INPE - Brazil  
São José dos Campos - SP - 12227-010 - Brazil  
Phone (55-12)3456201 - Fax (55-12)3456226  
E-mail: PRADO@DEM.INPE.BR

In the present paper a description of the close approach maneuver is made in the three-dimensional space. Analytical equations are derived to calculate the variation in velocity, angular momentum, energy and inclination of the spacecraft that realizes this maneuver. From the general equations derived it is possible to obtain expressions for particular cases, like the planar and the polar maneuver.

For the mathematical model it is assumed that the system is formed by two main bodies that are in circular orbits around their center of mass and a massless third body that is moving under the gravitational attraction of the two primaries. Its motion is not constrained to the orbital plane of the two primaries and the main goal of this research is to quantify the effects of the third dimension in this maneuver. The standard canonical system of units is used and it implies that the unit of distance is the distance between the two primaries and the unit of time is chosen such that the period of the orbit of the two primaries is 2π.

Several properties are derived and demonstrated. The most interesting ones are: i) for the planar maneuver the variation in inclination can have only three possible values: 180° and -180°, for a maneuver that reverse the sense of its motion, or 0° for maneuver that does not reverse its motion. Those results are explained by the physical model, since a planar maneuver does not allow values for the inclination other than 0° or 180°. ii) The change in inclination goes close to zero at the poles. Then, in the case where the variation in inclination is Δi = 180° or -180°, the change in inclination starts in zero at the poles, increases in magnitude until the planar maneuver is reached and then it starts decreasing again until zero at the other pole. When Δi = 0° for the planar maneuver, the behavior of Δi oscillates with two maximum for the magnitude and three zeros. The variation in inclination is symmetric with respect to the out of plane angle. iii) A passage by the poles keeps the inclination of the trajectory almost unchanged. iv) When α, the angle between the perapsis line and the line connecting the two primaries is zero or 180° there is no change in the inclination. This is in agreement with the fact that a maneuver with this geometry does not change the trajectory at all.