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ALFVÉN INTERMITTENT TURBULENCE DRIVEN BY TEMPORAL CHAOS

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ABSTRACT

The temporal evolution of Alfvén intermittent turbulence in cosmic plasmas is studied. The chaotic dynamics of a driven-dissipative Alfvén system is determined by numerically solving the derivative nonlinear Schrödinger equation in the low-dimensional limit. Two types of Alfvén intermittent turbulence are identified: Pomeau-Manneville intermittency and crisis-induced intermittency. The significance of this theory for Alfvénic intermittent turbulence observed in the solar wind is discussed.

Subject headings: chaos — MHD — plasmas — solar wind — turbulence

1. INTRODUCTION

Alfvén waves are of fundamental importance in astrophysical and space plasmas (see Chian et al. 1995 for a collection of review papers). For example, Alfvén waves may be a major mechanism for the production of stellar winds and extragalactic jets, and they may contribute to the formation of quasar clouds (Jatenco-Pereira 1995). Pulsar microstructures can evolve from nonlinear modulation of Alfvén waves in the pulsar magnetosphere (Chian 1992). The dissipation of Alfvén and magnetohydrodynamic (MHD) turbulence might also be responsible for interstellar scintillation of radio sources (Spangler 1991).

In space plasmas, gas models of Alfvén solitons have been proposed to describe MHD turbulence in the solar wind (Ovenden, Shah, & Schwartz 1983; Ponce Dawson & Fontán 1990). Turbulent heating of the solar corona by nonlinear Alfvén waves has been studied through numerical simulations by Pettini, Nocera, & Vulpiani (1985). MHD parametric instabilities induced by a large-amplitude standing Alfvén waves in the planetary magnetosphere were investigated by Chian & Oliveira (1994, 1996) and Oliveira & Chian (1996). Alfvén waves can be nonlinearly generated by Langmuir waves in auroral plasmas and solar active regions (Chian, Lopes, & Alves 1994; Chian et al. 1997). Marsch & Tu (1997) showed that effective heating of the solar corona and acceleration of the solar wind are achieved by high-frequency Alfvén waves.

A proper understanding of the nonlinear dynamical evolution of Alfvén waves is essential for most studies of Alfvén waves in astrophysical and space plasmas. In particular, the question of how nonlinear Alfvén waves evolve into Alfvén turbulence must be addressed. Recently, significant progress in this subject has been achieved through theoretical analysis of chaos in Alfvén systems. Ghosh & Papadopoulos (1987) studied the onset of Alfvén turbulence via chaos by solving numerically a driven-dissipative derivative nonlinear Schrödinger equation using the spectral method. They found that the onset of turbulence occurs via two different routes, depending on the number of modes in the system: in the reduced (7 wave) system it occurs via an infinite series of period-doubling bifurcations to a stranger attractor, whereas in the complete (32 wave) system it occurs via the Ruelle-Taken route, involving the destruction of a two-dimensional toroidal surface. Hada et al. (1990) investigated the chaos in driven Hamiltonian (conservative) and dissipative Alfvén systems. By assuming stationary wave solutions, they reduced the problem to 3 degrees of freedom and showed that this simplified model is capable of reproducing the chaotic properties of the more complicated high-dimensional model of Ghosh & Papadopoulos (1987). Buti (1992, 1997) applied the formalism of Hada et al. (1990) to study Alfvén chaos in multispecies and dusty plasmas. He showed that heavier ions tend to reduce the chaos, and even a small fraction of dust grains can eliminate the chaos in Alfvén systems that are chaotic in the absence of dust particles. Oliveira, Rizzato, & Chian (1997) used the spectral method to solve a set of nonlinearly coupled MHD wave equations. Their results show that the degree of Alfvén chaos is a function of the wave amplitude, plasma β, and the dispersive parameter.

The aim of this paper is to apply the model of Hada et al. (1990) to investigate the phenomenon of Alfvén intermittent turbulence driven by temporal chaos. We point out, for the first time, that the onset of Alfvén turbulence can occur via two new routes to chaos: Pomeau-Manneville intermittency and crisis-induced intermittency. The nature of these two types of Alfvén chaos will be analyzed in detail by the Poincaré methods, and we will discuss the application of this theory to understanding the observation (Marsch & Liu 1993; Tu & Marsch 1995) of Alfvénic intermittent turbulence in the solar wind. In particular, the fractal characteristics of either type of Alfvén intermittency will be determined by calculating the fractal dimension of the respective strange attractor. It is expected that the new concept of Alfvén chaos treated in this paper will be beneficial to the study of MHD turbulence in the cosmos.

2. THEORY

The spatiotemporal dynamics of a nonlinear Alfvén wave propagating along an ambient magnetic field in the x-direction is governed by the derivative nonlinear Schrödinger equation (Mjølhus 1976; Ghosh & Papadopoulos 1987; Kennel et al. 1988; Hada et al. 1990; Ponce Dawson & Fontán 1990)

$$\partial_t b + \alpha \partial_x(|b|^2 b) - i(\mu + i\eta)\partial_x^2 b = S(b, x, t) , \quad (1)$$

where $b = b_y + ib_z$ is the complex transverse magnetic field normalized to the constant ambient magnetic field B_0, time t is normalized to w_{ci}^{-1} (where $w_{ci} = eB_0/m_i$ is the ion cyclotron frequency), space x is normalized to c_A/w_i (where $c_A = B_0/(\mu_0 \rho_0)^{1/2}$ is the Alfvén velocity), $\alpha = 1/[4(1 - \beta)]$, $\beta =$

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c_S^2/c_A^2 (where $c_S = (P_0/\gamma\rho_0)^{1/2}$ is the acoustic velocity), $\mu = 1/2$, and η is the dissipative scale length. The external driving force $S(b, x, t) = A \exp(ik\phi)$ is a monochromatic circularly polarized wave with a wave phase $\phi = x - Vt$, where V is a constant wave velocity. By defining k to be real, we consider the driver to be nongrowing and undamped. Note that equation (1) can be extended to include kinetic effects of resonant particles as well as to three spatial dimensions (Mjølhus & Wyller 1986, 1988; Spangler 1990).

Two different approaches can be adopted to analyze the nonlinear evolution of the derivative nonlinear Schrödinger equation (1). First, one can treat it as a system with high degrees of freedom and solve this partial differential equation numerically using the spectral method (Ghosh & Papadopoulos 1987; de Oliveira, Rizzato, & Chian 1995; Chian 1997; Oliveira et al. 1997). This approach provides information on the spatiotemporal dynamics of a high-dimensional system. Alternatively, one can reduce equation (1) to a system of ordinary differential equations and obtain information on the temporal dynamics of a low-dimensional system (Hada et al. 1990; Chian, Lopes, & Abalde 1996; Chian 1997; Rizzato, Lopes, & Chian 1997). In this paper, we adopt the second approach.

For stationary waves with $b = b(\phi)$, the first integral of equation (1) yields a set of ordinary differential equations (Hada et al. 1990):

$$\dot{b}_y - v\dot{b}_z = \frac{\partial H}{\partial b_z} + a \cos\theta \tag{2}$$

$$\dot{b}_z + v\dot{b}_y = -\frac{\partial H}{\partial b_y} + a \sin\theta \tag{3}$$

$$\dot{\theta} = \Omega\,, \tag{4}$$

with

$$H = \frac{(\boldsymbol{b}^2 - 1)^2}{4} - \frac{\lambda}{2}(\boldsymbol{b} - \hat{\boldsymbol{y}})^2\,, \tag{5}$$

where the overdot denotes a derivative with respect to the temporal variable $\tau = \alpha b_0^2\,\phi/\mu$, $b \to b/b_0$ (where b_0 is an integration constant), $\theta = \Omega\phi$, $\Omega = \mu k/(\alpha b_0^2)$, $a = A/(\alpha b_0^2 k)$, $v = \eta/\mu$, $\lambda = -1 + V/(\alpha b_0^2)$, and $\alpha > 0$ (i.e., $\beta < 1$) is assumed. In the absence of a driver ($a = 0$), the dimension of equations (2)–(5) reduces to 2, and all solutions are regular, representing periodic Alfvén waves, Alfvén solitons, and Alfvén shocks, respectively (Kennel et al. 1988; Hada et al. 1990). In the presence of a driver ($a \neq 0$), the dimension of equations (2)–(5) increases to 3, making possible chaotic solutions (Ott 1993; Hada et al. 1990).

A bifurcation diagram for nonlinear Alfvén waves can be constructed from equations (2)–(5) by varying the driver amplitude (a), while keeping other control parameters fixed ($\Omega = -1$, $v = 0.02$, and $\lambda = 1/4$). A left-hand driver is chosen. Figure 1 illustrates a small region of the numerically computed bifurcation diagram, which elucidates nonlinear dynamical features of the system. Two types of temporal Alfvén intermittency can be identified in Figure 1: Pomeau-Manneville intermittency (Manneville & Pomeau 1979) and crisis-induced intermittency (Grebogi & Ott 1983). A chaotic region terminates at $a \sim 0.32138$. At that point, the Pomeau-Manneville intermittency sets in, and the solutions become periodic, with a period of 3. Beyond a certain driver amplitude ($a \sim 0.32692$), a period-doubling cascade occurs.

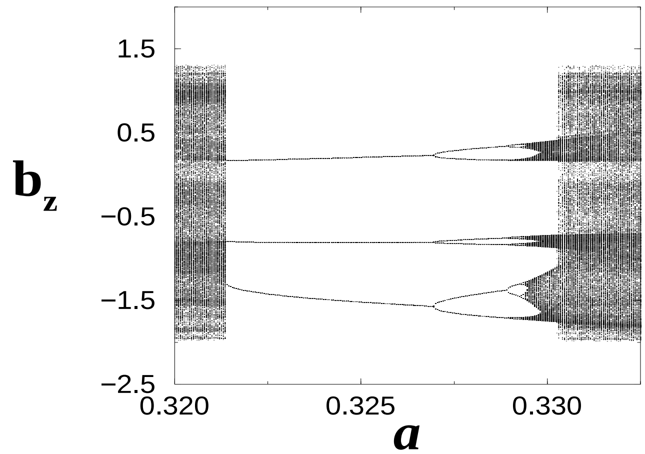

FIG. 1.—Bifurcation diagram $b_z(a)$ for $\Omega = -1$, $v = 0.02$, $\lambda = 1/4$, and $\alpha > 0$.

This process continues until band merging appears, which leads to the formation of a chaotic continuum at $a \sim 0.330249$ and the onset of the crisis-induced intermittency. The full bifurcation diagram contains many other regions of intermittency similar to that in Figure 1, indicating that Alfvén intermittent turbulence can readily appear in nature.

The Pomeau-Manneville intermittency is characterized by time series containing nearly periodic laminar phases that are randomly interrupted by chaotic (irregular) bursts (Manneville & Pomeau 1979; Ott 1993), as exemplified by Figure 2*a*. This example shows the transition from chaos to period 3 in Figure 1, and belongs to the intermittency of type I. Type I Pomeau-Manneville intermittency occurs when a dynamical system is close to a tangent (saddle-node) bifurcation arising from the coalescence of pairs of stable and unstable orbits. At the onset of the period 3 window ($a \sim 0.321382105$), the diagonal line $(b_z)_{n+3} = (b_z)_n$ is tangent to three points of the curve in the third-order return map. For values of the driver amplitude just below the onset of the period 3 window, the third-order return map is no longer tangent to the diagonal line, but remains close to it, resulting in laminar regions (no loss of correlations) that are interrupted by random turbulent bursts (loss of correlations). A better way to see these intermittent features is to plot the time series in terms of the driver cycles, as done in Figure 2*b*, corresponding to the same interval of the time series of Figure 2*a*. The power spectrum for the times series of Figures 2*a* and 2*b* is shown in Figure 2*c*. An example of strange attractor (Poincaré map) of the Pomeau-Manneville intermittency is shown in Figure 3.

The crisis-induced intermittency is characterized by time series containing weakly chaotic laminar phases that are randomly interrupted by strongly chaotic bursts (Grebogi & Ott 1983; Ott 1993), as depicted in Figure 4*a*. The corresponding time series plotted in terms of the driver cycle is given in Figure 4*b*. This example demonstrates how three separate narrow chaotic bands merge to form the single wide chaotic continuum in Figure 1, belonging to a type of chaotic transition called interior crisis. The interior crisis occurs when a dynamical system is near a crisis point ($a \sim 0.330249$), where the unstable periodic orbits collide with chaotic attractors. The power spectrum for the times series of Figures 4*a* and 4*b* is shown in Figure 4*c*. Figure 5*a*

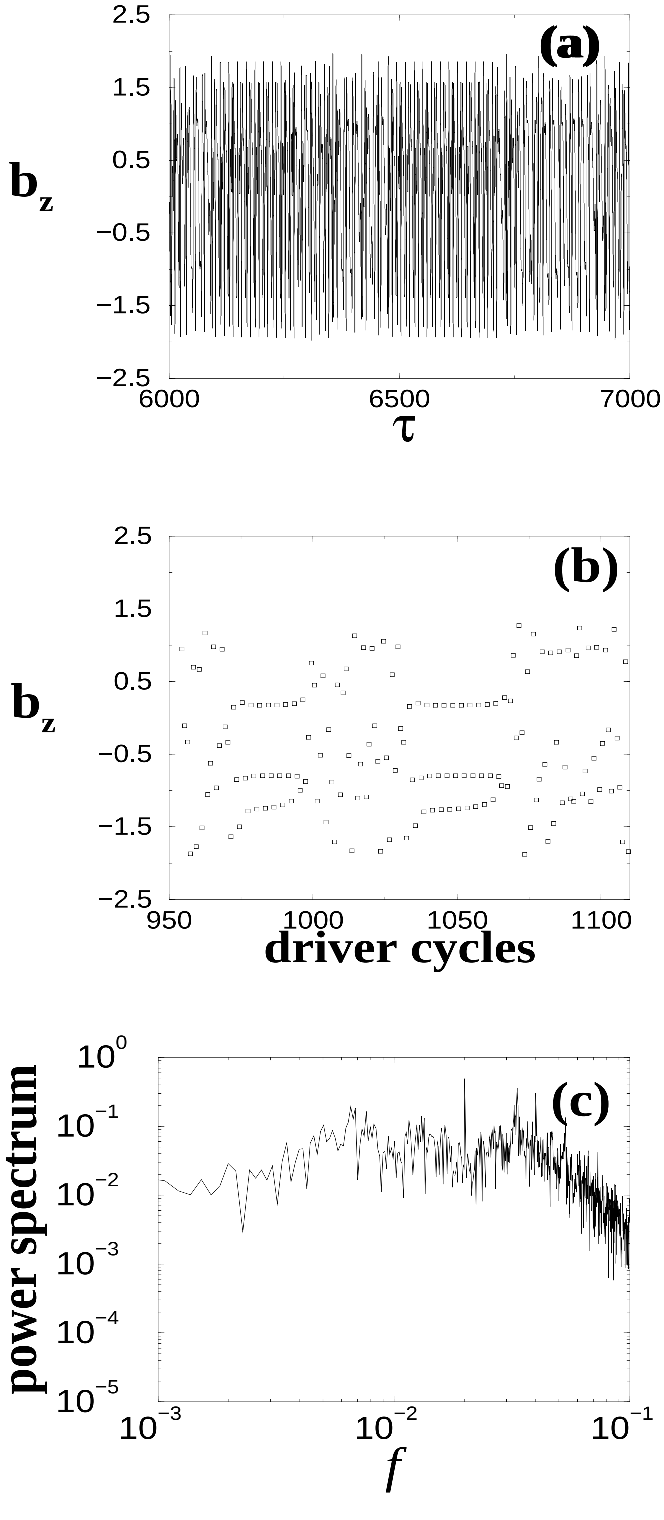

FIG. 2.—Example of Pomeau-Manneville intermittent turbulence for $a = 0.3213795$ for (*a*) $b_z(\tau)$, (*b*) variation of b_z with driver cycles, and (*c*) $|b_z|^2$ as a function of *f*.

shows an example of a weak strange attractor just below the crisis point; Figure 5*b* shows a strong strange attractor just beyond the crisis point. The three separate regions of the weak strange attractor, marked by the three arrows in Figure 5*a*, correspond to the three separate narrow chaotic bands of Figure 1 mentioned above. In Figure 5*c*, the two strange attractors of Figures 5*a* and 5*b* are superposed.

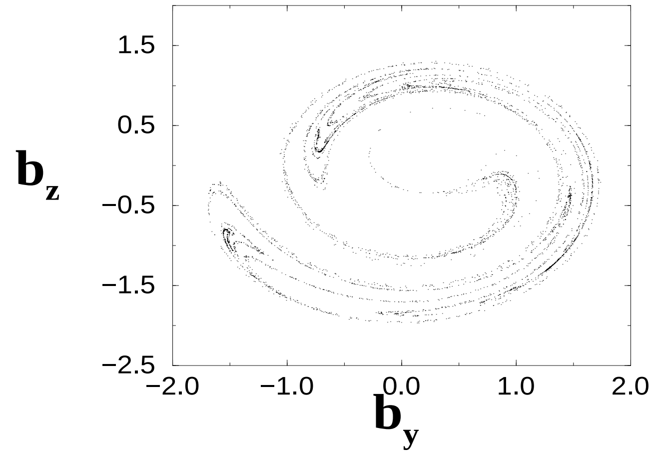

FIG. 3.—Example of a strange attractor of the Pomeau-Manneville intermittent turbulence for $a = 0.3213795$.

Note that for the sake of clarity, the attractor of Figure 5*a* has been artificially darkened in Figure 5*c*. It can be seen from Figures 4*b* and 5*c* that the crisis-induced intermittency consists of long intervals (laminar phases) of chaotic motion near the precrisis attracting loci, broken by brief excursion into extended regions (turbulent bursts) of the postcrisis attractor added by the crisis.

3. DISCUSSION

Interplanetary MHD fluctuations are found in two distinct states. In low-speed streams, the fluctuations are typically non-Alfvénic, showing features of standard turbulence in the sense that waves with a definite sense of propagation are hard to identify and the fluctuation spectra obey a Kolmogorov power law (Tu & Marsch 1995; Velli & Pruneti 1997). On the other hand, in high-speed streams originating from coronal holes, the fluctuations are dominated by large-amplitude Alfvén waves propagating freely away from the Sun. The outward-propagating nature of interplanetary Alfvénic waves was first observed by Coleman (1968) and Belcher & Davies (1971), and subsequently confirmed by the *Helios*, *Pioneer*, and *Voyager* spacecraft in a vast range of radial distances in or near the ecliptic plane (Tu & Marsch 1995). Recent *Ulysses* observations of Alfvénic waves in the polar regions provided further evidence that these fluctuations are outward-propagating waves in both solar hemispheres (Smith et al. 1995; Velli & Pruneti 1997). Since the Alfvénic fluctuations in the solar wind are propagating waves, the derivative nonlinear Schrödinger equation (1) is an appropriate model for describing the spatiotemporal dynamics of these nonlinear waves.

There is some observational evidence of chaos and nonlinear dynamical phenomena in both Alfvénic and non-Alfvénic fluctuations in the solar wind. Turning first to the non-Alfvénic chaos (Burlaga 1995), the formation of ordered large structures from irregular smaller structures was detected in the interplanetary magnetic field data of *Voyager 1* and *2* between 1 AU and 9.5 AU, which provided the first example of "order out of chaos" in the outer heliosphere. Combined data of *IMP 8* at 1 AU and *Voyager 2* at 15 AU indicated period doubling (from ∼13.4 days to ∼25 days) of the period of the corotating interaction regions. Simultaneous data from *ISEE 3* at 1 AU and *Voyager 1* in the outer heliosphere measured period doubling (from 6.5

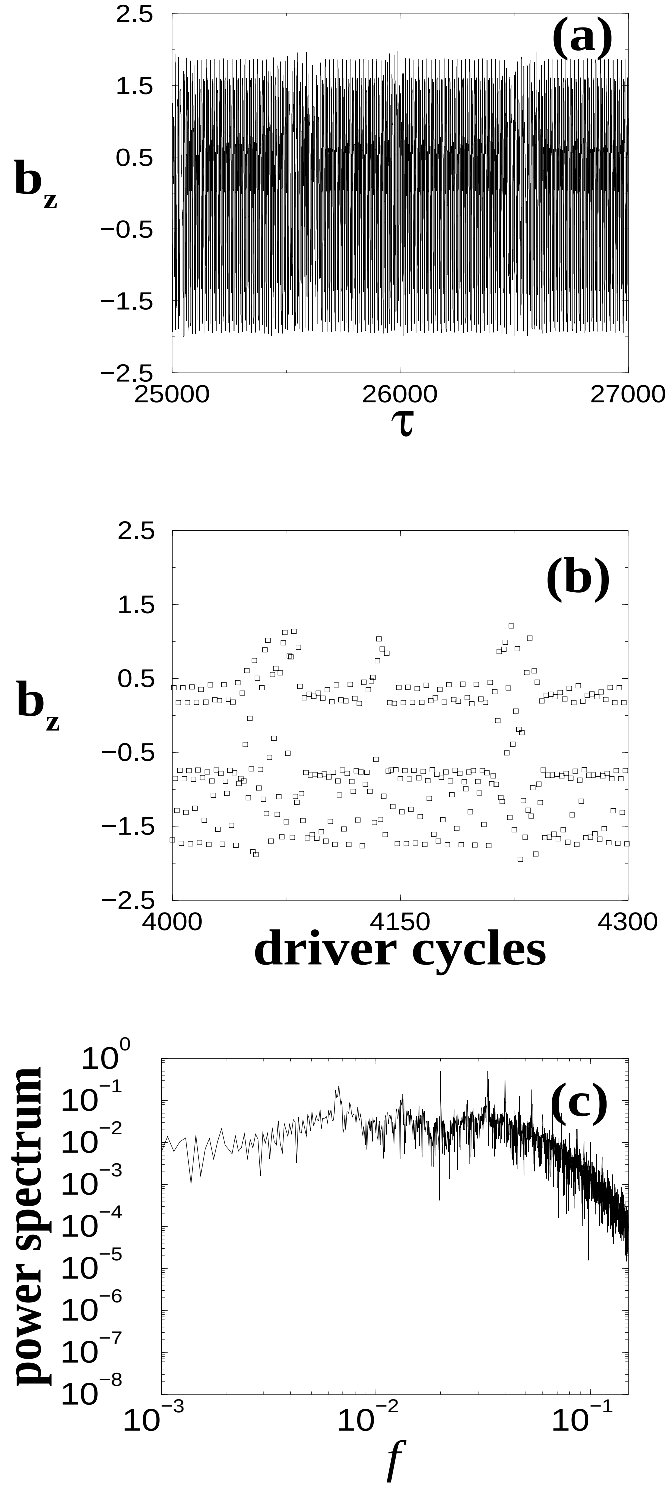

FIG. 4.—Example of the crisis-induced intermittent turbulence for $a = 0.33029$ for (*a*) $b_z(\tau)$, (*b*) variation of b_z with driver cycles, and (*c*) $|b_z|^2$ as a function of f.

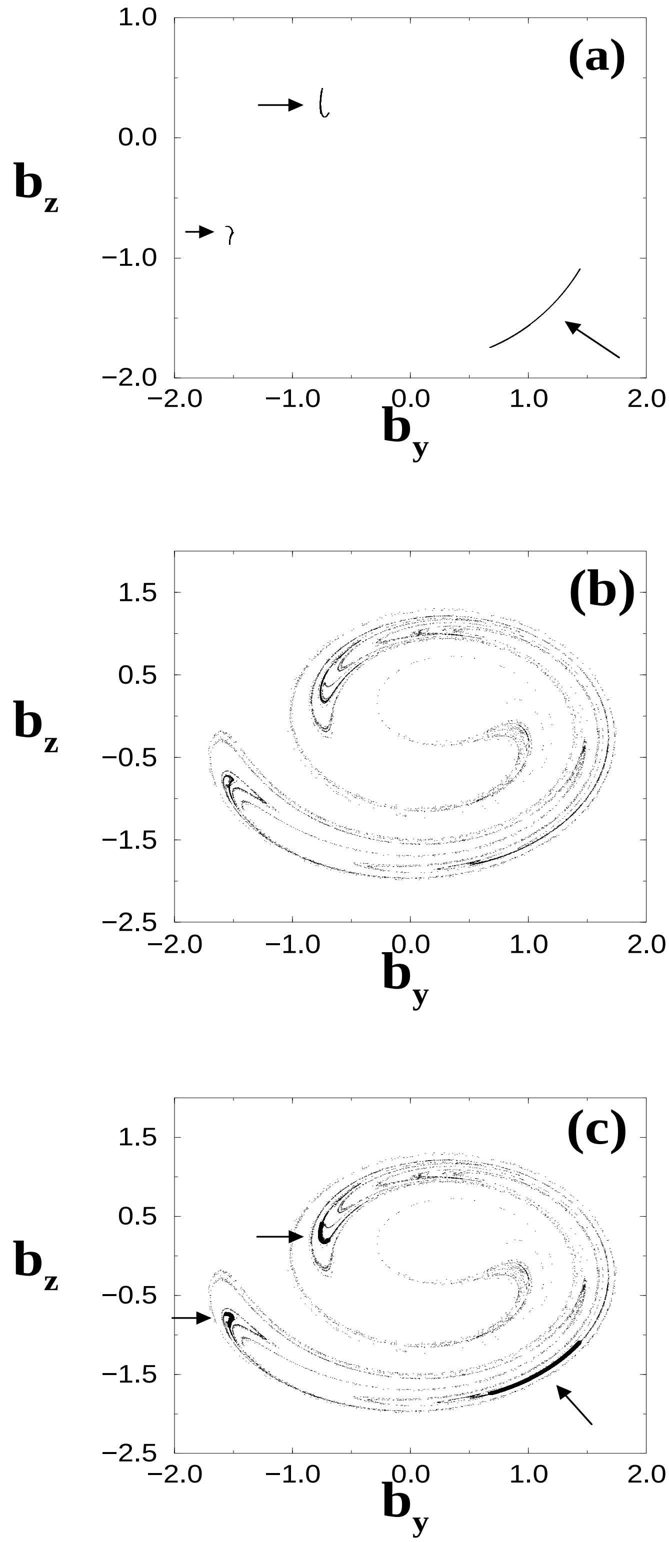

FIG. 5.—Examples of strange attractors of the crisis-induced intermittent turbulence for (*a*) $a = 0.33022$ (precrisis), (*b*) $a = 0.332$ (postcrisis), and (*c*) a superposition of (*a*) and (*b*).

days to 13 and 26 days) of the period of large-scale interplanetary structures. Pavlos et al. (1992) used the magnetic field data of *IMP 8* to present evidence for strange attractor structures in the solar wind, and found a fractal dimension of 4.5. Macek & Obojska (1997) applied the fractal analysis to the *Helios 1* data at 0.3 AU to show that the radial velocity fluctuations in the solar slow-speed stream have chaotic temporal behavior, with a fractal dimension of about 3.5. The aforementioned observations of non-Alfvénic chaos in the solar wind should serve as motivation to search for evidence of chaos in the interplanetary Alfvénic fluctuations.

Alfvénic turbulence, consisting of microscale fluctuations with periods ranging from fractions of minutes to several hours, and showing a high correlation between solar wind velocity and solar wind magnetic field fluctuations, is frequently observed in the solar wind. For example, Bavassano & Bruno (1989) presented evidence of the local generation of Alfvénic turbulence at the stream shear regions of the solar wind. Grappin, Velli, & Mangeney (1991) detected the coexistence of Alfvénic turbulence and standard weakly compressive MHD turbulence in the fast hot streams of the inner heliosphere. Tsurutani et al. (1994) used the *Ulysses* data to show that high-speed streams emanating from solar coronal holes at high heliographic latitudes are dominated by nonlinear Alfvén waves. Large-amplitude Alfvén waves were measured in the corotating interaction regions formed by high-speed corotating streams interacting with slow-speed streams at midlatitudes of the heliosphere (Tsurutani et al. 1995). Nonlinear interplanetary Alfvén-wave trains may cause geomagnetic storms, leading to high-intensity, long-duration, continuous auroral activity (HILDCAA) (Gonzalez, Gonzalez, & Tsurutani 1995).

Observations of Alfvénic intermittent turbulence in the solar wind were reported by Marsch & Liu (1993) and Tu & Marsch (1995). Using the *Helios 2* data in the inner solar wind between 0.3 and 1.0 AU, they identified the multifractal nature of interplanetary Alfvénic fluctuations and the dependence of Alfvénic intermittent turbulence on stream speed and radial distance from the Sun. It is worth mentioning that the multifractal character of small-scale (0.85–13.6 hr) velocity fluctuations, related to the presence of MHD (non-Alfvénic) intermittent turbulence, were seen in the solar wind by Burlaga (1991). Moreover, MHD intermittent turbulence and multifractal structures are common features of large-scale (non-Alfvénic) fluctuations, with periods varying from several hours to the solar rotation period throughout the interplanetary medium (Burlaga 1995). The direct link between the temporal intermittent turbulence and the fully developed spatiotemporal intermittent turbulence remains an open question (Paladin & Vulpiani 1987; Marsch & Liu 1993). Nonetheless, the temporal model of Alfvén intermittent turbulence formulated in this paper is capable of exhibiting the fractal nature of the Alfvénic intermittency. In Figures 3 and 5, we show that the temporal Alfvén intermittency is characterized by strange attractors. Continuous blow-up of a tiny portion of these Poincaré maps shows that these strange attractors have self-similar structures on arbitrarily small scales. This scale invariance is the fundamental property of fractal objects (Ott 1993). The fractal dimension (Russell, Hanson, & Ott 1980) of the strange attractor of the Pomeau-Manneville intermittency of Figure 3 is found to be 2.44, while those of the weak and strong strange attractors of the crisis-induced intermittency in Figures 5*a* and 5*b* are 2.176 and 2.513, respectively. The power spectra (Figs. 2*c* and 4*c*) of chaos-driven Alfvén intermittencies present features similar to the power spectra of MHD turbulence in the solar wind (Burlaga 1995).

4. CONCLUSION

There is abundant observational evidence of chaotic phenomena in laboratory experiments. For example, experimental observations of chaotic behavior and period doubling have been performed on a laboratory plasma discharge (Cheung & Wong 1987). In addition, in a plasma laboratory experiment of drift waves in a triple-plasma device, it has been shown that as the control parameter is increased, the transition from a stable state to chaos and intermittent turbulence occurs via successive Hopf bifurcations (Klinger et al. 1997). The two chaotic phenomena studied in this paper, Pomeau-Manneville intermittency and crisis-induced intermittency, have been observed in numerous laboratory experiments. For example, the transition from half-harmonic oscillation to chaos via the type I Pomeau-Manneville intermittency has been observed in a biological experiment of pacemaker neurons (Hayashi, Ishizuka, & Hirakawa 1983); the type I Pomeau-Manneville intermittence between chaos and period 3 attractors has been observed in a leaky-faucet experiment (Sartorelli, Gonçalves, & Pinto 1994). Intermittency induced by interior crisis has been observed in a CO_2 laser experiment (Dangoisse, Glorieux, & Hennequin 1986) and in a mechanical experiment of magnetoelastic ribbon (Ditto et al. 1989). The observation of chaos and intermittency in plasma and laboratory experiments supports the validity of modeling chaos-driven Alfvén intermittency by the derivative nonlinear Schrödinger equation (1).

It is worth pointing out that chaotic behavior appears not only in the highly reduced equations, but also readily in the primitive equations. It has been shown by various works that chaotic solutions can be found in primitive plasma equations, such as the conservative (Hamiltonian) Zakharov equations, in the absence of driving and dissipation (de Oliveira et al. 1995; Chian et al. 1996; Chian 1997; Rizzato et al. 1997). In fact, Oliveira et al. (1997) showed that Alfvén chaos appears in the numerical solutions of primitive MHD equations that are generalizations of the derivative nonlinear Schrödinger equation. It is interesting to note that the simplified low-dimensional system adopted by the present paper retains the intrinsic properties of the Alfvén chaos contained in the high-dimensional system studied by Ghosh & Papadopoulos (1987) and Oliveira et al. (1997). The choice of a steady state (neither growing nor damped) sinusoidal circularly polarized driver in equation (1) is a good representation of an undepleted large-amplitude Alfvén driver, which is a steady state solution of the undriven derivative nonlinear Schrödinger equation. The results from the simplified model of temporal Alfvén intermittency provide a good basis for future investigations of spatiotemporal Alfvén intermittency.

In summary, we demonstrated that Alfvén intermittent turbulence driven by temporal chaos can evolve via two distinct routes; Pomeau-Manneville intermittency and crisis-induced intermittency. Both types of chaotic transitions involve an episodic switching between different states of temporal behavior. In the Pomeau-Manneville route, the time series of magnetic fluctuations switches intermittently between nearly periodic and chaotic states. In the crisis-induced route, the time series of magnetic fluctuations switches intermittently between weakly chaotic and strongly chaotic states. These two types of intermittency, along with quasi-periodicity and period doubling to chaos, are intrinsic nonlinear dynamical features of Alfvénic turbulence in astrophysical and space plasmas.

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