Impulsive Transfers to/from the Lagrangian Points in the Earth-Sun System

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The well-known Lagrangian points that appear in the planar restricted three-body problem are very important points for astronautical applications. They are five points of equilibrium in the equations of motion, what means that a particle located at one of those points with zero velocity will remain there indefinitely. The collinear points (L_1, L_2 and L_3) are always unstable and the triangular points (L_4 and L_5) are stable in the case studied in this paper (Earth-Sun system). They are all very good points to locate a space-station, since they require a small amount of ΔV (and fuel) for station-keeping. The triangular points are specially good for this purpose, since they are stable equilibrium points. In this paper, the planar restricted three-body problem is regularized (using Lamaître regularization) and combined with numeric integration and gradient methods to solve the two point boundary value problem (the Lambert's three-body problem), that can be formulated as:

"Find an orbit (in the three-body problem context) that makes a spacecraft to leave a given point A and goes to another given point B, arriving there after a specified time of flight". Then, by varying the specified time of flight it is possible to find a whole family of transfer orbits and study them in terms of the ΔV required, energy, initial flight path angle, etc. To solve this problem the following steps are used: i) Guess a initial velocity $\vec{V}_i$, so together with the initial prescribed position $\vec{r}_i$ the complete initial state is known; ii) Guess a final regularized time τ_f and integrate the regularized equations of motion from $\tau_0 = 0$ until τ_f; iii) Check the final position $\vec{r}_f$ obtained from the numerical integration with the prescribed final position and the final real time with the specified time of flight. If there is an agreement (difference less than a specified error allowed) the solution is found and the process can stop here. If there is no agreement, an increment in the initial guessed velocity $\vec{V}_i$ and in the guessed final regularized time is made and the process goes back to step i). The method used to find the increment in the guessed variables is the standard gradient method.

This combination is applied to the search of families of transfer orbits between the Lagrangian points and the two primaries of the Earth-Sun system, with the minimum possible energy. The regularization is required to avoid problems during the numeric integration close to a singularity. This paper is related with the previous papers: Broucke[1], that studied transfer orbits between the Lagrangian points and the Moon; and Prado[2], that studied transfer orbits between the Lagrangian points and the Earth in the Earth-Moon system.

REFERENCES

[1] Broucke, R., "Traveling Between the Lagrange Points and the Moon", *Journal of Guidance and Control*, Vol. 2, N° 4, July-Aug. 1979, pp. 257-263.

[2] Prado, A.F.B.A., "Traveling Between the Lagrangian Points and the Earth". Acta Astronautica, Vol. 39, No. 7 (October/96), pp. 483-486.