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BATCH AND SEQUENTIAL DATA SMOOTHING AND COMPRESSION BY CURVE FITTING USING AND ADAPTIVE STATE ESTIMATION TECHNIQUE

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12. Revised by

Roberto V. Fonseca Lopes

9. Authorship: Valcir Orlando, Atair Rios Neto, Hélio Koiti Kuga

Responsible author

13. Authorized by

Marco Antonio Raupp
Director General

14. Abstract/Notes

The problem of data smoothing and compression by curve fitting is formulated such as to allow the application of Kalman filtering and a state noise estimation technique to its solution. This leads to a procedure that can be used to process the data either in batch or sequentially. The capability of sequential processing opens the possibility or curve fitting in real time. Tests done with digitally simulated satellite tracking data indicate that the procedure has a satisfactory performance, being a potentially promising tool for data preprocessing.

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1 - INTRODUCTION

In the handling and analysis of data obtained from measurements, data smoothing by curve fitting procedures are commonly used with one or more of the following objectives: (i) elimination of high frequency measurement noise to evaluate the behaviour of the physical phenomenon under observation; (ii) data compression to reduce the amount of data to be stored, processed or transmitted; (iii) interpolation of intermediate values, as in the case when it is necessary to synchronize data from several sources.

The curve fitting procedures more frequently used are based on Least Square or Chi-Square criteria (Wertz, 1978; Meyer, 1975; Bendat and Piersol, 1971). Usually the linear case is considered and the curve selected to approximate the behavior of the observed variable is taken as a linear combination of known orthogonal functions of time. If the choice of the fitting curve is done properly, the result is to be a good approximation along time of the true occurred values of the variable being observed, that is, the influence of unbiased measurement errors is averaged out. The Chi-Square procedure, under the necessary assumption of gaussian measurement errors, also gives an approximation of the error in the smoothed values. For measurement errors having a Gaussian distribution the Least Square procedure is equivalent to the Chi-Square procedure. The Least-Square procedure, however, applies to any measurement errors distribution.

A common characteristic of existing procedures is that all data points have to be collected before starting the curve fitting, thus eliminating the possibility of real time processing. In what follows, the curve fitting problem is treated such as to make feasible the use of Kalman filtering (e.g., Jazwinski, 1970; Sorenson, 1966) combined with a technique of state noise adaptive estimation (Jazwinski, 1969; Rios Neto and Kuga, 1985). This leads

to a procedure that besides giving estimates for the fitting error also has the feature of allowing the sequential processing of data. This characteristic together with an adequate choice of the fitting curve opens the possibility of doing the fitting in real time.

The use of the procedure in the case of preprocessing data from low altitude satellites (Orlando, 1983) showed encouraging results. Polynomial fitting was adopted and the tests were conducted with digital computer simulation of range data from the orbit of the TD-1A satellite (ESA, 1977).

In the following sections, the paper is organized starting with the data smoothing by curve fitting problem presentation, in section 2, followed by the procedure description in section 3, the testing results in section 4 and the conclusions in section 5.

2 - PROBLEM PRESENTATION

A set of measurements $y(k)$ of a physical variable, $y_r(k)$, in discrete times t_k, is given by:

$$y(k) = y_r(k) + v(k) \quad , \qquad k = 1,2,\ldots,n \quad , \tag{1}$$

where $v(k)$ is the measurement noise, assumed zero mean Gaussian with:

$$E\,[v(k)v(j)] = R(k)\delta_{kj} \quad , \tag{2}$$

where δ_{kj} is the Kronecker delta function and $E[.]$ represents the expectation operator.

The problem at hand is that of fitting to the given data points a curve of the form:

$$f(\underline{x},t) = x_1 g_1(t) + x_2 g_2(t) + \ldots + x_n g_n(t) \ , \qquad (3)$$

where $g_j(t)$, $j = 1,2,\ldots,n$ are known orthogonal functions; x_j are real coefficients to be determined in the fitting process; and $\underline{x}=[x_1\ x_2\ \cdots\ x_n]^T$. Assuming $f(\underline{x},t)$ to be, for a certain set $\underline{x}$, a good approximation for $y_r(t)$ in the interval $[t_1, t_m]$, the problem posed is that of solving for $\underline{x}$, in the equation:

$$y(k) = f(\underline{x}, k) + v(k) \quad , \quad k = 1,2,\ldots,m \ . \qquad (4)$$

3 - PROPOSED PROCEDURE

3.1 - BATCH PROCESSING

In this case, the problem of determining the vector of coefficients $\underline{x}$ in equation (4) is characterized as one of estimating the state of a single stage linear dynamic process, as follows:

$$\underline{x}\ (1) = \underline{x}\ (0) \ + \ \underline{w}(0) \quad , \qquad (5)$$

where $\underline{w}(0)$ expresses the uncertainties in the mathematical modelling adopted and is taken as a vector of zero mean Gaussian noise with

$$E\ [\underline{w}(0)\underline{w}^T(0)] = \underline{Q}(0) = \text{diag.}[q_i(0),\ i = 1,2,\ldots,n] \ ,$$

$\underline{x}(0)$ is a random vector that approximates the coefficients, with statistics given by:

$$E[\underline{x}(0)/Y(0)] = \hat{\underline{x}}(0/0) \ , \qquad (6)$$

$$E\{[\underline{x}(0) - \hat{\underline{x}}(0/0)][\underline{x}(0) - \hat{\underline{x}}(0/0)]^T/Y(0)\} = \underline{P}(0/0) \ , \qquad (7)$$

where $\hat{\underline{x}}(0/0)$ and $\underline{P}(0/0)$ are the conditional mean and covariance respectively; and $Y(0)$ means all past information used in

establishing the conditioned a priori information of $\underline{\hat{x}}$ (0/0) and $\underline{P}(0/0)$.

The observations to be processed are taken as measurements of the response of the dynamical process equation (5) and are modeled by:

$$\underline{Y}(1) = \underline{M}(1)\ \underline{x}(1) + \underline{V}(1) \quad , \tag{8}$$

where:

$$\underline{Y}^T(1) \triangleq [y(1) \quad y(2) \quad \ldots \quad y(m)] \tag{9}$$

is the vector of measured values along the instants $t_1, t_2, \ldots, t_m$; $\underline{M}(1) \triangleq [M_{ij} \triangleq g_j(t_i)]$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$ is the matrix of the functions for m measurements, assuming in equation (1) that the curve $f(\underline{x},t)$ is in the form taken in equation (3); and

$$\underline{V}^T(1) \triangleq [v(1) \quad v(2) \quad \ldots \quad v(m)] \tag{10}$$

is the vector of zero mean measurement noises with:

$$E\ [\underline{V}(1)\underline{V}^T(1)] = \underline{R}(1) = \text{diag}\ [R(k),\ k = 1,2,\ldots,m]. \tag{11}$$

With the representation adopted the problem of curve fitting has been reduced to one of estimating the state of a dynamic process. The optimal solution is thus a Kalman filtering given by:

$$\underline{\hat{x}}(1/1) = \underline{\hat{x}}(1/0) + \underline{K}(1)\ [\underline{Y}(1) - \underline{M}(1)\ \underline{\hat{x}}(1/0)] \quad , \tag{12}$$

$$\underline{P}(1/1) = [\underline{I} - \underline{K}(1)\ \underline{M}(1)]\ \underline{P}(1/0) \quad , \tag{13}$$

$$\underline{\hat{x}}(1/0) = \underline{\hat{x}}(0/0) \ , \quad \underline{P}(1/0) = \underline{P}(0/0) + \underline{Q}(0) \ , \tag{14}$$

$$\underline{K}(1) = \underline{P}(1/0)\ \underline{M}^T(1)[\underline{M}(1)\ \underline{P}(1/0)\ \underline{M}^T(1) + \underline{R}(1)]^{-1} \ , \tag{15}$$

where the state noise covariance matrix, $\underline{Q}(0)$, is estimated using the procedure proposed by Rios Neto and Kuga (1985), summarized in the Appendix.

Notice that if the vector of observations is processed at once, the determination of the Kalman gain implies a m x m matrix inversion. However, since the noises in the observations are uncorrelated random variables, these observations can be processed one by one, thus avoiding the need of matrix inversion (Sorenson, 1966). Also notice that the matrix of covariance errors in $\underline{\hat{Y}}(1)$ is estimated as:

$$\underline{P}_{\hat{y}} \triangleq E\{[\underline{Y}(1) - \underline{\hat{Y}}(1)][\underline{Y}(1) - \underline{\hat{Y}}(1)]^T\} = \underline{M}(1)\ \underline{P}(1/1)\ \underline{M}^T(1) \ , \tag{16}$$

where

$$\underline{\hat{Y}}(1) = \underline{M}(1)\ \underline{\hat{x}}(1/1) \ .$$

3.2 - SEQUENTIAL PROCESSING

If now the curve fitting problem is characterized as one of estimating the state of a multistage linear dynamic process, where each stage is associated with a particular instant of measurement, it results:

$$\underline{x}(k+1) = \underline{x}(k) + \underline{w}(k) \ , \tag{17}$$

$$y(k+1) = \underline{m}(k+1)\underline{x}(k+1) + v(k+1) \ , \tag{18}$$

$$\underline{m}(k+1) \triangleq [g_1(k+1) \quad g_2(k+1) \ \ldots \ g_n(k+1)] \ , \tag{19}$$

where K = 0,1,2,...,m-1 represents the discretization times t_k.

The problem is thus reduced to the case of estimating the state of a discrete linear dynamic system, and the Kalman filtering algorithm of equations (11)-(14) applies directly by replacing 1 by k and 0 by k-1.

To characterize this dynamic system the following assumptions are made: (i) the curve adopted in equation (18) is assumed to have a set of parameters that give a good fitting approximation in the interval of measurements, $[t_1, t_m]$; (ii) It is also assumed that this happens outside this interval for a virtual set of data that, for any real time, complements, on the left and on the right, the measured data. With these assumptions it is then reasonable to consider that the model of equation (17) is only an approximation and a bias exists. In other words, when including one additional data point in the fitting process, the corresponding occurred coefficient variations may be bias type variations, which was not considered in the adopted coefficients model (Equation 17). The model that includes the bias is given by:

$$\underline{x}(k+1) = \underline{x}(k) + \underline{\psi}(k) + \underline{w}(k) \quad , \tag{19}$$

where $\underline{\psi}(k)$ represents the bias. This bias is certainly uniformily bounded and the system of equations (17), (18) is uniformly completely observable and uniformly completely controllable (Orlando, 1983), when the set of complementary virtual data is considered. Then, the true error variance in the estimates $\underline{\hat{x}}(k)$ is bounded by the filter computed error variance when the model of equations (17) and (18) is used, as long as conservative values are adopted for P(0/0), Q(k) and R(k) (Jazwinski, 1970).

3.3 - CALIBRATION OF PARAMETERS AND INITIAL GUESSES

The polynomial curve fitting case is assumed, and it is also assumed, either by previous experience or by previous simulation, that there is available enough knowledge of the phenomenon to evaluate in advance the appropriate curve degree, n, to be used for $f(\underline{x},t)$, in equation (3). In a post facto basis, the correctness of the degree choice can always be checked over by analysing the observation residues. Along the fitting, monitoring can be done by analysing the filter estimated variances of the errors in the estimates of the vector of coefficients, $\underline{x}$.

Both in the batch and sequential case the initial estimates for $\hat{\underline{x}}(0/0)$, $\underline{Q}(0)$ and $\underline{P}(0/0)$ are taken as:

$$\hat{x}_1(0/0) = y(1),\ \hat{x}_j(0/0) = 0\ ,\quad j = 2,\ldots n\ , \tag{20}$$

$$\underline{P}(0/0) = \text{diag}\ (p_i(0/0)\ ,\ i = 1,2,\ldots,n)\ ,$$

$$\underline{P}^q(0/0) = \text{diag}\ (p_i^q(0/0) = p_i^2(0/0)\ ,\ i = 1,2,\ldots,n)\ , \tag{21}$$

$$\underline{Q}(0/0) = 0\ ,$$

where $\underline{Q}(0/0)$ is the a priori value assumed for $\underline{Q}(0)$; the meaning of $\underline{P}^q(0/0)$ is defined in the Appendix and the values of the diagonal elements $p_i(0/0)$ are taken as:

$$P_1(0/0) = R(1)\ , \tag{22}$$

$$P_j(0/0) = \{\sigma_2/[\Delta T^{(j-2)}]\}^2\ ,\quad j = 2,\ldots,n \tag{23}$$

where, in the case of batch processing or nonreal time sequential processing,

$$\sigma_2 = \max_{\{i:i = 2,\ldots,n\}} |[y(i) - y(1)]/(t_i - t_1)| \qquad (24)$$

and ΔT is the value of $t_i - t_1$ in equation (24) such that the value of $|y(i) - y(1)|$ is maximum in the fitting interval.

In the case of real time sequential processing a σ_2 value may be guessed based on previous knowledge of time variation of the physical phenomenom under measurement.

Clearly the initial values given by equations (20)-(24) are conservative and intended to guarantee the filter convergence and to force $f(\underline{\bar{x}},t)$ to behave as a convergent series. To reinforce this characteristic by the adaptive noise procedure (described in the appendix), one estimates only the first two diagonal terms, q_1 and q_2, of the state noise matrix ($\underline{Q}(0)$ in equation (14) for batch processing and $\underline{Q}(k)$ for sequential processing). For the other terms one takes:

$$\bar{q}_i = \bar{q}_2 / [\Delta T^{2(i-2)}] \quad , \qquad i = 3,\ldots,n \qquad (25)$$

in the case of having the curve degree, n, greater than two. For sequential processing this can give a significant reduction in the processing time. In this case, the adaptive noise procedure is applied every time that a data point is processed, in the kalman filter update step. Thus, in each step, independently of state dimension the adaptive noise procedure only estimates the noise parameters q_1 and q_2 related to the first two state vector terms. The other terms are calculated by equation (25).

4 - PROCEDURE TESTS

The sequential and batch procedures were both tested using simulated artificial satellite range tracking data. The range

data was calculated from simulation of the TD-1A satellite orbit (ESA, 1977), considering a fictitious tracking station and a one second sample rate. To each data was added a Gaussian random error with standard deviation of 50 meters. The initial conditions for curve coefficients estimation were taken as described in section 3.2.

Figure 1 presents the simulated range data curve for an interval of 40 seconds. In Figure 2 the results obtained using the batch fitting procedure and fourth degree time power polynomial series to 40 simulated data points are presented. In this figure the following parameters were superimposed: (i) the real error of the fitted curve; (ii) the limits given by estimated errors (the square root of the diagonal terms of $\underset{\sim}{P}_{\underline{y}}$ matrix defined in equation (16)); (iii) the discrete values of the randon error added to each simulated range data along the data simulation process. All error values presented were normalized with respect to the standard deviation of the random error in the simulated data: σ = 50 m.

One can see in Figure 2 that the real error of the fitted curve stays within the limits given by estimated error and that the later remains smaller than one standard deviation of random error in simulated data. One can also see that the limits given by estimated errors are greater in the beginning and end of the fitting time interval. This happens due to the asymmetrical treatment of the data in the curve fitting procedures.

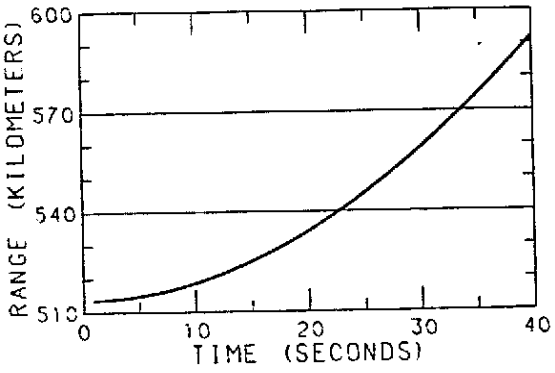

Fig. 1 - Range Data Curve

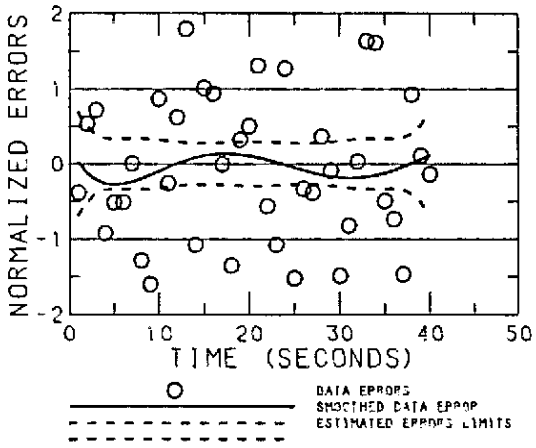

Fig. 2 - Batch Processing Fitting Procedure: Application Results.

Figure 3 shows the results obtained when the sequential procedure is applied to the same simulated data points. One can see by comparing this figure with Figure 2 that the sequential processing has an equivalent performance when compared to batch processing.

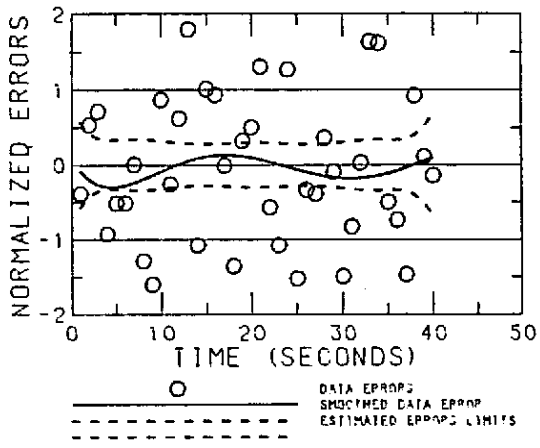

Fig. 3 - Sequential Processing Fitting Procedure: Application Results.

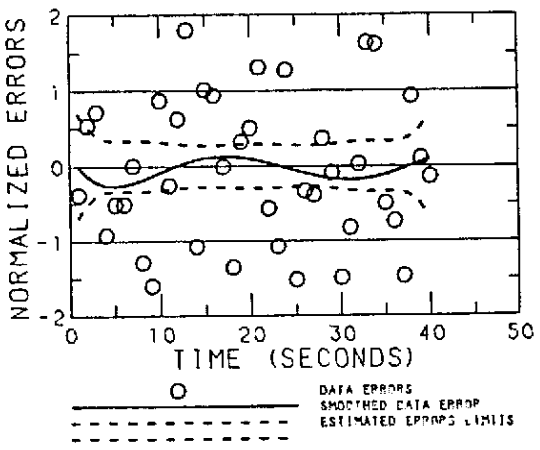

Fig. 4 - Least Square Procedure: Application Results.

Finally, Figure 4 shows the results of the application to the simulated data of the Least Square curve fitting procedure. Considering the statistical properties of the simulated data error (zero mean Gaussian noise), this method is in this case equivalent to the Chi-square procedure. One can observe by comparing Figure 4 with Figures 2 and 3 that the proposed procedure showed the same performance quality as that obtained with the Least Square procedure.

5 - CONCLUSIONS

The Kalman filtering application to the problem of data smoothing and compression by curve fitting has allowed the development of two smoothing technique versions: the batch and the sequential ones. The batch processing version requires that all the data set be collected before the processing is started. However, if the data random errors are uncorrelated, the data points can be processed one by one through Kalman Filter update step. This avoids the need of matrix inversion for Kalman gain calculation. In this fashion, both sequential and batch procedures can be applied without the need of matrix inversion. For the Least-Square and Chi-Square curve fitting procedures the matrix inversion operation is always necessary (Meyer, 1975).

In the sequential fitting procedure version there is another relevant advantage: the real time curve fitting possibility if the degree of the curve to be adjusted is known a priori.

In terms of performance the sequential and the batch processing versions showed in the tests a behavior equivalent to the Least Square procedure.

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APPENDIX

STATE NOISE ESTIMATION TECHNIQUE

To estimate the diagonal elements of $\underline{Q}(0)$ and $\underline{Q}(k)$, in both the batch and the sequential processing, one imposes consistency between the observations residues and their statistics (Jazwinski, 1969). To do so, the diagonal terms, which correspond to the variances of the state noise, are imposed to assume those values that maximize the probability of occurrence of the true residue of each observation (Rios Neto and Kuga, 1985). For the sake of understanding, the resulting procedure is reproduced in what follows.

Defining, for K = 1,2,...,m, the residue as (see equation (18))

$$r(k+1) = y(k+1) - \underline{m}(k+1)\underline{\tilde{x}}(k+1/k) \tag{A.1}$$

and the true residue as:

$$r^{v}(k+1) = r(k+1) - v(k+1) \tag{A.2}$$

then, under the hypothesis of the residue having normal distribution the criterion of statistical consistency is realized by imposing

$$[r^{v}(k+1)]_{o}^{2} = E\{[r^{v}(k+1)]^{2}\} \tag{A.3}$$

which is the condition to maximize the probability of occurrence of the value $[r^{v}(k+1)]_{o}$ of the random variable $r^{v}(k+1)$. Using equations (17)and(18) of section 3.2 and after some algebraic manipulations, it results:

$$[r^2(k+1) - 2r(k+1)v(k+1) + v^2(k+1)]_o = \underline{m}(k+1)\underline{P}(k/k)\underline{m}^T(k+1) +$$

$$+ \underline{m}(k+1)\underline{Q}(k)\underline{m}^T(k+1). \qquad (A.4)$$

It is thus reasonable to define the noise:

$$\eta(k+1) \triangleq - 2[r(k+1)]_c\, v(k+1) + v^2(k+1) - R(k+1)\ , \qquad (A.5)$$

$$E[\eta(k+1)] = 0,\ E[\eta^2(k+1)] = N(k+1) = 4[r^2(k+1)]_c R(k+1) + 2R^2(k+1), \qquad (A.6)$$

where $[r(k+1)]_c$ is the calculated value, corresponding to the occurred value of the random variable $r(k+1)$.

Having $\eta(k+1)$ defined and assuming a diagonal $Q(k)$ matrix whose elements $q(k+1)_i$ are components of the $\underline{q}(k+1)$ vector, the following model can be taken to represent the condition of equation (A.3):

$$z(k+1) = \underline{h}(k+1)\ \underline{q}(k+1) + \eta(k+1)\ , \qquad (A.7)$$

where

$$h(k+1) = [m_1^2(k+1) : m_2^2(k+1) : \ldots : m_n^2(k+1)]\ ,$$

$$z(k+1) = r^2(k+1) + R(k+1) - \underline{m}(k+1)\ \underline{P}(k/k)\ \underline{m}^T(k+1).$$

The vector $q(k+1)$ is then estimated using the following algorithm:

(i) prediction or characterization of a priori information for $k > 1$.

$$\hat{q}(k+1)/k) \;=\; \hat{q}(k/k)\ , \tag{A.8}$$

$$\underline{P}^{q}(k+1/K) = \underline{P}^{q}(k/k) + \text{diag}\ (\frac{1}{9}\ \hat{q}_j^2(k/k),\ j = 1,2,\ldots,n); \tag{A.9}$$

(ii) filtering

$$\hat{q}(k+1/k+1) = \underline{\hat{q}}(k+1/k) + K^{q}(k+1)[z(k+1) - \underline{h}(k+1)\underline{\hat{q}}(k+1/k)]\ , \tag{A.10}$$

$$\underline{P}^{q}(k+1/k+1) \;=\; [I - K^{q}(k+1)\underline{h}(k+1)]\ \underline{P}^{q}(k+1/k)\ , \tag{A.11}$$

$$K^{q}(k+1) = \underline{P}^{q}(k+1/k)\underline{h}^{T}(k+1)[\underline{h}(k+1)\underline{P}^{q}(k+1/k)\underline{h}^{T}(k+1) + N(k+1)]^{-1}. \tag{A.12}$$

The matrix $\underline{Q}_k$ is then approximately given by:

$$\underline{Q}_k \cong \text{diag}\ \{\underline{q}(k+1) = \underline{\hat{q}}\ (k+1/k+1) \geq 0\}\ .$$